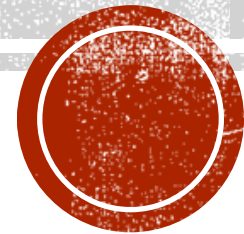




**Indian Institute of Information Technology Allahabad**

# Data Structures and Algorithms

## Depth First Search (DFS)



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Associate Professor

Department of Information Technology

Indian Institute of Information Technology, Allahabad

Email: [srdubey@iiita.ac.in](mailto:srdubey@iiita.ac.in)

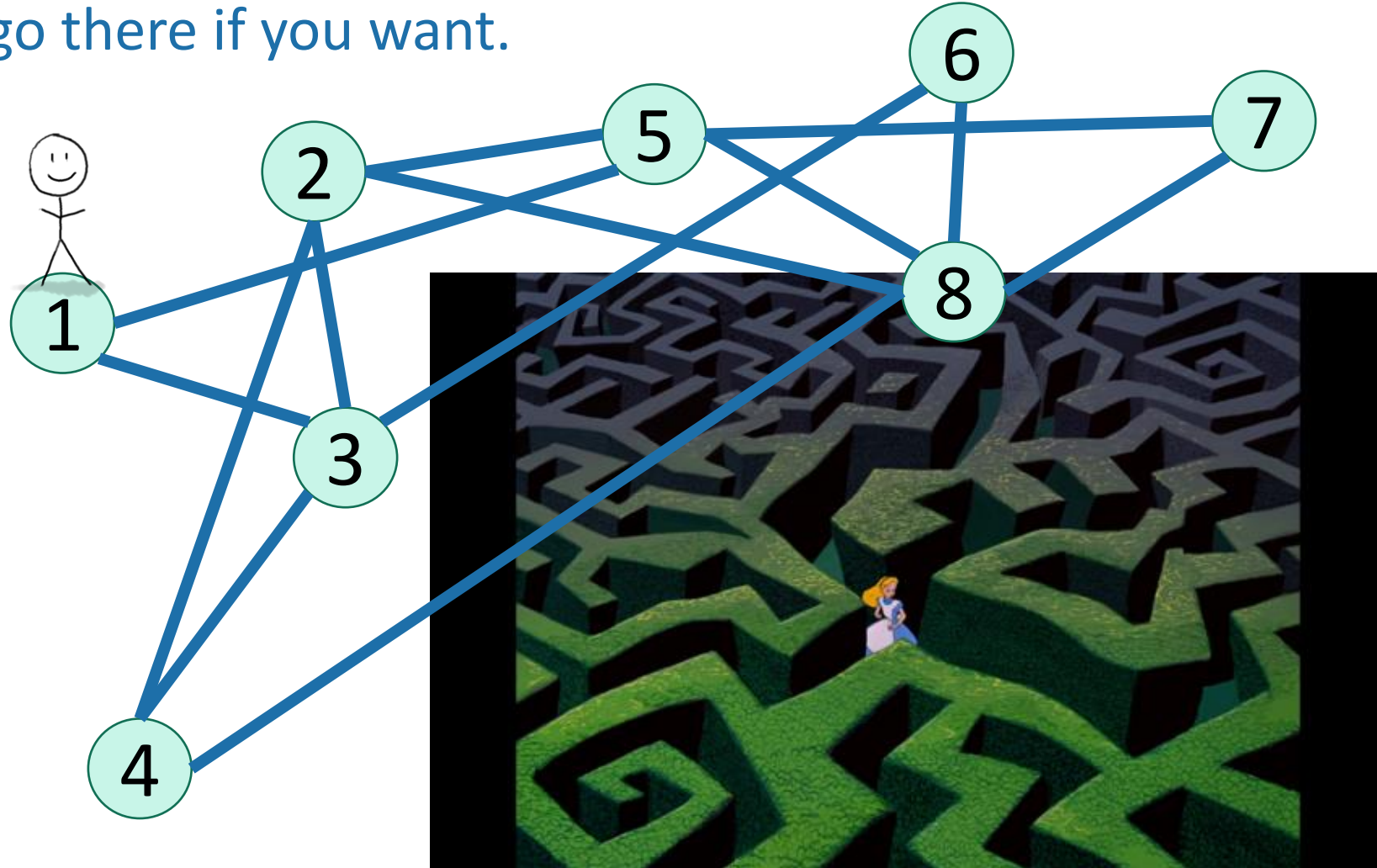
Web: <https://profile.iiita.ac.in/srdubey/>

# DISCLAIMER

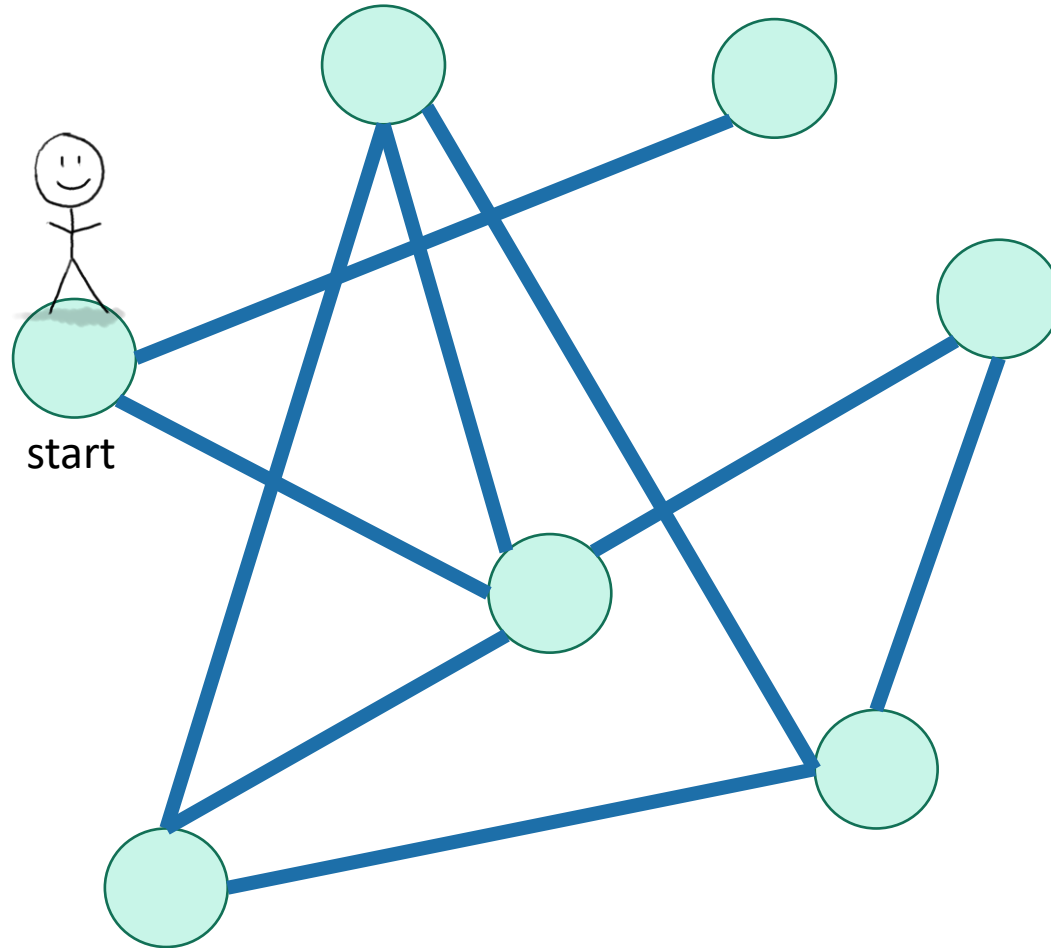
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# How do we explore a graph?

At each node, you can get a list of neighbors, and choose to go there if you want.

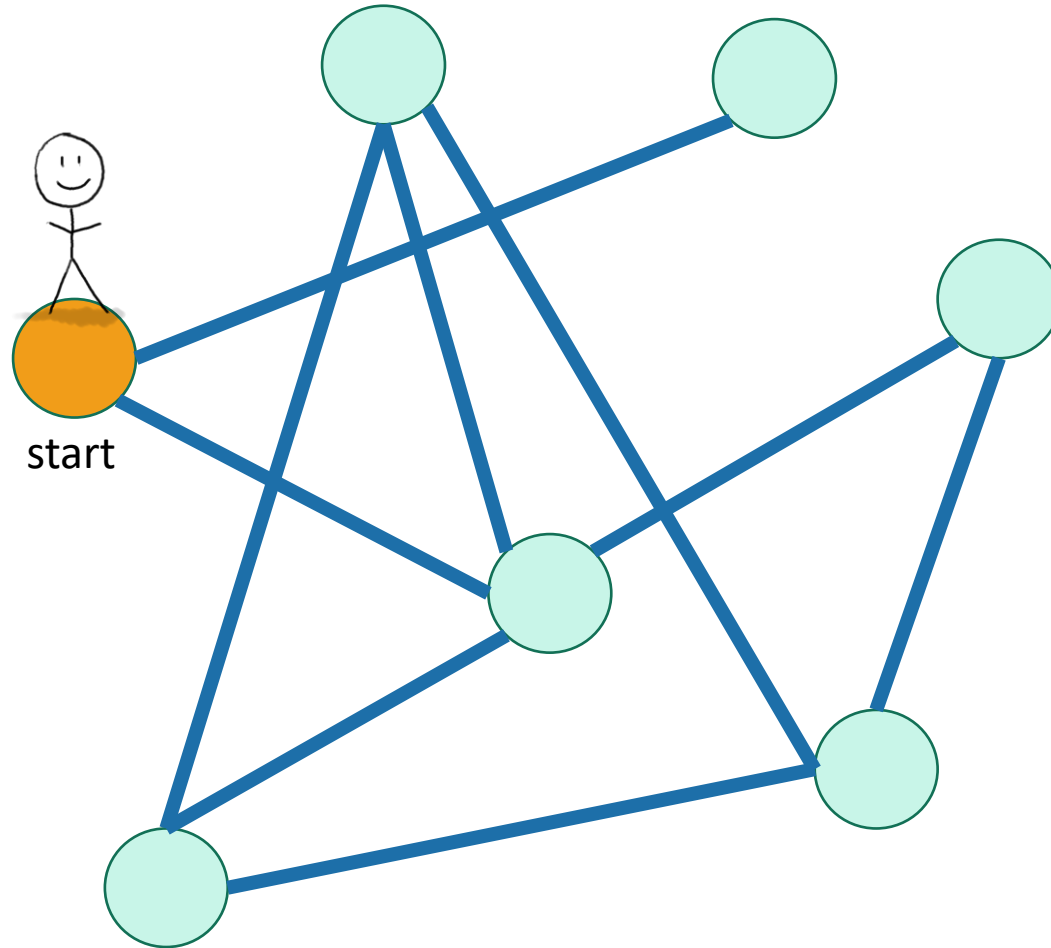





# Depth First Search



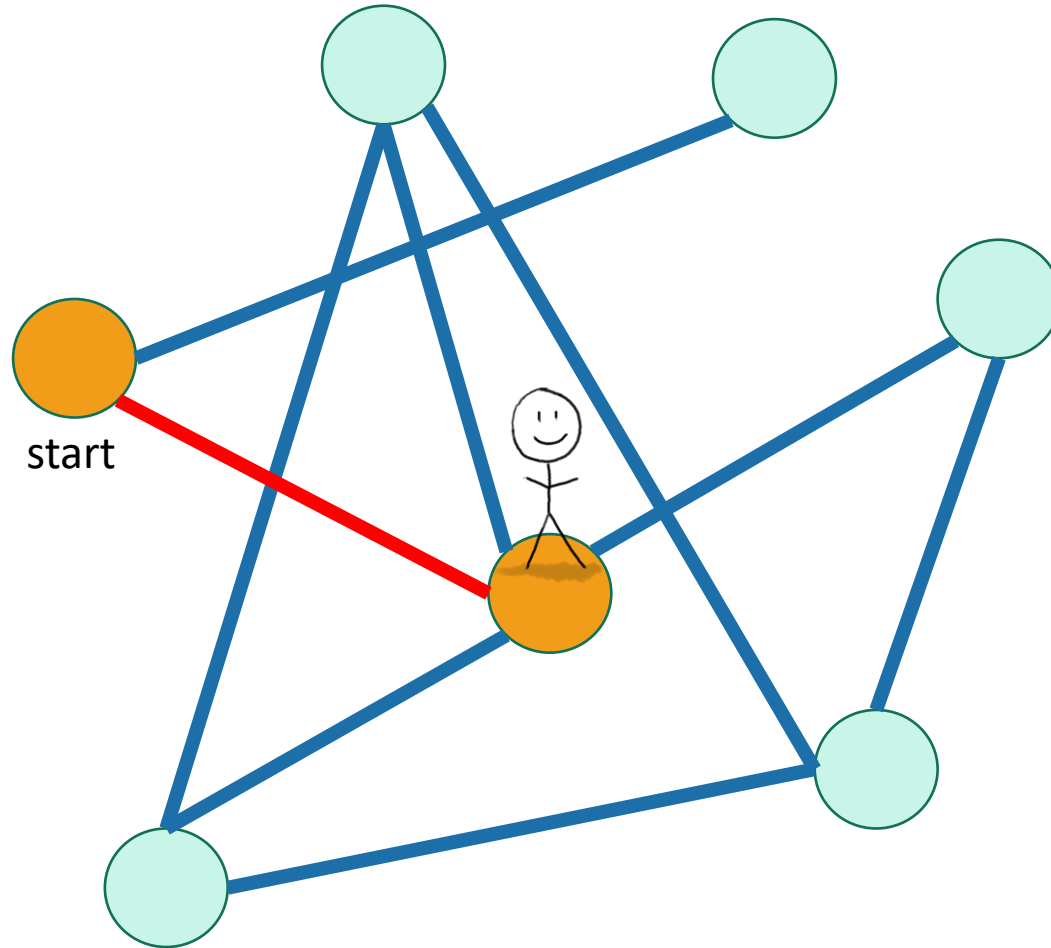
-  Not been there yet
-  Been there, haven't explored all the paths out.
-  Been there, have explored all the paths out.




