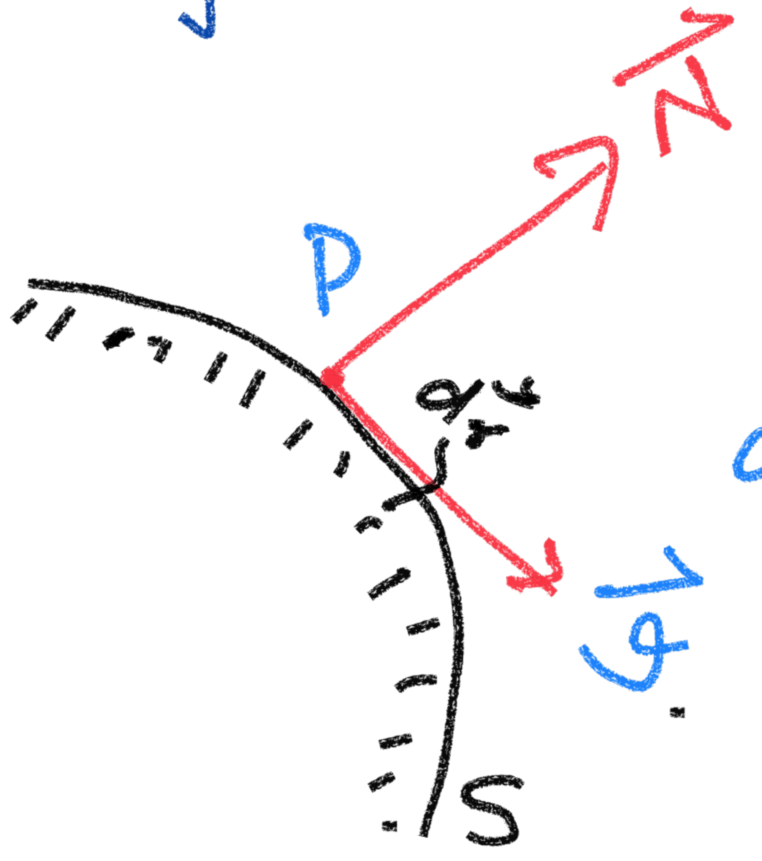
Examples of Virtual Displacement :-

Ex-1



$d\vec{r} = \vec{v}\, dt$

$d'\vec{r} = \vec{v}\,' dt$

A particle is in motion on a fixed surface S.

A possible displacement is obtained by considering the tangent vector at P. $d\vec{r} = \vec{v}\, dt$

$\vec{v}$ → a possible velocity

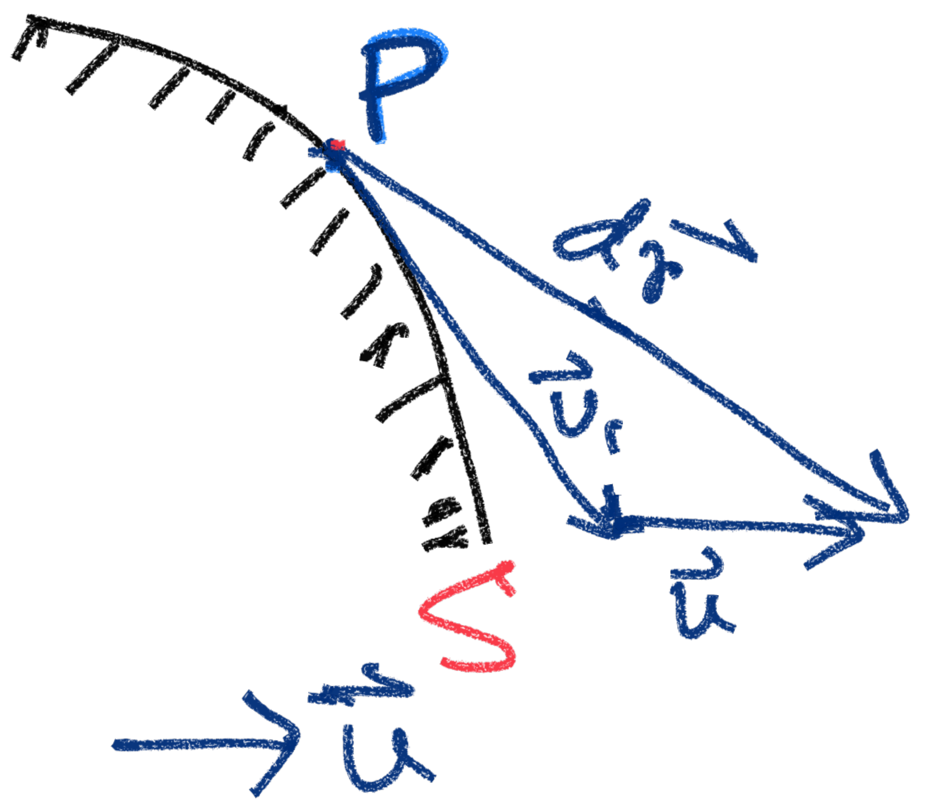
$d\vec{r}$ and $\vec{v}$ lie in the tangent plane at P.

