Screening Tests and Bayes' Theorem

- Clinical tests are important part of Health Science
- For proper Diagnosis efficient clinical tests are almost indispensable now a days.

Few standard notions related to screening tests:

False Positive: When a test indicates a positive status when the true status is negative.

False Negative: when a test indicates a negative status when the true status is positive.

Definitions

- **Sensitivity:** The sensitivity of a test is the probability of a positive test result (or presence of the symptom) given the presence of the disease.

- **Specificity:** The specificity of a test (or symptom) is the probability of a negative test result (or absence of the symptom) given the absence of the disease.

- **Predictive value positive:** The predictive value positive of a screening test (or symptom) is the probability that a subject has the disease given that the subject has a positive screening test result (or has the symptom).

- **Predictive value negative:** The predictive value negative of a screening test (or symptom) is the probability that a subject doesn't have the disease given that the subject has a negative screening test result.

Screening test results	Truly diseases (cases)	Truly non-diseases	Totals
Positive	a	b	a+b
Negative	c	d	c+d
totals	a+c	b +d	a+b+c+d

Sensitivity = a/(a+c)

Specificity = d/(b+d)

PV+ = a/(a+b)

PV- = d/(c+d)

Definitions

Report from an agency suggests: three weeks after onset of symptoms 1000 people had antibody tests. 50 of them really had COVID-19.

58 people would test positive for COVID-19. Of these, 12 people would not have COVID-19. So 12 are false positives.

942 people would test negative for COVID-19. Of course, 4 people (0.4%) would actually have COVID-19.

D： Person has disease

T：Person is tested positive

Sensitivity:

$$P(T \mid D) = \frac{46}{50} = .92$$

Specificity:

$$P(\bar{T} \mid \bar{D}) = \frac{938}{950} = .987$$

Predictive value positive:

$$P(D \mid T) = \frac{P(T \mid D)P(D)}{P(T \mid D)P(D) + P(T \mid \bar{D})P(\bar{D})} = \frac{.92 \times .05}{.92 \times .05 + .0126 \times .95} = .793\,; \quad P(T \mid \bar{D}) = 12/950$$

Predictive value negative:

$$P(\bar{D} \mid \bar{T}) = \frac{P(\bar{T} \mid \bar{D})P(\bar{D})}{P(\bar{T} \mid \bar{D})P(\bar{D}) + P(\bar{T} \mid D)P(D)} = .996;\ P(\bar{T} \mid D) = 4/50$$

Screening test results	Disease (yes)	Disease (No)	Total
Positive	(a) 46	(b) 12	(a+b) 58
Negative	(c) 4	(d) 938	(c+d) 942
Total	50 (a+c)	950 (b+d)	(a+b+c+d) 1000
Sensitivity=46/50	Specificity=938/950	PV^{+}= 46/58	PV^{-}=938/942