# Depth First Search



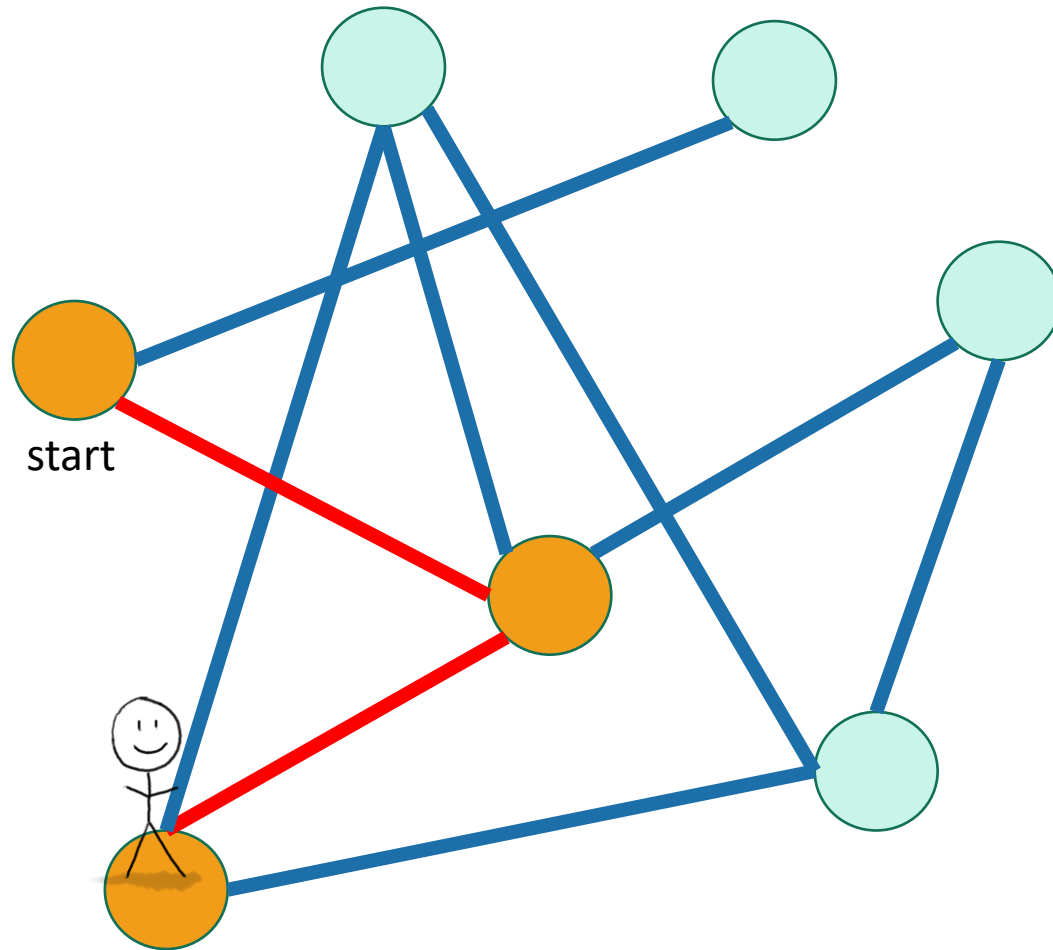
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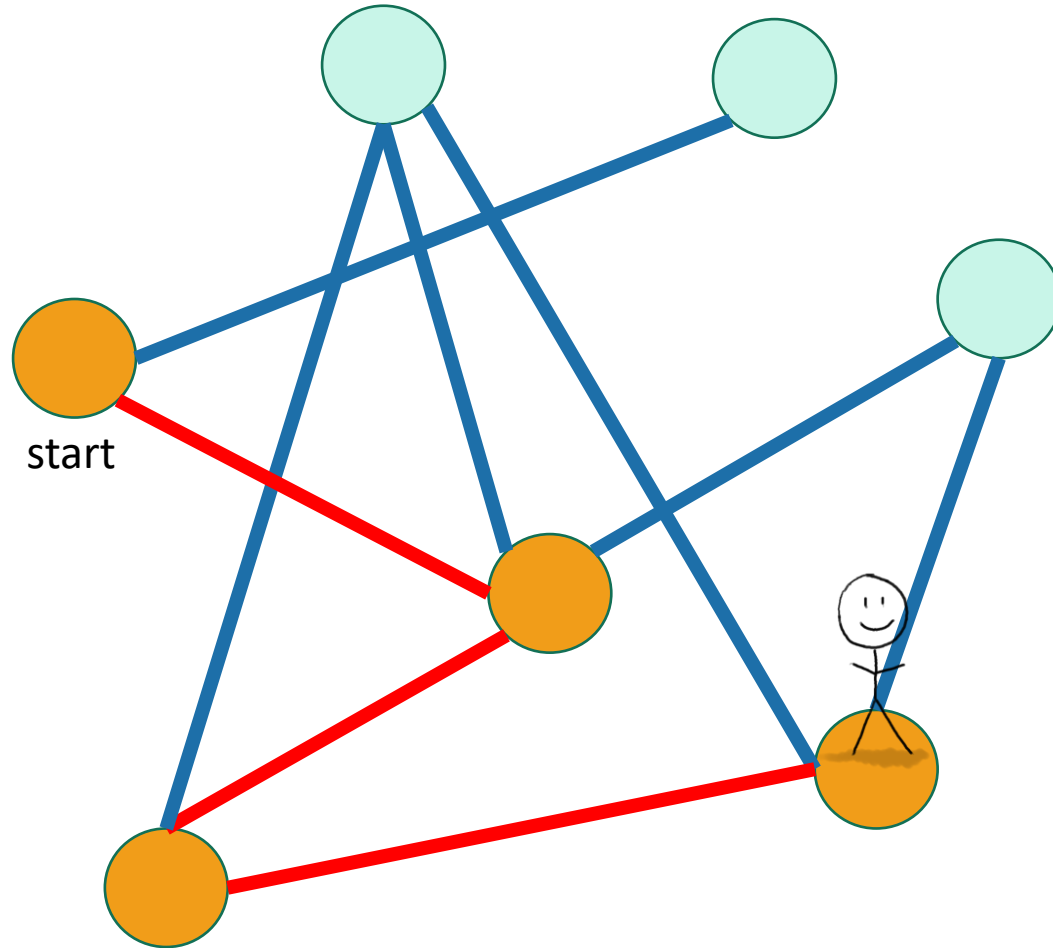
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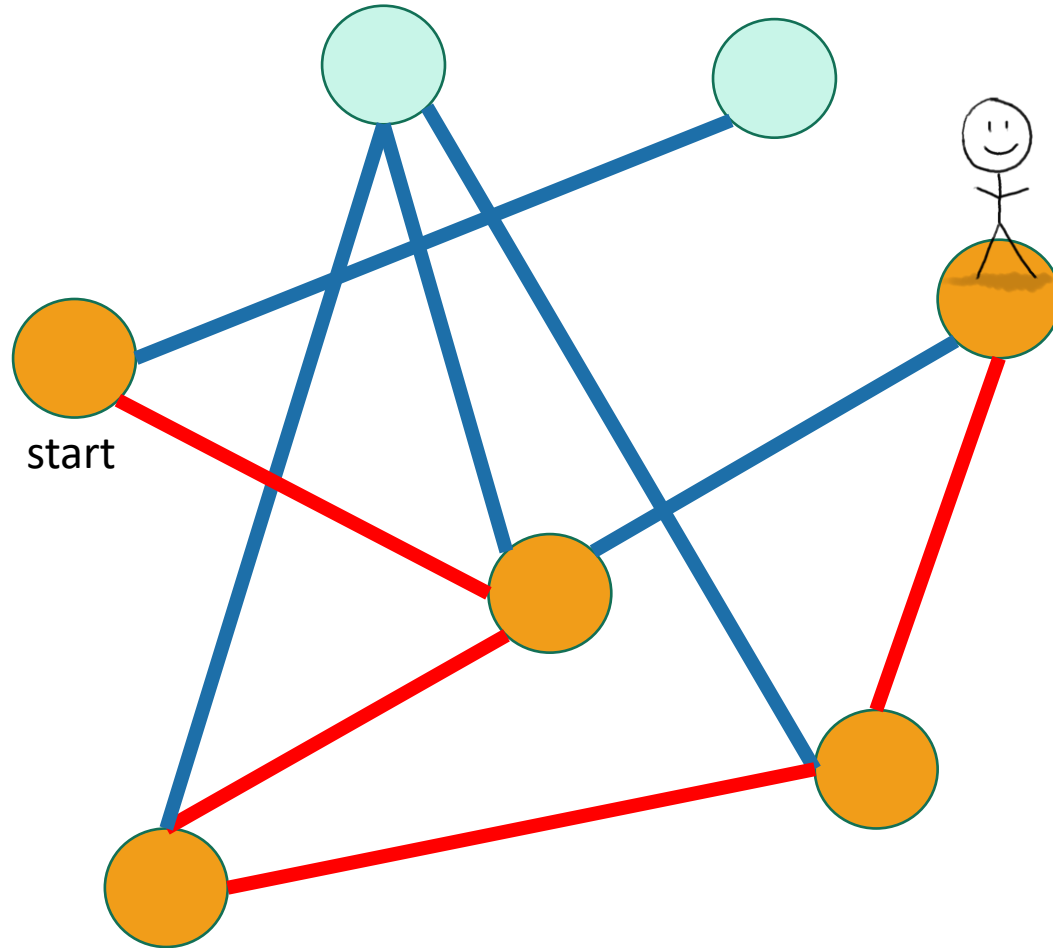
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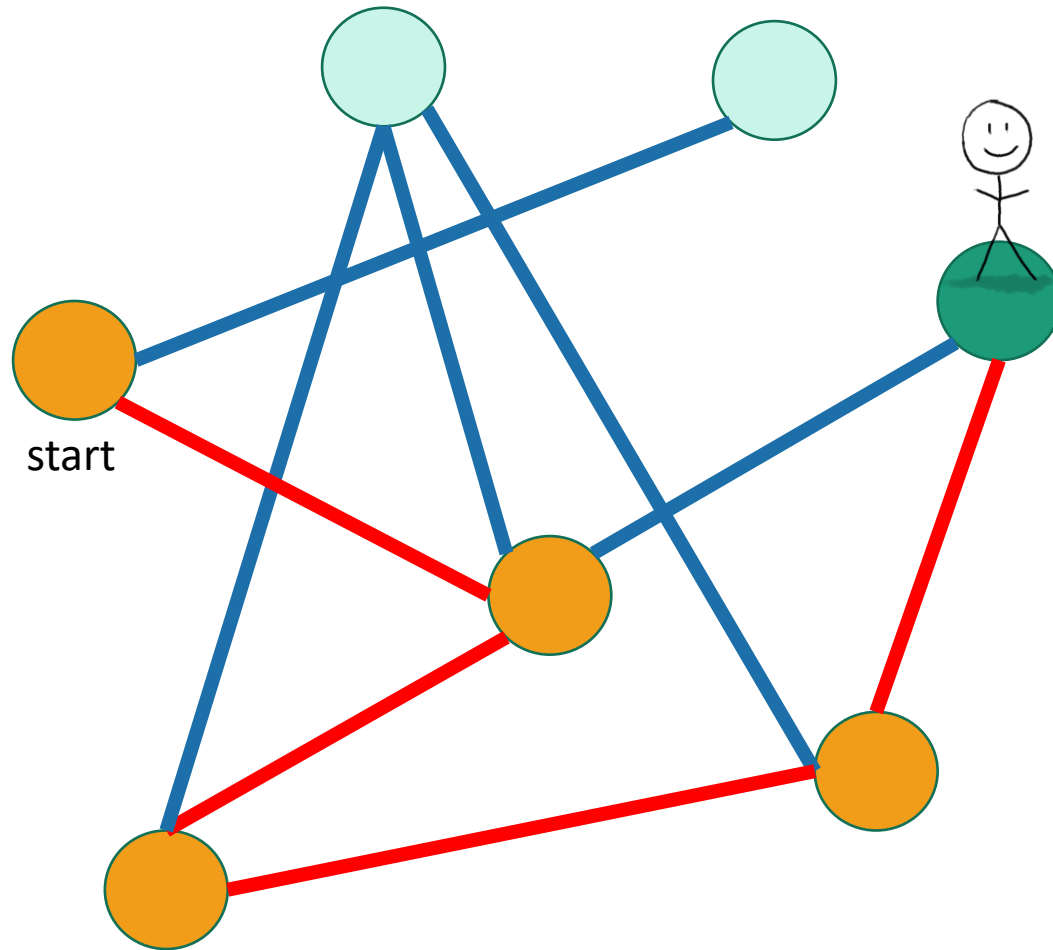
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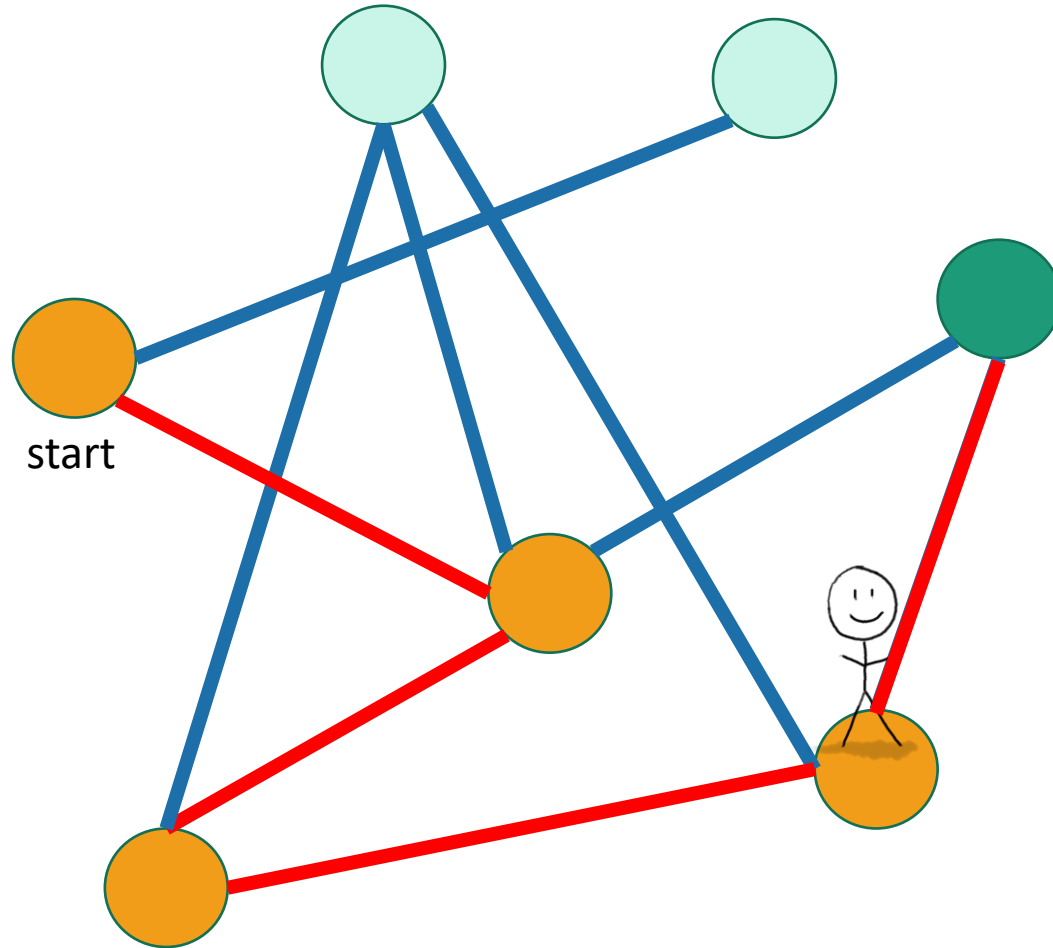