* Let $d\vec{r}\,'$ is another possible displacement at P. The virtual displacement $\delta\vec{r} = d\vec{r}\,' - d\vec{r}$ lies at the same tangent plane. The virtual displacement coincides with possible displacement.

$\vec{N}\cdot d\vec{r} = \vec{N}\cdot\delta\vec{r} = 0$

Ex-2

A particle is in motion on a moving surface



S is moving with velocity $\vec{u}$ w.r.t. an origin (fixed).

Possible velocity is obtained by adding a tangent vector (arbitrary) $\vec{v}_1$ at P with $\vec{u}$.

$$\vec{v} = \vec{v}_1 + \vec{u}$$

$$\therefore \quad \vec{v}\,dt = d\vec{r} = \vec{v}_1\,dt + \vec{u}\,dt$$

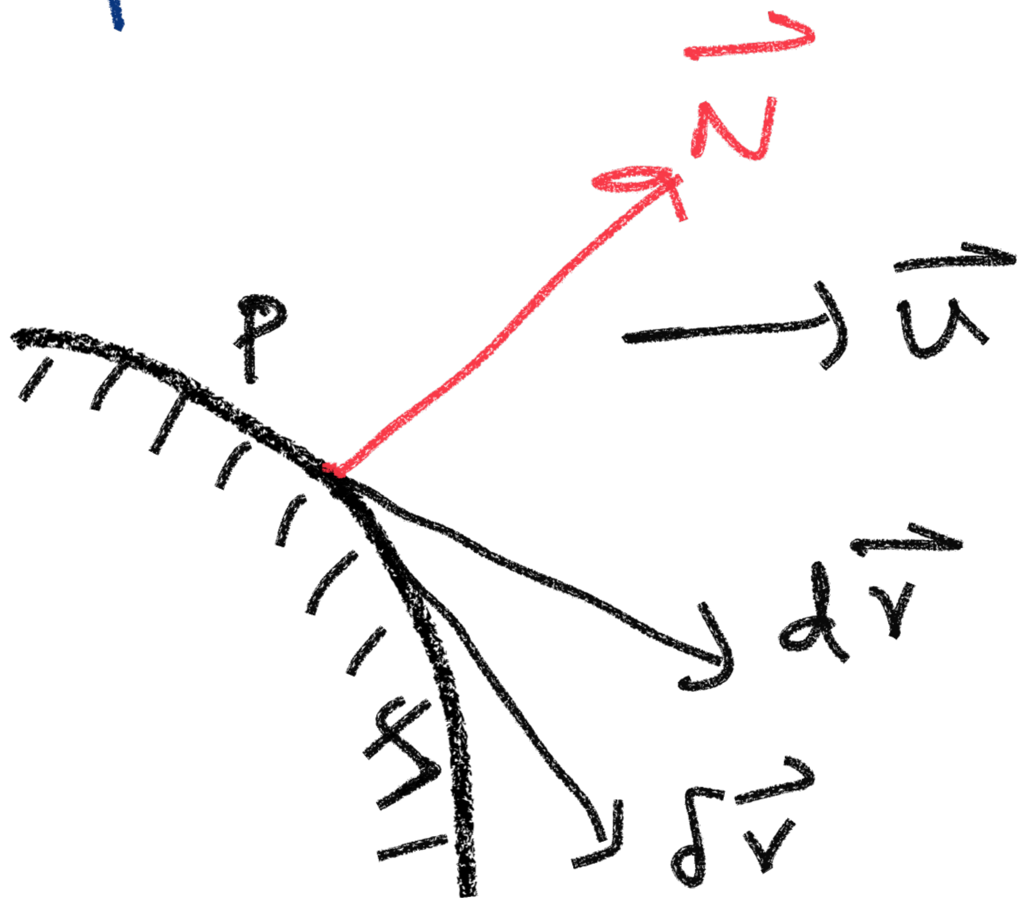
Consider another displacement at P at same instant $d'\vec{r}$

