Quantum Computing & Information

Assignment-1

Dated: 03-09-24 FM: 20

Due date: 17-09-24, 3 PM

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1. Suppose $\frac{1}{3}\left((1+2i)|0>+(ai)|1>\right)$ is a quantum state or a qubit. What is the value of a? [2]

2. Let $|+>=\frac{1}{\sqrt{2}}(|0>+|1>)$, and $|->=\frac{1}{\sqrt{2}}(|0>-|1>)$ are two basis vectors. Consider a state $|\psi>=a|+>+b|->$. What are the probabilities of measuring $|0>$ and measuring $|1>$? [2]

   If the state $|\psi>$ is represented as $|\psi>=\alpha|0>+\beta|1>$, then what is the probability of measuring $|+>$? What is the probability of measuring $|->$? [2]

3. Show that $trace(A|u><u|) =<u|A|u>$. [2]

4. Express $|\psi>=(3,5)^T$ in terms of the projection operators P_1 and P_2. Where the basis vectors are- $|b_1>=\frac{1}{\sqrt{2}}(1,1)^T$ and $|b_2>=\frac{1}{\sqrt{2}}(1,-1)^T$. [2]

5. Consider a linear operator X. The operation of X on basis vectors is as follows: $X|0>=|1>$ and $X|1>=|0>$. Construct the operator X in matrix form. [2]

6. If $X|0>=|0>$ and $X|1>=e^{i\phi}|1>$, then construct the matrix operator X. What will be the form of X for $\phi=\pi$. Identify the quantum gate. [2]

7. Which quantum gate upon acting on $|+>$ and $|->$ kets (as given in Q. 2) produces $|0>$ and $|1>$ respectively? [3]

8. Consider the gate $H_2 = H \otimes H$. Show that it creates a quantum state that has an equal probability of being observed in any of its four possible outcomes; $|00\rangle, |01\rangle, |10\rangle$, and $|11\rangle$ when acted on the state $|00\rangle$. [3]