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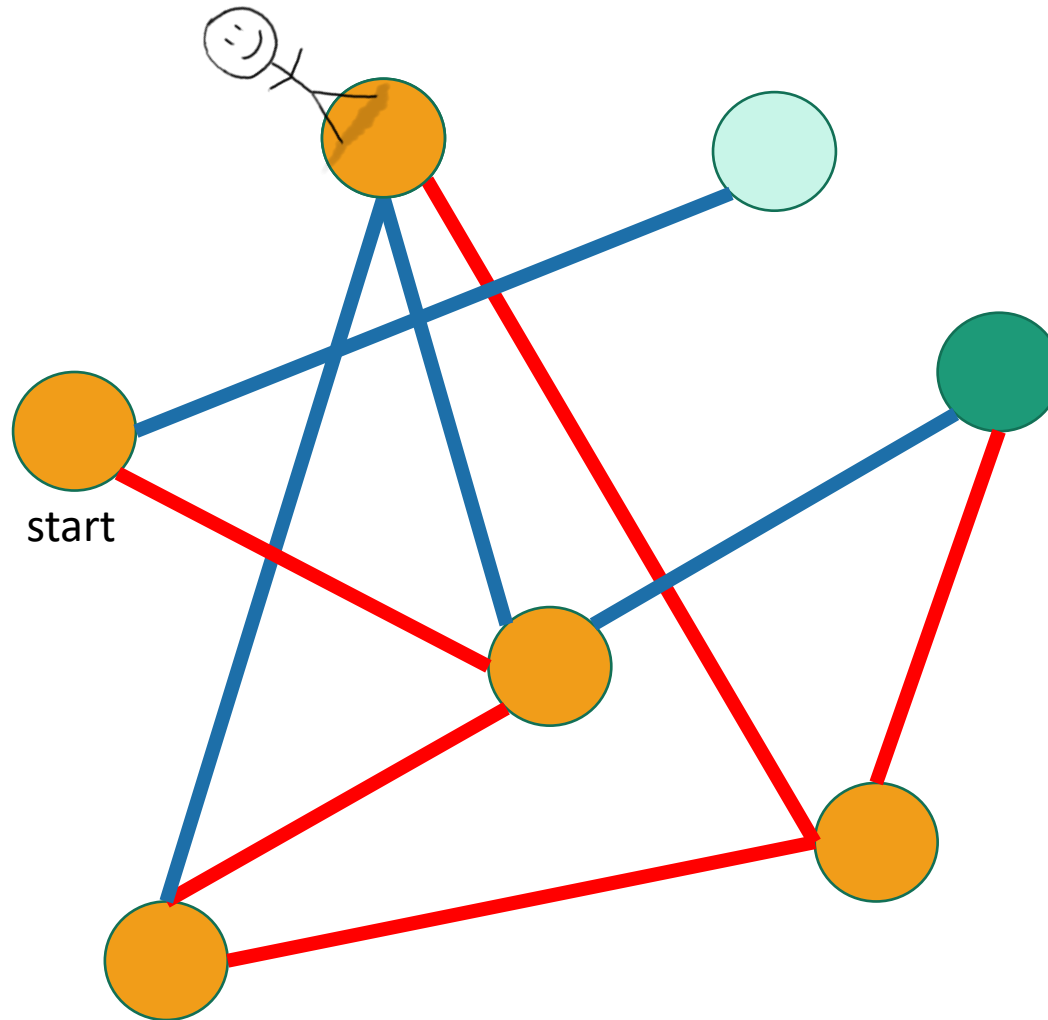
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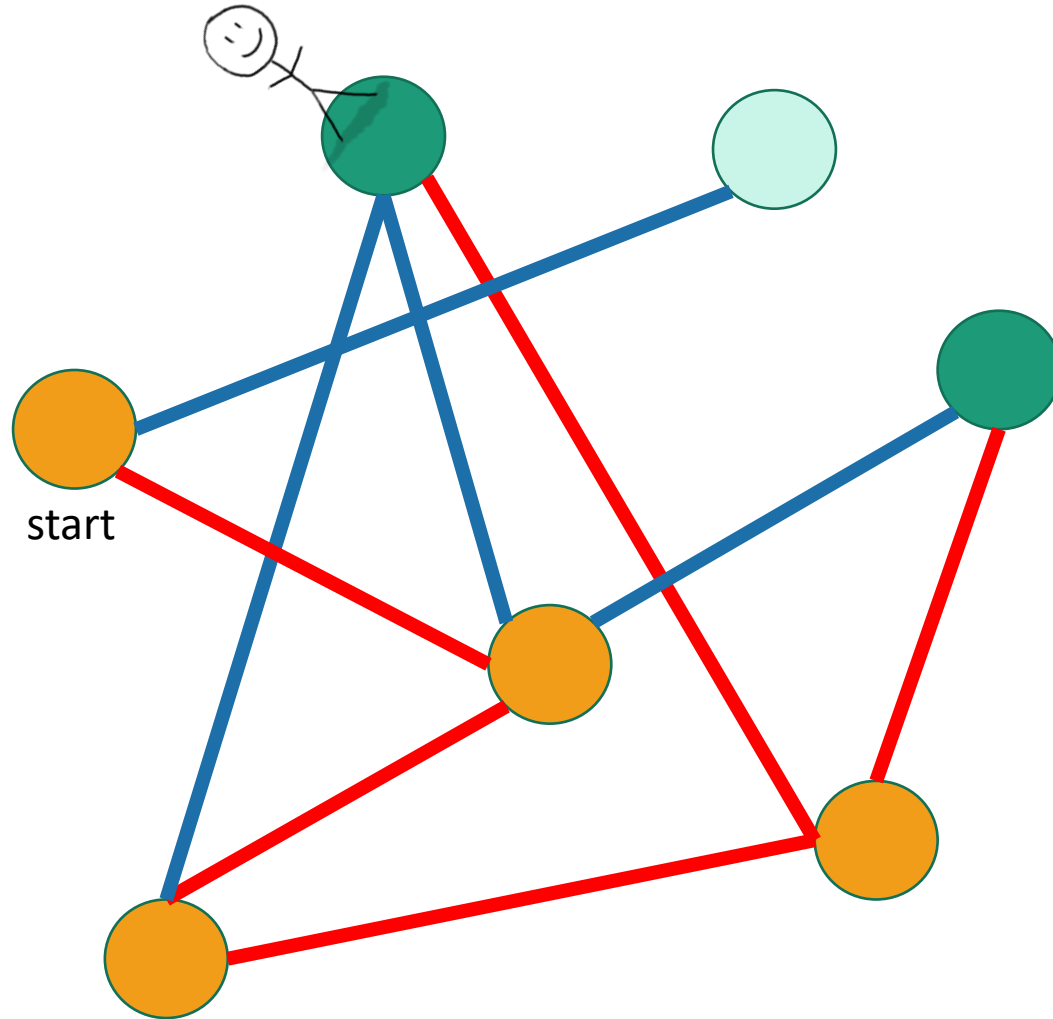
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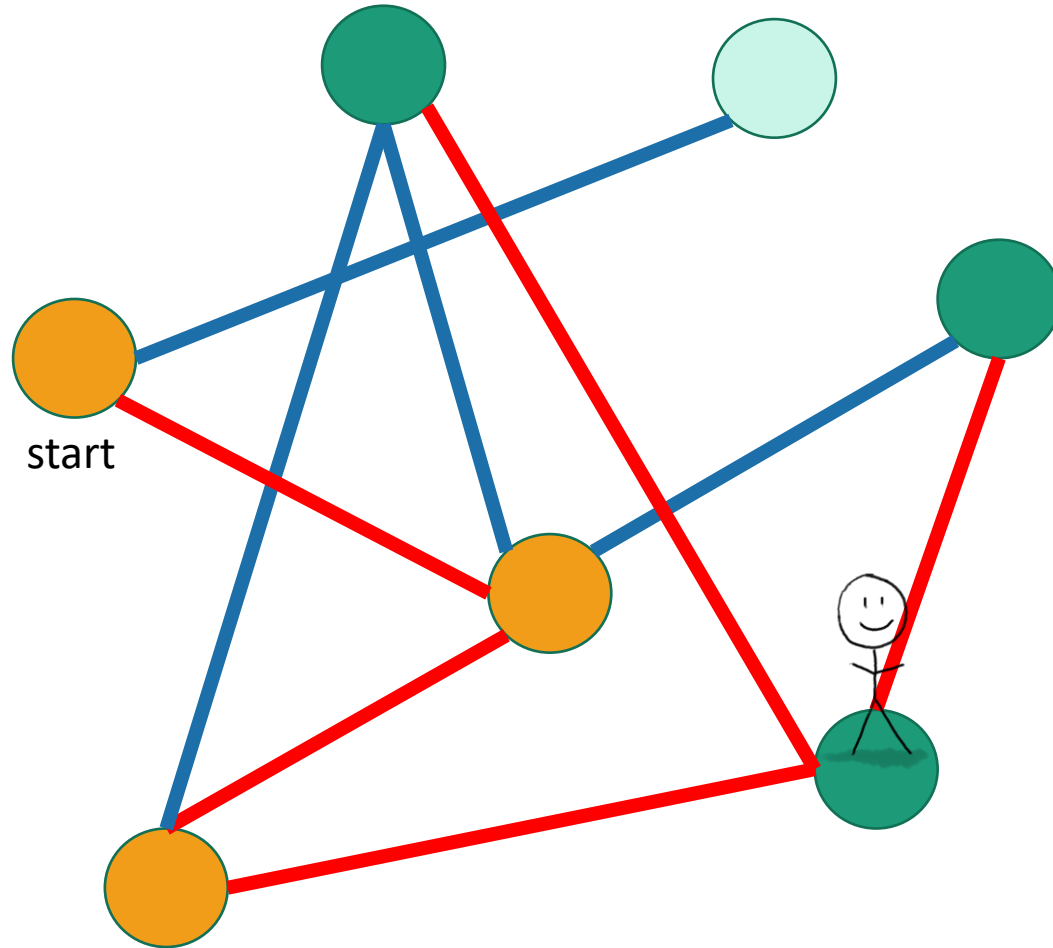
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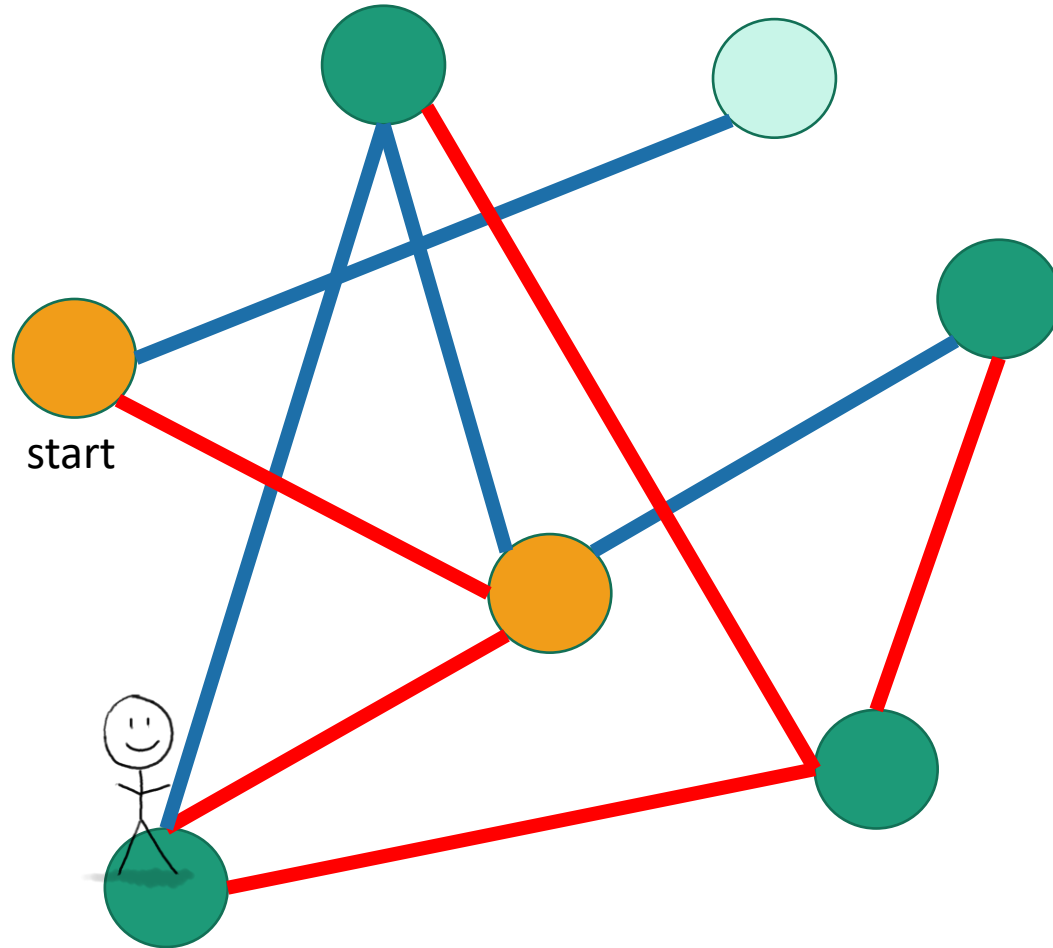