$$d'\vec{r} = \vec{v}_1{}'\,dt + \vec{u}\,dt$$

$$\delta\vec{r} = d'\vec{r} - d\vec{r}$$
$$= (\vec{v}_1{}' - \vec{v}_1)\,dt$$

$\delta\vec{r}$ is lying at the same tangent plane as the $\vec{v}_1$ or $\vec{v}_1'$ velocity vector lies.

but $d\vec{r}$ and $\delta\vec{r}$ are not coplanar this time.



But the normal force (force of constraint) is $\perp^r$ to $\delta\vec{r}$.

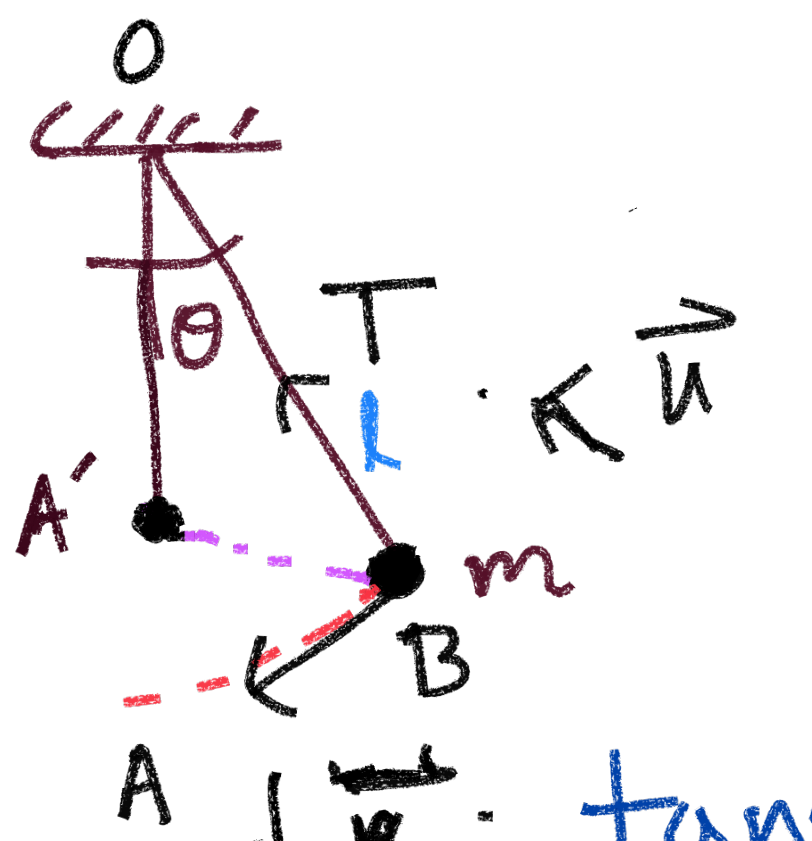
So, $\vec{N} \cdot \delta\vec{r} = 0$

Although $\vec{N} \cdot d\vec{r} \neq 0$

Why virtual displacement is needed?

