More on Binomial RV

- Tossing of a coin three times. Let **p** be the probability of obtaining a head. Then what is the probability of the outcome- **HTH?**

- **Ans.** **$p(1-p)p$**

- What is the probability of obtaining 2 heads?

- **Ans.** **${}^{3}C_{2}\ p^{2}(1-p)$; $X \sim Bin(3,p)$**

- What is the probability of having at least two heads?

- **Ans.** **$1-P(X=0)-P(X=1)$**

Poisson random variable

- continuous-time analog of the Bernoulli process
- Number of message received in a mobile phone in a given interval of time.

An arrival process is called a Poisson process with rate λ if it has the following properties:

(1) (**Time-homogeneity**) The probability of arrivals is the same for all intervals of the same length δt.

(2) (**Independence**) The number of arrivals during a particular interval is independent of the number of arrivals in the intervals preceded that.

(3) The probability of the single occurrence of the event in a given interval is proportional to the length of the interval.

The PMF:

$$P(\lambda) = e^{-\lambda}\frac{\lambda^x}{x!};\ x = 0,1,2,\cdots$$

Mean: λ and Variance: λ

X~Po(λ)

Example of Poisson process:

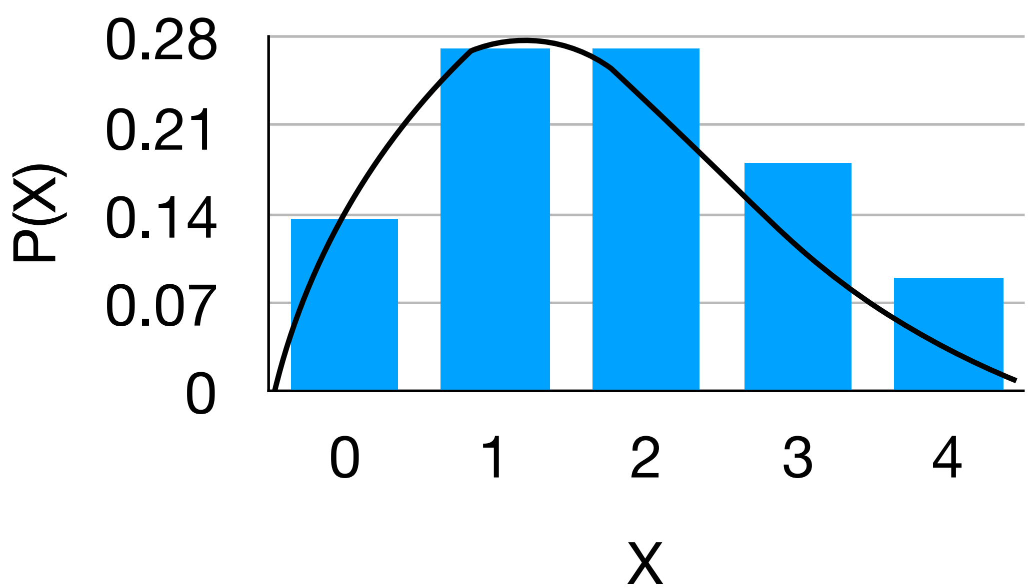
- In a study of drug-induced anaphylaxis among patients taking rocuronium bromide as part of their anesthesia, Laake and Røttingen found that the occurrence of anaphylaxis followed a Poisson model with $\lambda = 12$ incidents per year in Norway. Find the probability that in the next year, among patients receiving rocuronium, exactly three will experience anaphylaxis.

- If X denotes the number of patients who will experience anaphylaxis in a year

$$P(X=3)=e^{-12}\frac{12^3}{3!} = .00177$$

- For a Poisson RV with $\lambda = 2$

P(X=0)=0.1353, P(X=1)=0.2706, P(X=2)=0.2706, P(X=3)=0.1804, P(X=4)=0.0902



Poisson distribution has a long tail

$$P(X = \lambda) = P(X = \lambda - 1)$$

Mean, Mode, Median

- Measures of central tendency

- Mean is synonymous as the Expectation value

- Mode: Outcome(s) with highest probability/frequency

- Poisson RV has two modal values for integer λ

- Median: The outcomes are arranged in increasing order of their frequency/probability. Then the probability of the middle value (for odd number of outcomes) is called the median. For even number of outcomes there is no middle value. Then, if there are **n** outcomes(data points), we calculate the average of **n/2** th and **(n+2)/2**th values.

Poisson PMF is a good approximation to the binomial as long as λ = np, n is very large, and p is very small.

$$
\begin{aligned}
P(X = k) &= \frac{n!}{k!(n-k)!} p^k (1-p)^k \\
&= \frac{n(n-1)\cdots(n-k+1)}{n^k} \frac{\lambda^k}{k!} \frac{(1-\lambda/n)^n}{(1-\lambda)/n)^k} \\
&\simeq e^{-\lambda} \frac{\lambda^k}{k!}, \; \lambda = np; \; n \to \infty, \; p \to 0
\end{aligned}
$$