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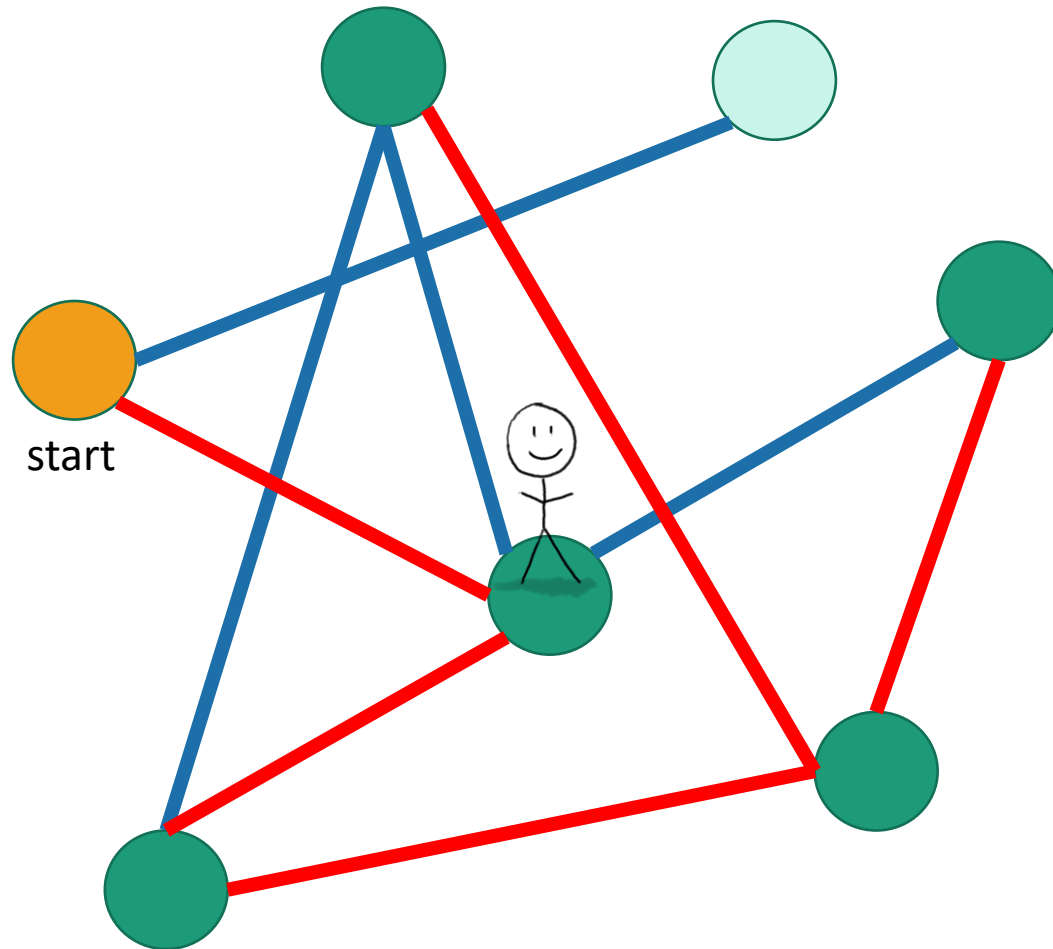
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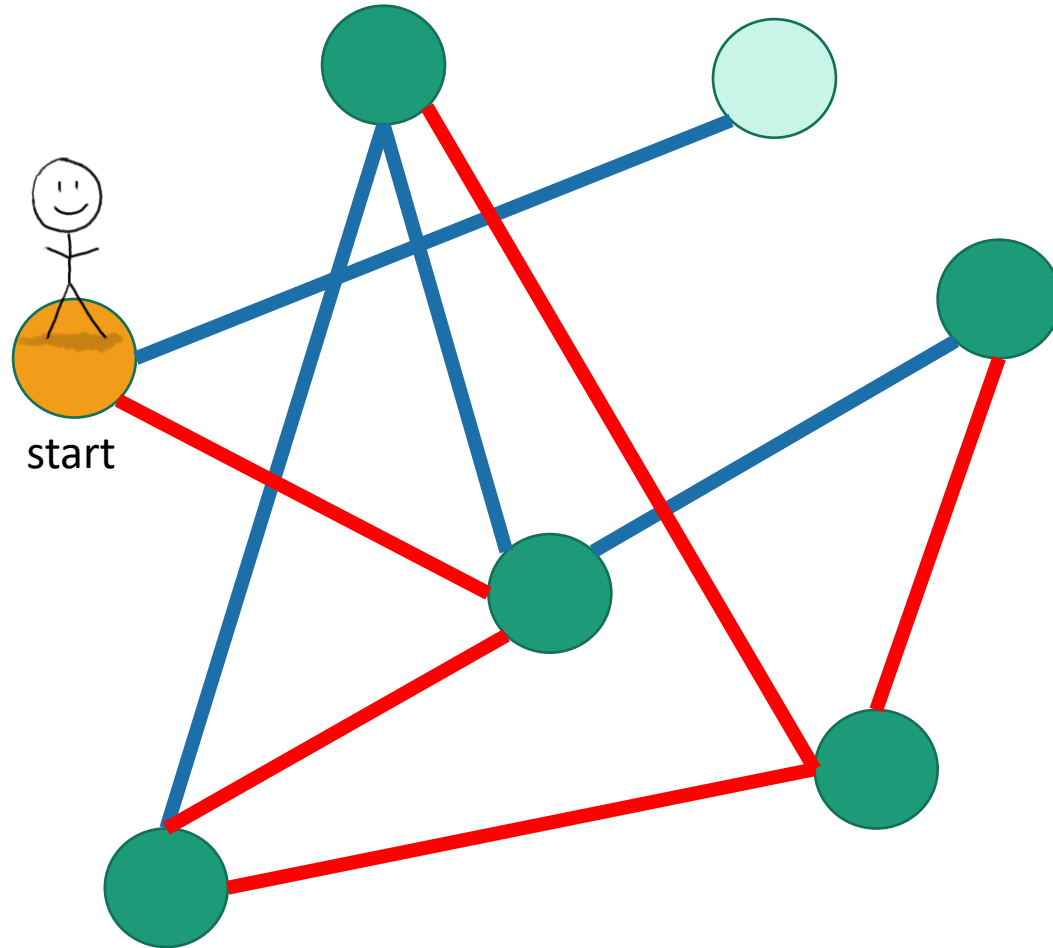
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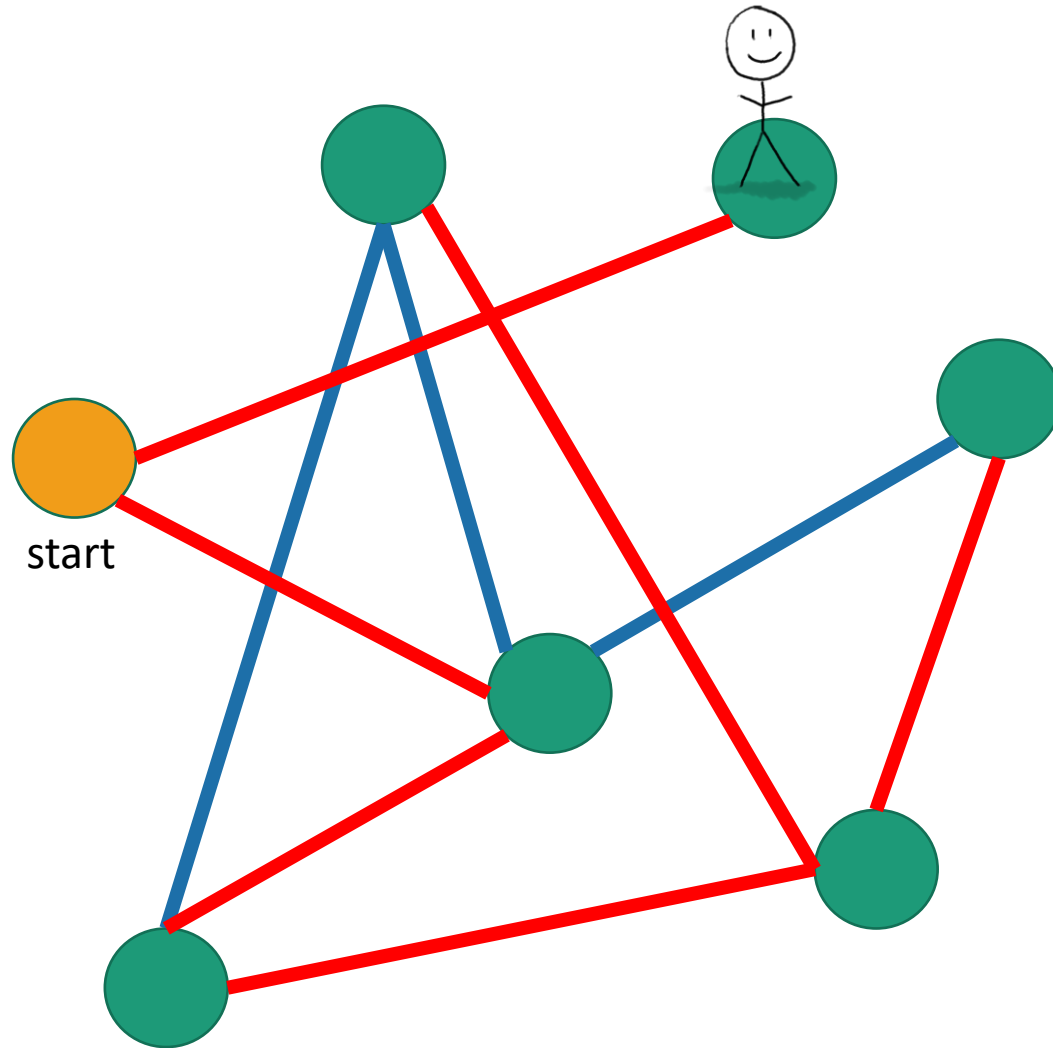
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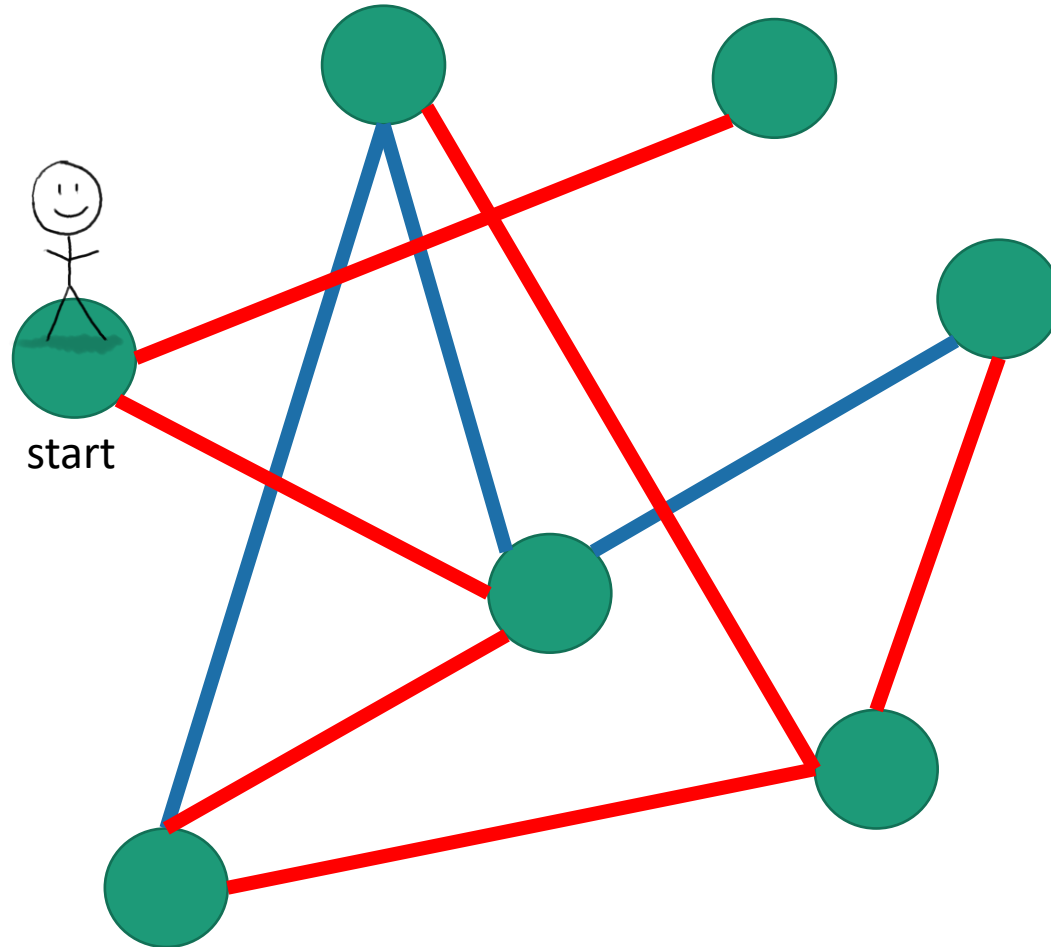
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# Depth First Search