Constraint forces do not work w.r.t. a virtual displacement.

Recall a simple pendulum



$d\vec{r}$: tangential to the circular arc AB,

T is force of constraint

$$\vec{T}\cdot d\vec{r} = 0 = \vec{T}\cdot\vec{v}dt$$

$$\vec{T}\cdot\delta\vec{r} = 0$$

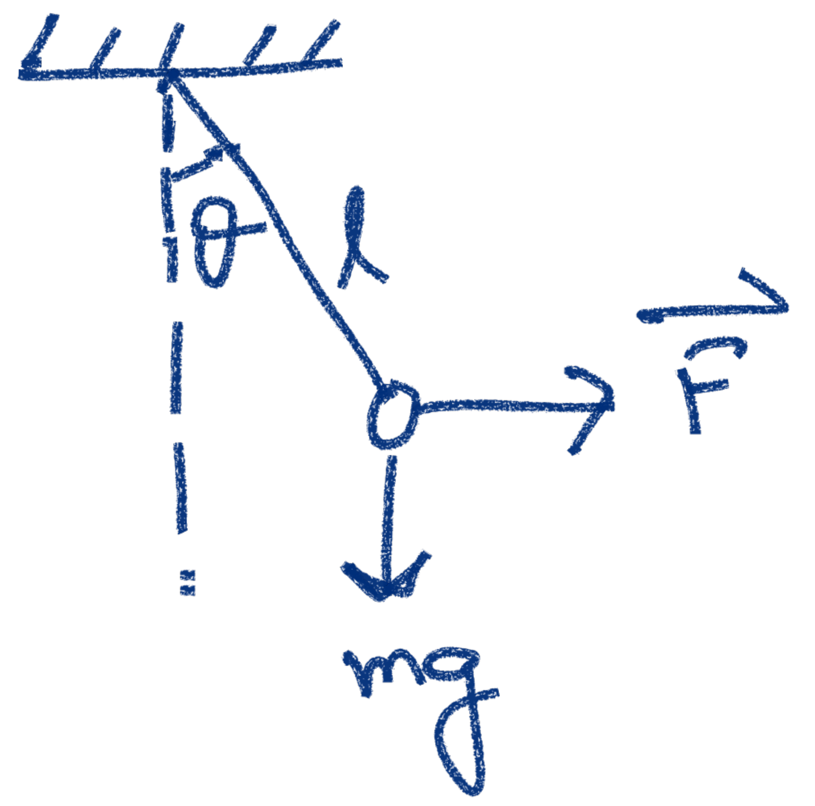
Now suppose the string of the pendulum is changing the length w.r.t. time. (see the pink arc) $l = l(t)$

This time the bob of the pendulum executes an arc A'B. Obviously now $\vec{T}\cdot d\vec{r} \neq 0$

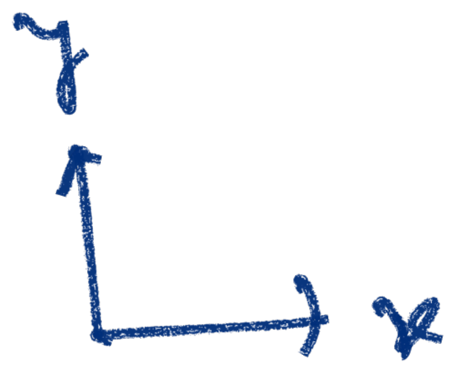
By a similar arrangement as before we can make $\vec{T}\cdot\delta\vec{r} = 0$

Problems on Principal of Virtual Work :-

(1)



Consider the pendulum. A force $\vec{F}$ is acting on the mass horizontally. What is the condition for equilibrium?



