Tutorial-1

1. Sample space(s) of tossing of a coin
2. Example of Multiplication Rule
3. Bayes' Theorem- discussion
4. Examples of Bayes' Theorem

E1. Three cards are drawn from an ordinary 52-card deck without replacement (drawn cards are not placed back in the deck). Find the probability that none of the three cards is a heart.

E2. Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1. Linda is a bank teller.
2. Linda is a bank teller and is active in the feminist movement.

Example on Bayes' Formula

E3. A laboratory blood test is 99 percent effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1 percent of the healthy persons tested. (That is, if a healthy person is tested, then, with probability .01, the test result will imply he or she has the disease.) If 0.5 percent of the population actually has the disease, what is the probability a person has the disease given that his test result is positive?

A new survey revealed only 0.1% of the population those are tested gives false positive. Now what is the probability a person has the disease given that his test result is positive?

E4. Suppose a 60-year-old man who has never smoked cigarettes presents to a physician with symptoms of a chronic cough and occasional breathlessness. The physician becomes concerned and orders the patient admitted to the hospital for a lung biopsy. Suppose the results of the lung biopsy are consistent either with lung cancer or with sarcoidosis, a fairly common, nonfatal lung disease. A survey tells that in a given population, 0.1% cases are false positives. Also the survey tells that 90% times the biopsy gives positive report among the patients having lung cancer, and the biopsy reveals sarcoidosis in 90% of the patients of having the disease.

The prevalences of lung cancer and sarcoidosis are 0.001 and 0.009 respectively.

What are the probabilities for

i) the person is not having either of the diseases given the symptom is present/a positive test result has been obtained.

ii) the person is having lung cancer given the symptom is present. iii) the person is having sarcoidosis given he has the symptom.

Tut-1

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Sample Space of a single coin

$S = \{H, T\}$

$\Omega = \{H, T, \phi, (H, T)\} \longrightarrow$ Power set of S 2^S

$P(S) = 1$

$S = \{HT, TH, HH, TT\}$

$\Omega = \{HT, TH, HH, TT, (HH, H), 5$ more $, (HH, TH, HT), 3$ more $, \phi, S\}$

Tut-2

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$$P(HT) = \frac{1}{4}$$

2. A_i = i^{th} card is not a heart

$$P(A_1) = \frac{39}{52}$$

$$P(A_2 | A_1) = \frac{38}{51}$$

$$P(A_3 | A_1 \cap A_2) = \frac{37}{50}$$

$$P(A_1 \cap A_2 \cap A_3) = P(A_1) \times P(A_2 | A_1) \times P(A_3 | A_1 \cap A_2)$$

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$A \wedge B$

Linda problem

$A \rightarrow$ bank teller. $P(A) = \cdot 02$

$B \rightarrow$ Activist $P(B) = \cdot 98$

$P(A \cap B) = \cdot 02 \times \cdot 98$ $\leftarrow P(A)/P(B)$

$P(A) > P(A \wedge B)$

4 .

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D → Disease

T → Positive

$P(T|D) = .99$

$P(D) = 0.005$

$P(\bar{T}|\bar{D})$

$$PV^+ = P(D|T) = \frac{P(T|D) \times P(D)}{P(T|D) \times P(D) + P(T|\bar{D})\,P(\bar{D})}$$

$, P(\bar{D}) = 1 - P(D) = 1 - .005 = .995$

$P(T|\bar{D}) = 1 - P(T|D)$

$= 0.01$

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T : Person has the symptom

D_1 : Not having disease

D_2 : having lung cancer

D_3 : " sarcoidosis

$P(D_2) = 0.001$; $P(D_3) = 0.009$

$P(D_1) = 0.99$

i) $P(D_1 | T)$

$P(T/D_2) = 0.9$ $P(T/D_1)$

$P(T/D_3) = 0.9$ $= 0.001$

$$= \frac{P(T|D_1) \cdot P(D_1)}{P(T|D_1)P(D_1) + P(T/D_2)P(D_2) + P(T/D_3)P(D_3)}$$

$= 0.00099 / .0099 = .099$