## Pseudocode

- Each vertex keeps track of whether it is:
  - Unvisited 
  - In progress 
  - All done 

# Depth First Search




## Pseudocode

- Each vertex keeps track of whether it is:
  - Unvisited 
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- Each vertex will also keep track of:
  - The time we **first enter it**.
  - The time we finish with it and mark it **all done**.



# Depth First Search

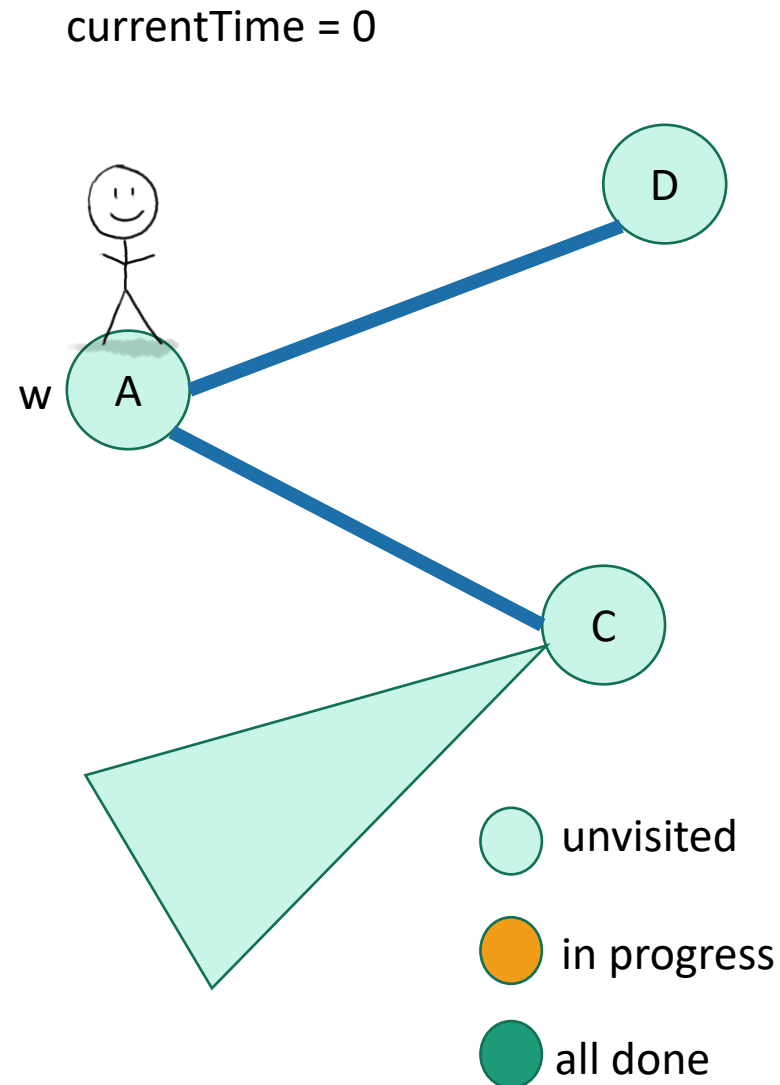
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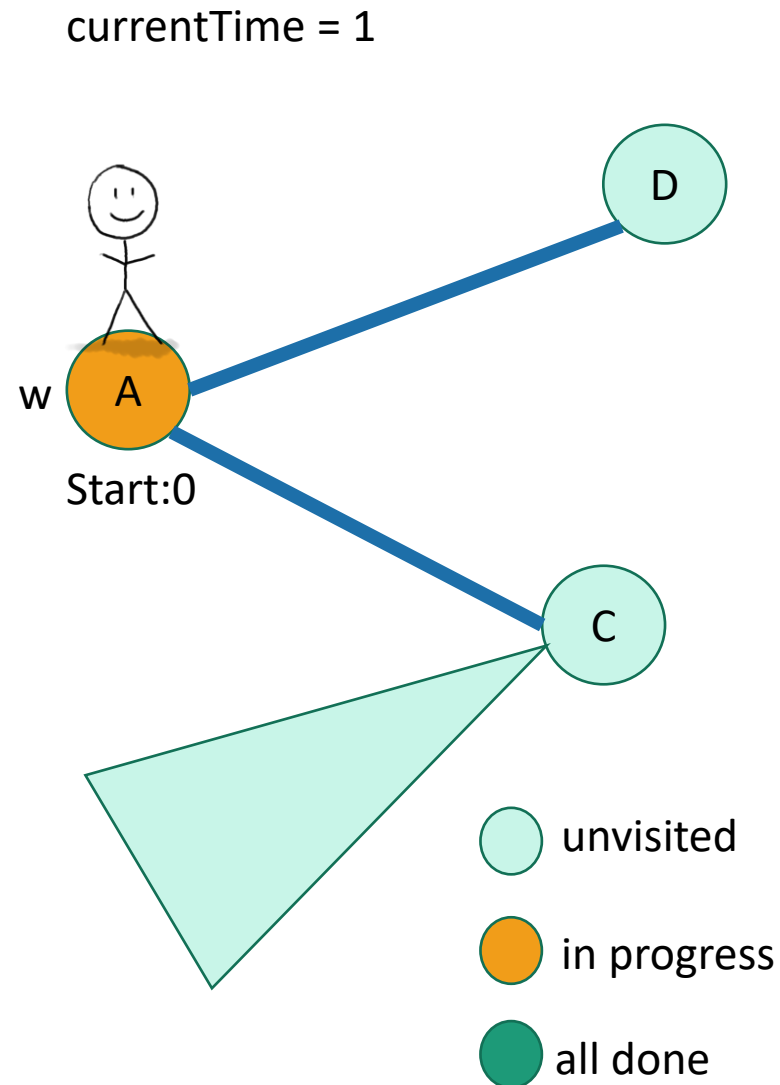
You might have seen other ways to implement DFS than what we are about to go through. This way has more bookkeeping – the bookkeeping will be useful later!

# Depth First Search



- **DFS**(w, currentTime):
  - w.startTime = currentTime
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  - **for** v in w.neighbors:
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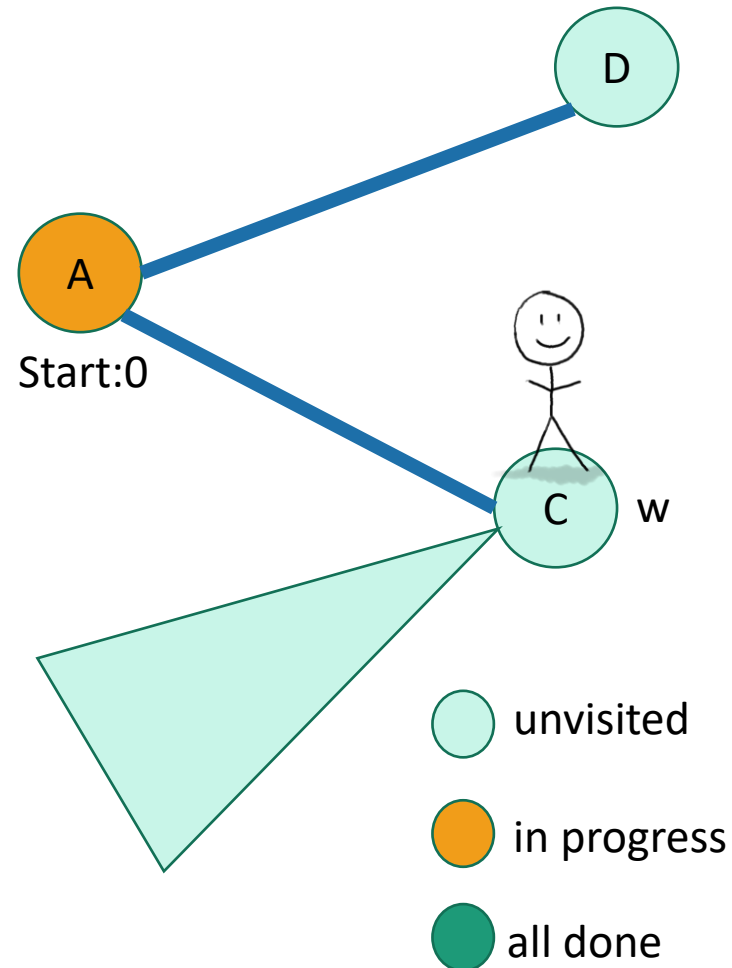
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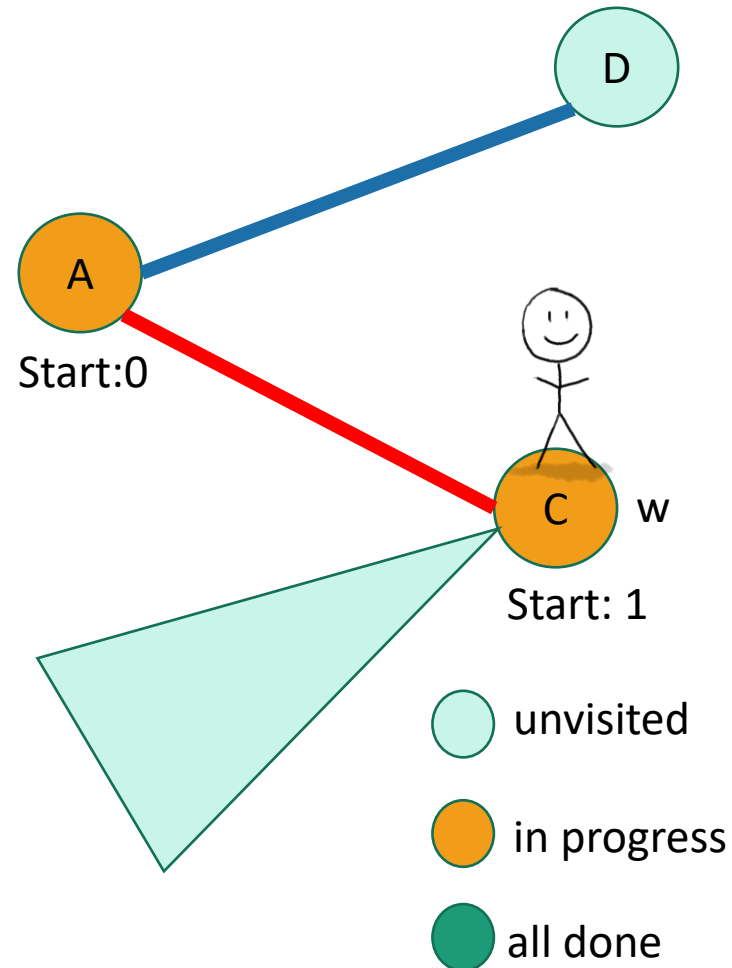
currentTime = 1



- **DFS**(w, currentTime):
  - w.startTime = currentTime
  - currentTime ++
  - Mark w as **in progress**.
  - **for** v in w.neighbors:
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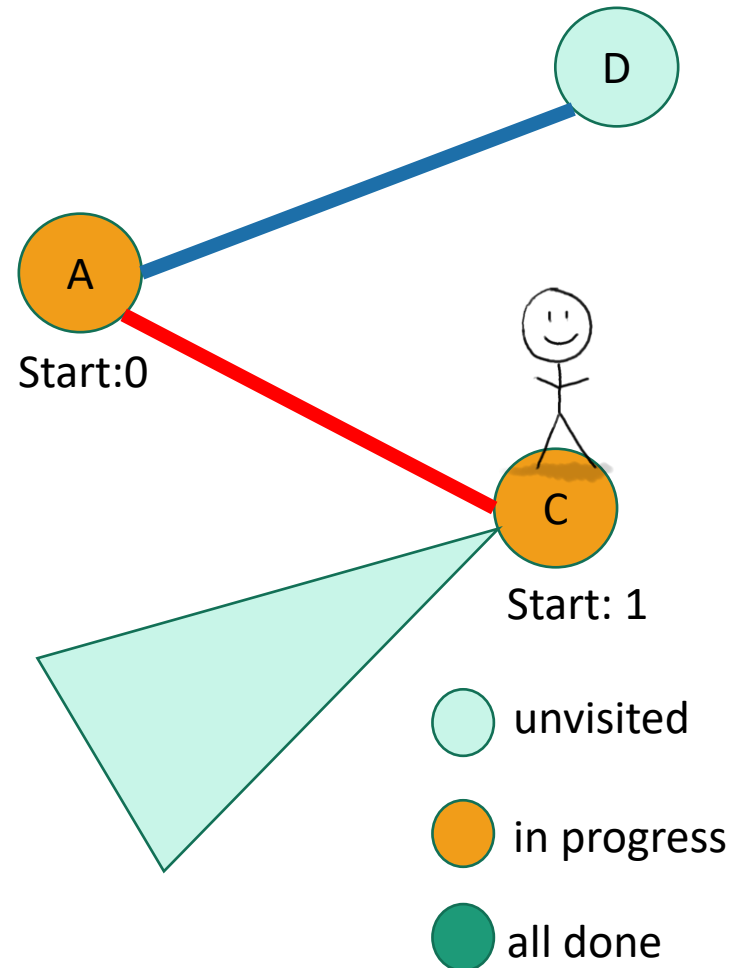
currentTime = 2