Let us consider a virtual displacement $\delta\theta$. Total virtual works:-

$x = l \sin\theta$

$y = l \cos\theta$

$$F\,\delta x = F l \cos\theta\,\delta\theta \quad (1)$$

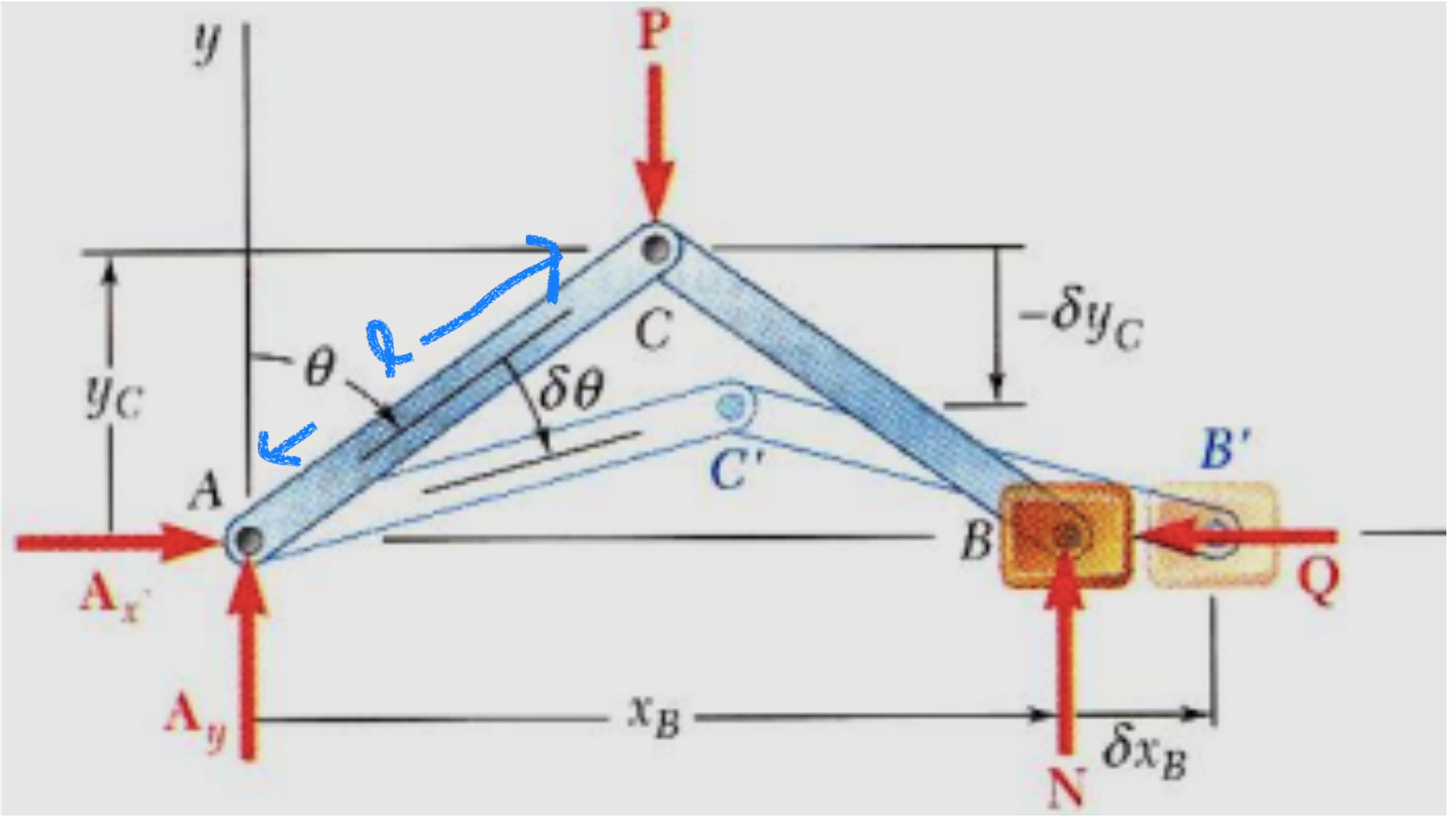
$$mg\,\delta y = -mg l \sin\theta\,\delta\theta \quad (2)$$

$$\Rightarrow \quad (F l \cos\theta - mg l \sin\theta)\delta\theta = 0$$

$$\Rightarrow \boxed{F = mg\tan\theta}$$

1

Toogle Vice



Determine the force **Q** exerted on the block B if a force **P** is applied at C

$\delta\theta$ is a virtual displacement.

'ℓ' is the length AC = CB

P and Q are applied forces.

x_B increases and y_C decreases.

$x_B = 2\ell \sin\theta \Rightarrow \delta x_B = 2\ell \cos\theta \, \delta\theta$

$y_C = \ell \cos\theta \Rightarrow \delta y_C = -\ell \sin\theta \, \delta\theta$

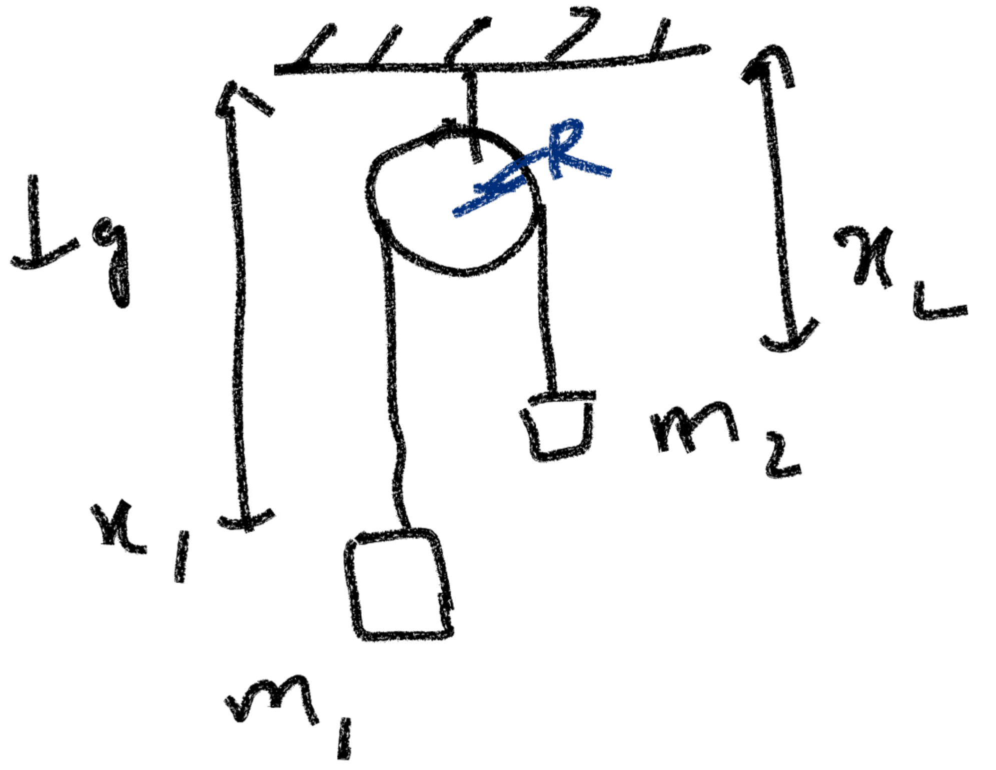
Virtual displacements

Principle of V. W. $\Rightarrow -Q\,\delta x_B + P(-\delta y_C) = 0$

$$\Rightarrow \boxed{Q = \frac{1}{2} P \tan\theta}$$

Examples on D'Alembert's Principle

1



$m_1 > m_2$

Atwood's Machine

Find the accn of m_1.