- **DFS**(w, currentTime):
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  - currentTime ++
  - Mark w as **in progress**.
  - **for** v in w.neighbors:
    - **if** v is **unvisited**:
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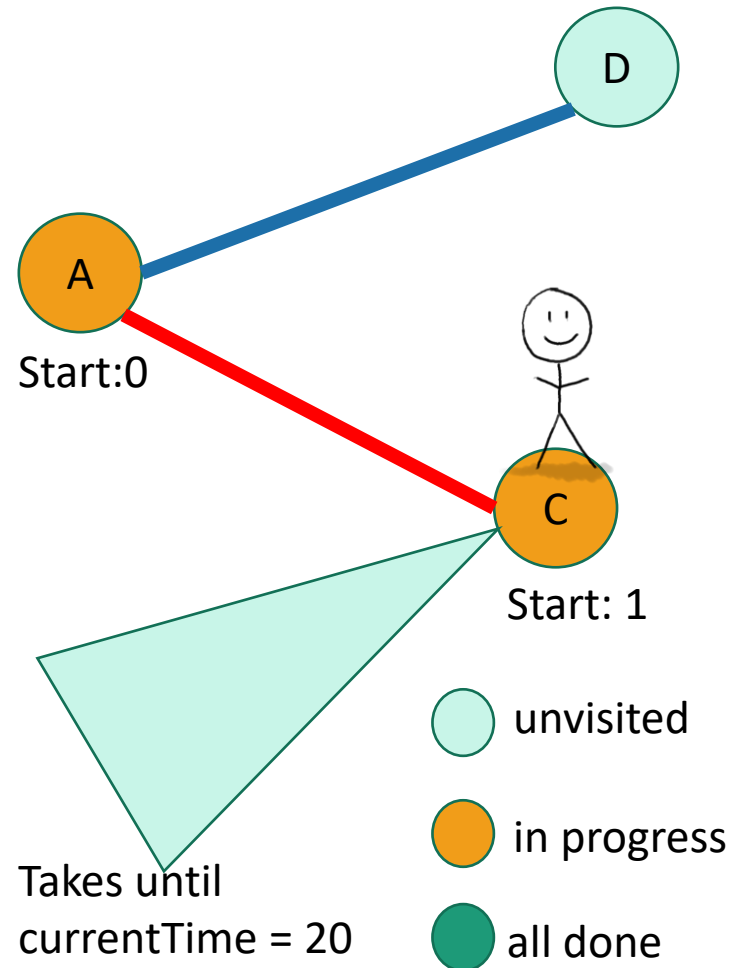
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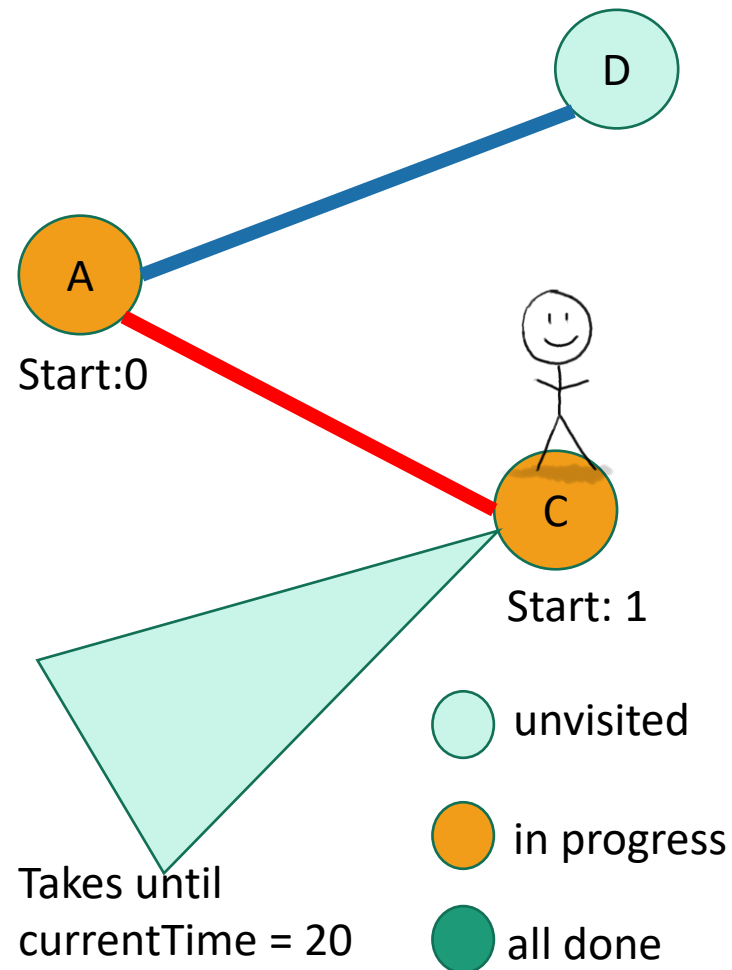
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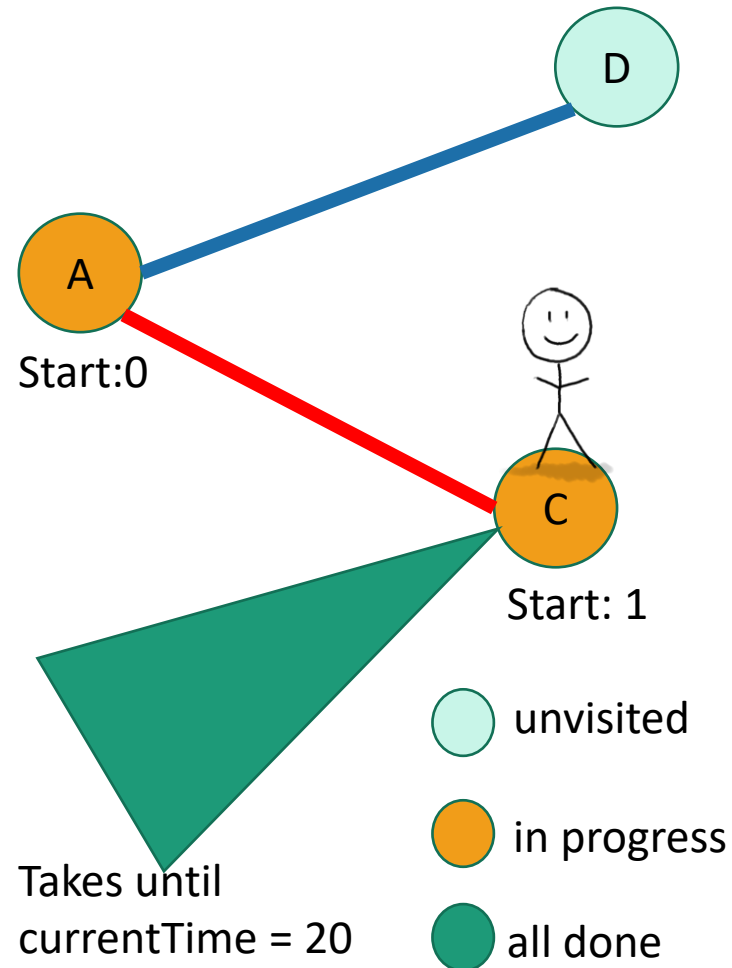
currentTime = 20



- **DFS**(w, currentTime):
  - w.startTime = currentTime
  - currentTime ++
  - Mark w as **in progress**.
  - **for** v in w.neighbors:
    - **if** v is **unvisited**:
      - currentTime  
= **DFS**(v, currentTime)
      - currentTime ++
  - w.finishTime = currentTime
  - Mark w as **all done**
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# Depth First Search

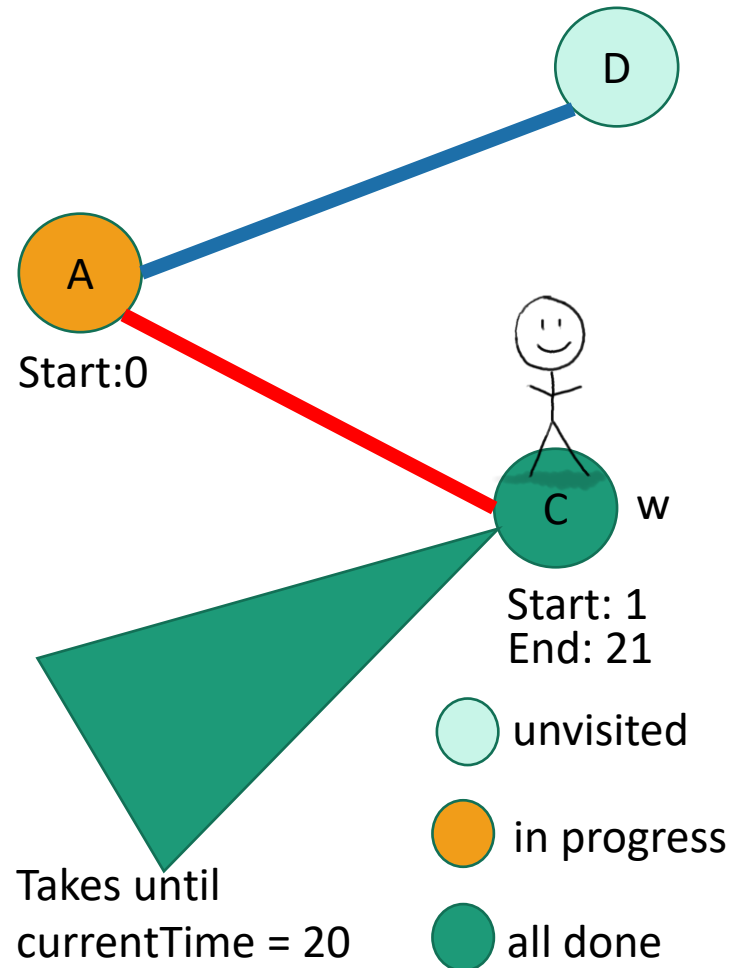
currentTime = 21



- **DFS**(w, currentTime):
  - w.startTime = currentTime
  - currentTime ++
  - Mark w as **in progress**.
  - **for** v in w.neighbors:
    - **if** v is **unvisited**:
      - currentTime = **DFS**(v, currentTime)
      - currentTime ++
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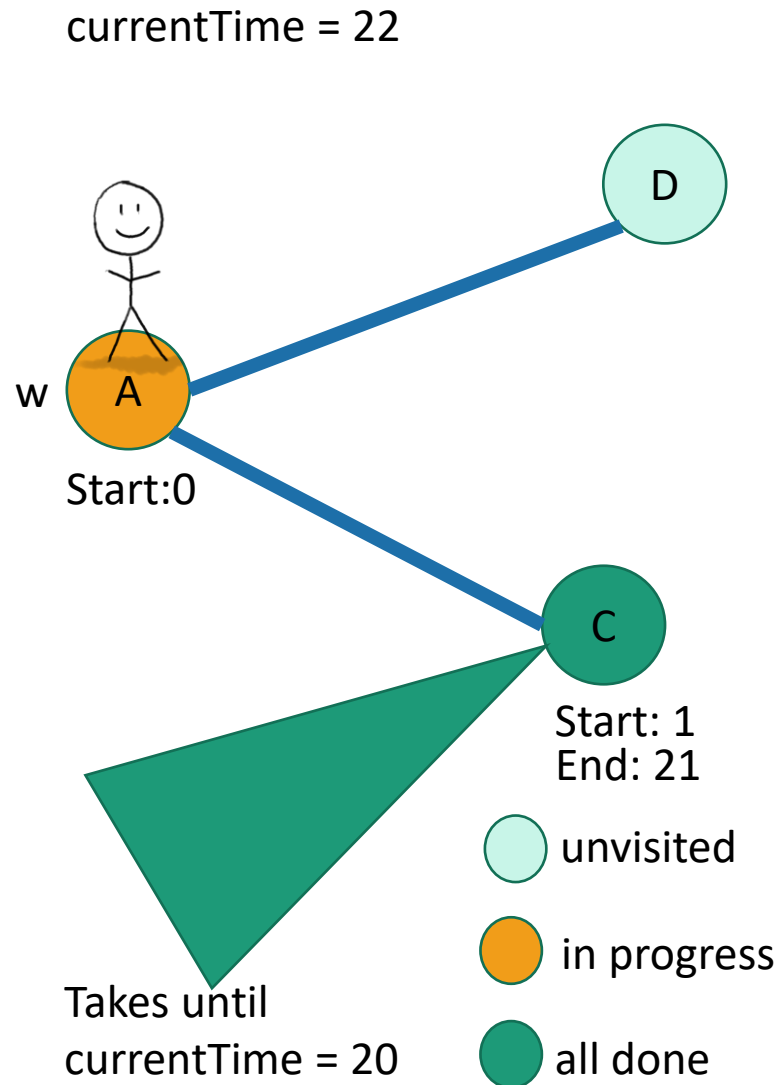
# Depth First Search

currentTime = 21



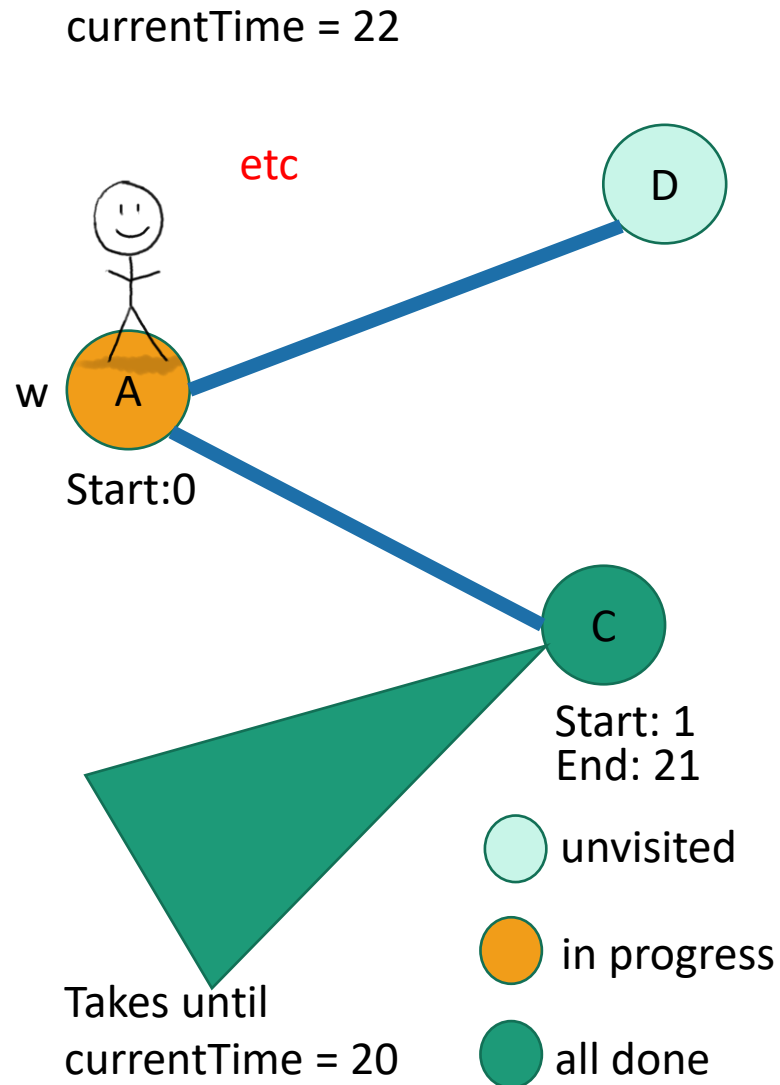
- **DFS**(w, currentTime):
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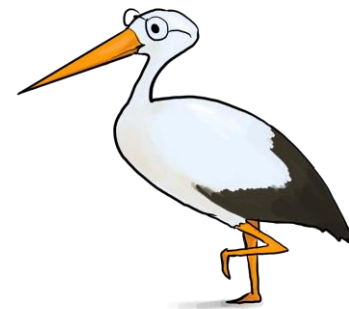
# Depth First Search