Since $x_1 + x_2 = l + \pi R \Rightarrow \ddot{x}_1 = -\ddot{x}_2$

$F^{(a)}_{m_1} = m_1 g$ Virtual displacement of $m_1 \equiv \delta x$

$\therefore W_1^a = m_1 g \delta x$. V.W. for inertial force on m_1

$W_1^{(i)} = -m_1 \ddot{x} \delta x$

Similarly $W_2^{(a)} = -m_2 g \delta x$

$W_2^{(i)} = -m_2 \ddot{x} \delta x$

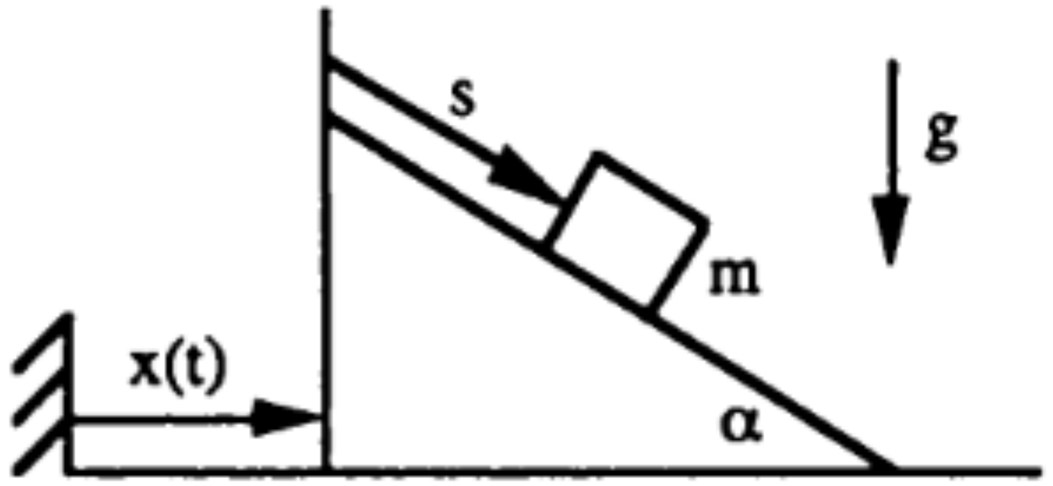
Inertial force on m_1 acting upwards.

Using D'Alembert's principle :- $(m_1 g - m_1 \ddot{x}) \delta x - (m_2 g + m_2 \ddot{x}) \delta x = 0$

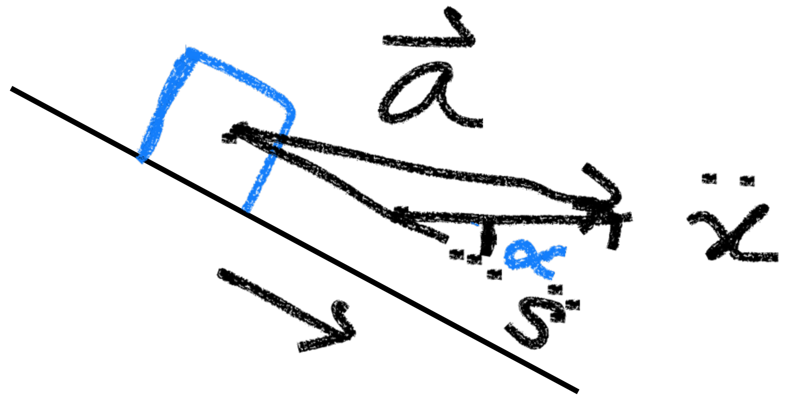
$\Rightarrow (m_1 - m_2) g = (m_1 + m_2) \ddot{x}$

$\Rightarrow \ddot{x} = a = \dfrac{m_1 - m_2}{m_1 + m_2} g$ Ans.

(2)



A block of mass m slides on a frictionless inclined plane.
The inclined plane is moving horizontally.
Find the eqn of motion for the block.



$\vec{a}$ is vector sum of horizontal accⁿ of the plane and the vertical component of the velocity of the block.

Imagine a VD along $s \rightarrow \delta s$

External force of gravity on the mass does a work $W_e = mg \sin\alpha \, \delta s$

Inertial force on the mass $-m\vec{a}$

The component of this in the direction of virtual displacement (VD) is $-m(\ddot{x}\cos\alpha + \ddot{s})$

Applying D'Alembert's principle:

$$\{mg\sin\alpha - m(\ddot{x}\cos\alpha + \ddot{s})\}\delta s = 0$$

$$\Rightarrow \boxed{\ddot{s} = g\sin\alpha - \ddot{x}\cos\alpha}$$