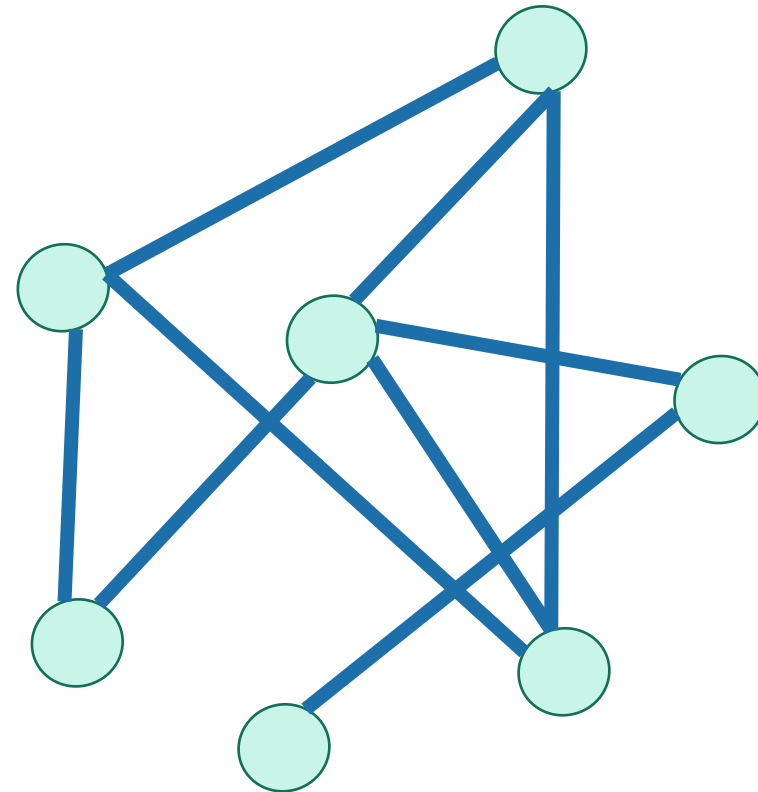
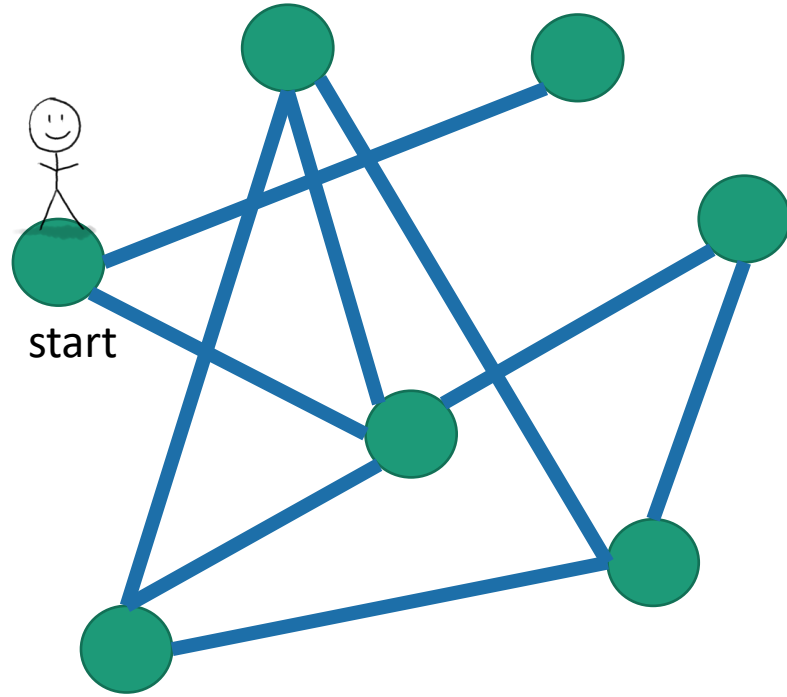
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# Fun exercise

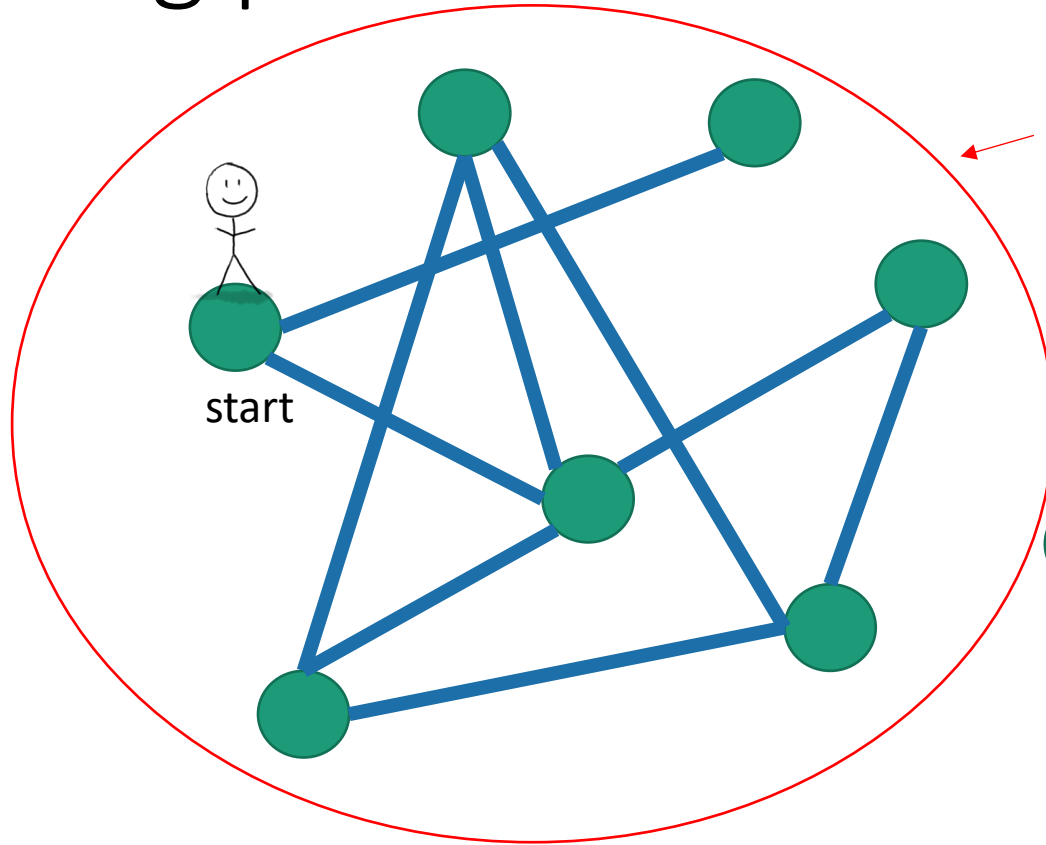
- Write pseudocode for an iterative version of DFS.



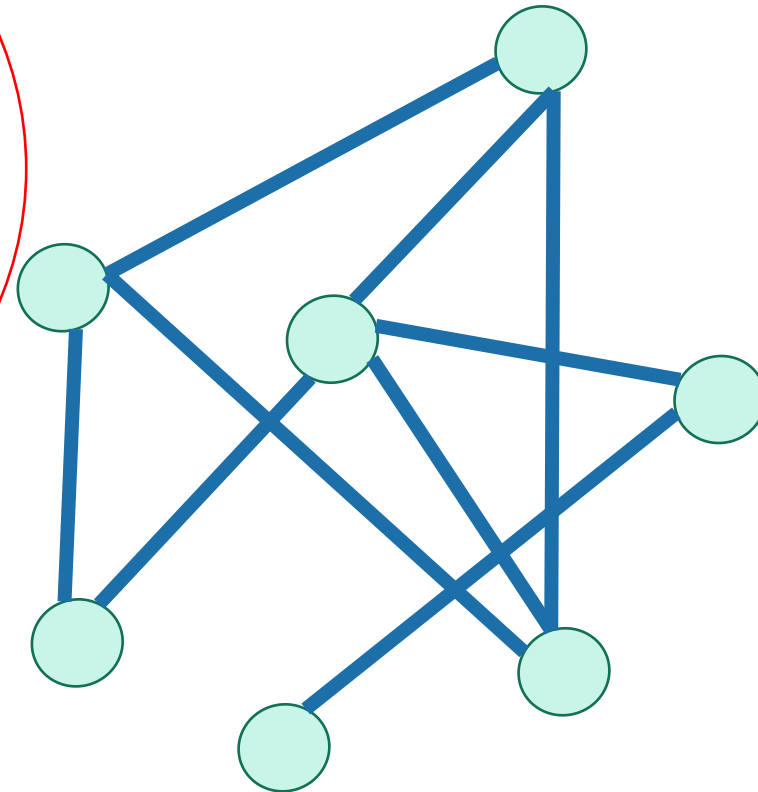
# DFS finds all the nodes reachable from the starting point



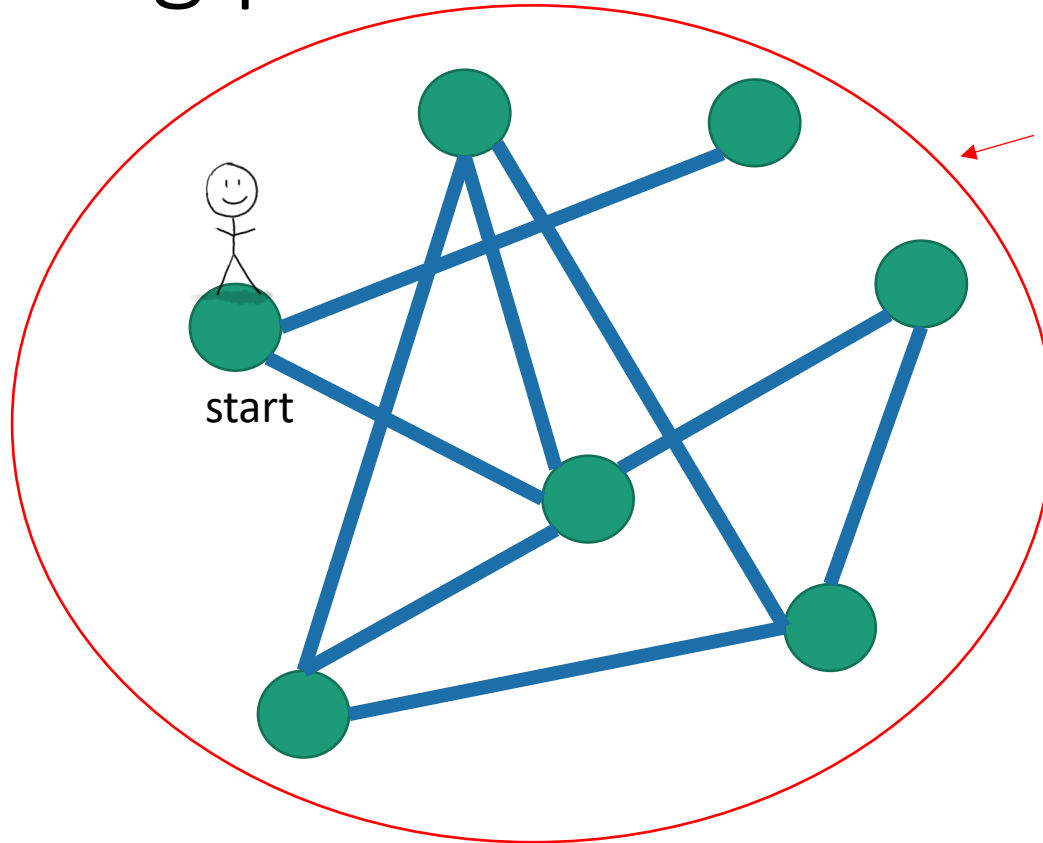
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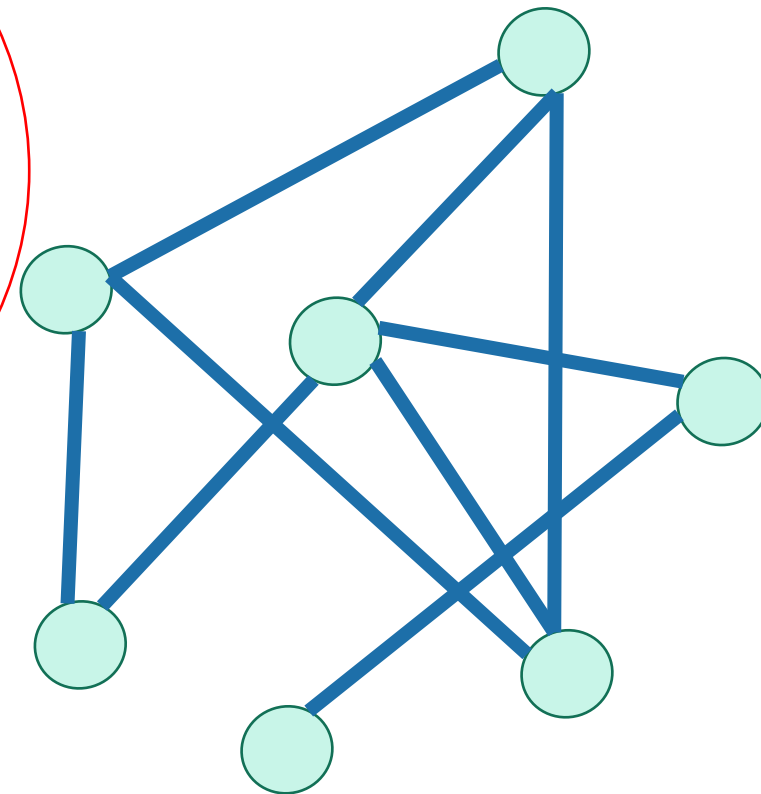
In an undirected graph, this is called a **connected component**.



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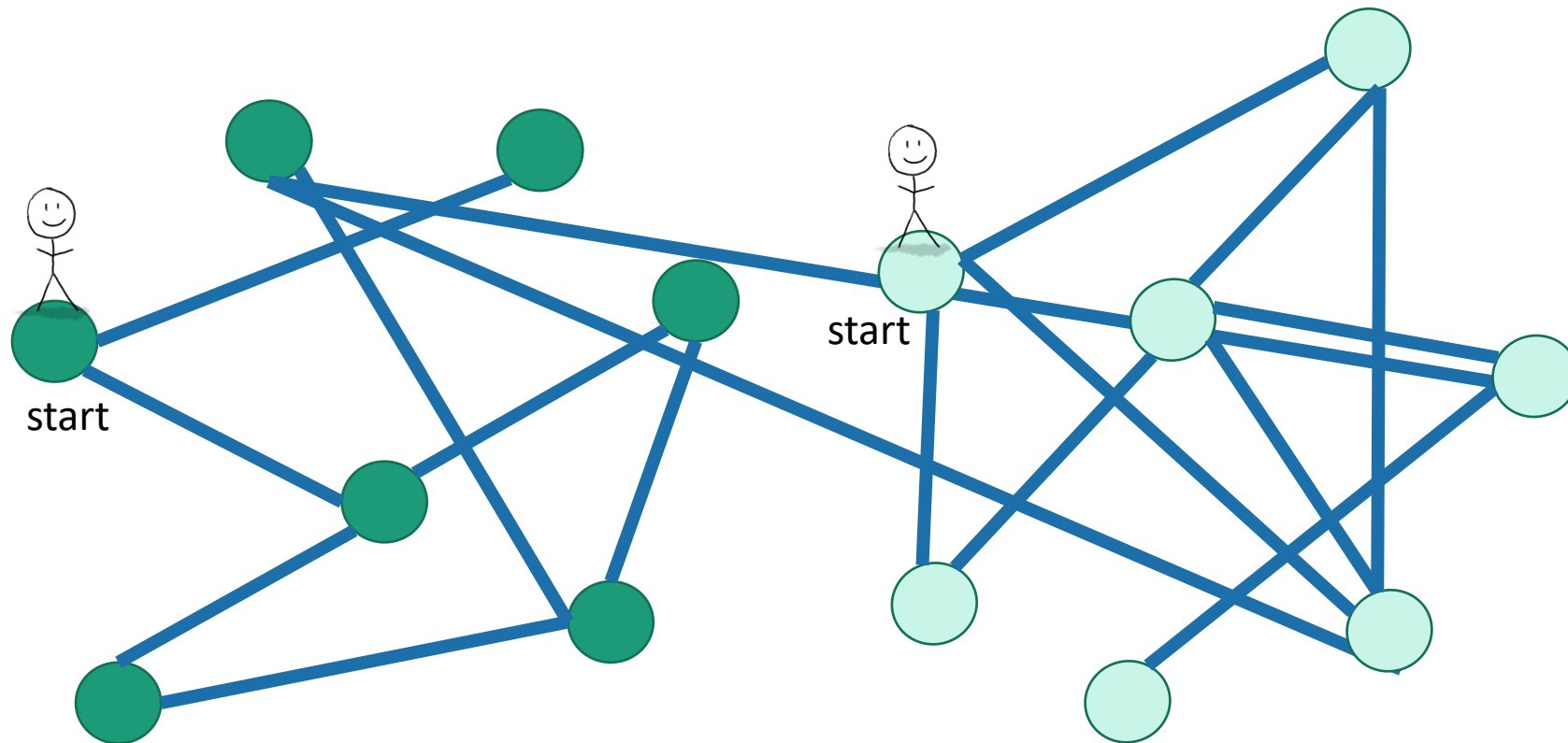
In an undirected graph, this is called a **connected component**.



**One application of DFS:** finding connected components.

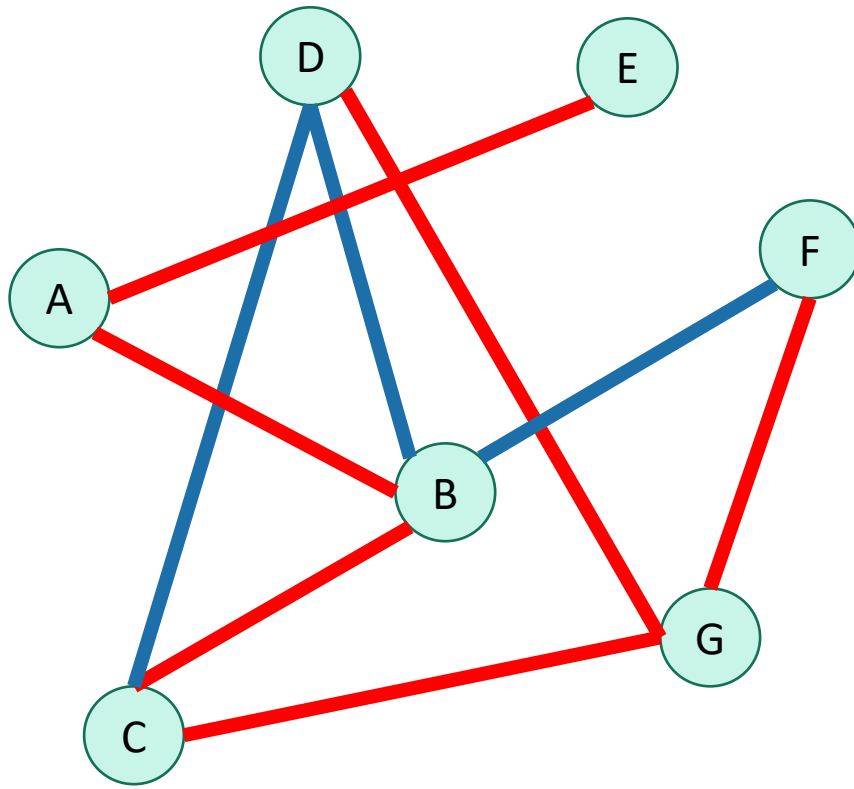
# To explore the whole graph

- Do it repeatedly!



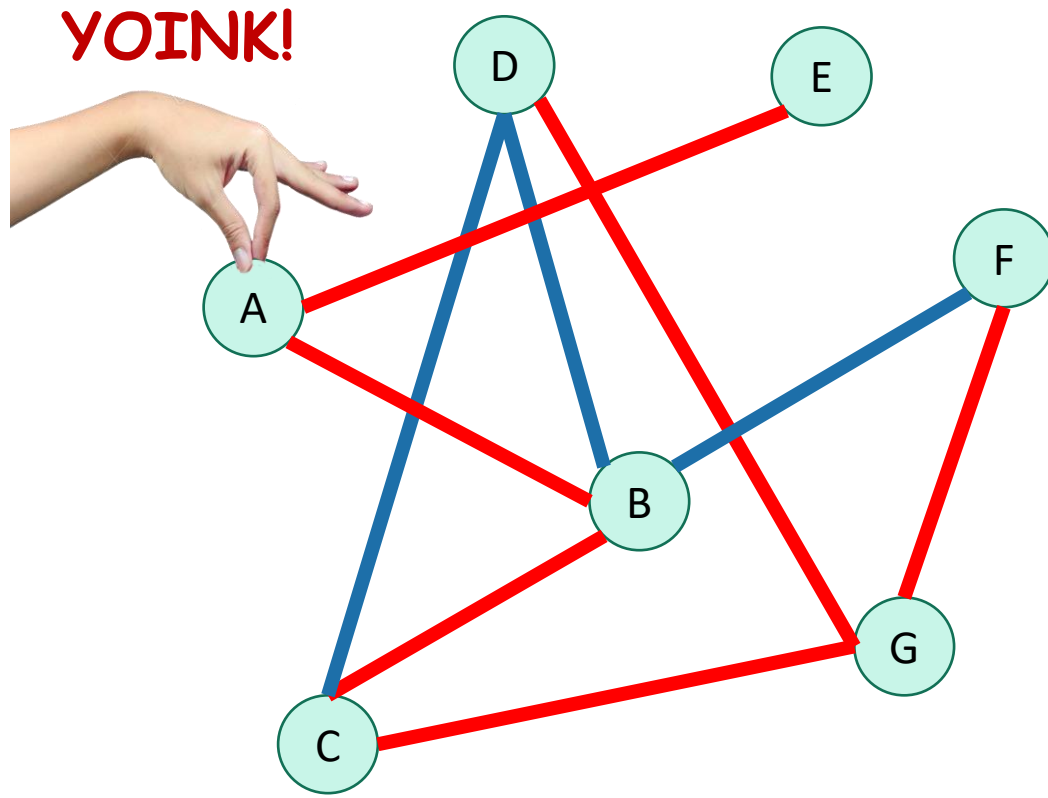
# Why is it called depth-first?

- We are implicitly building a tree:



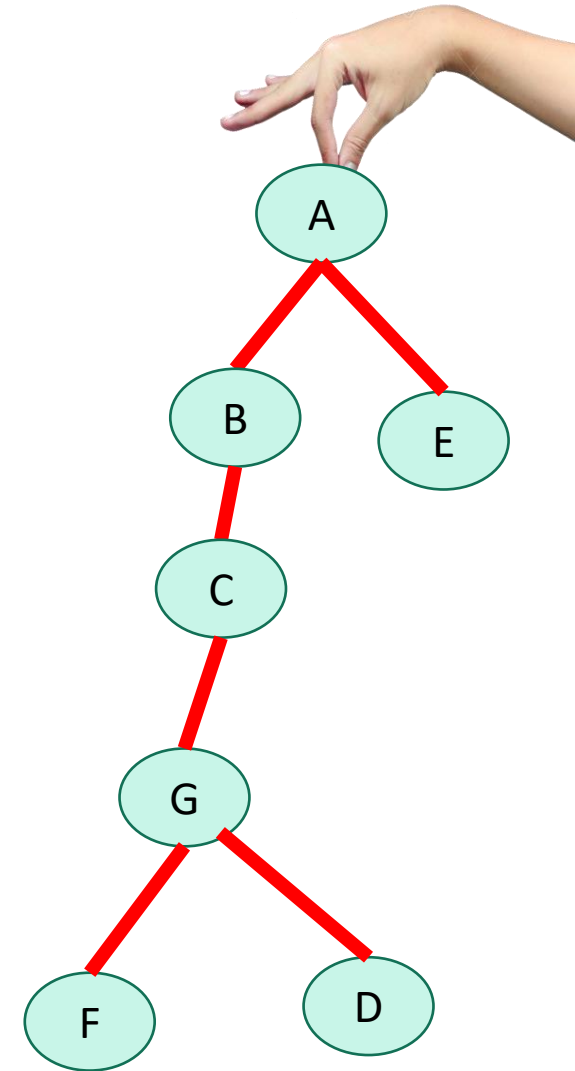
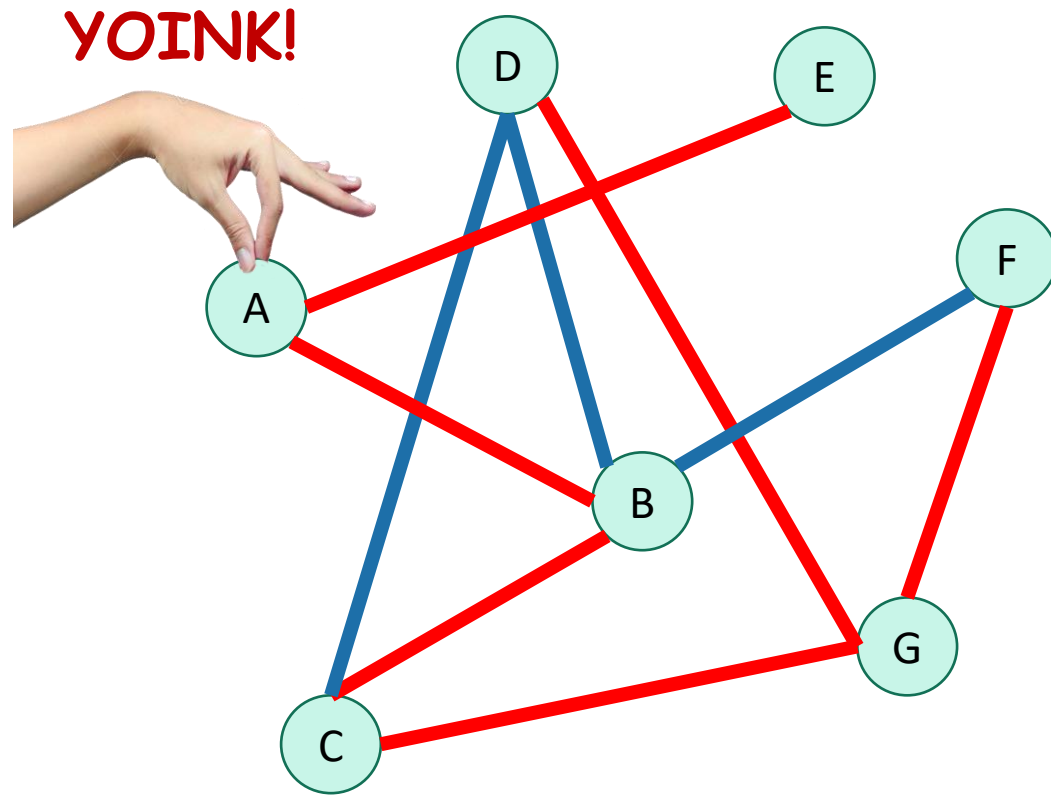
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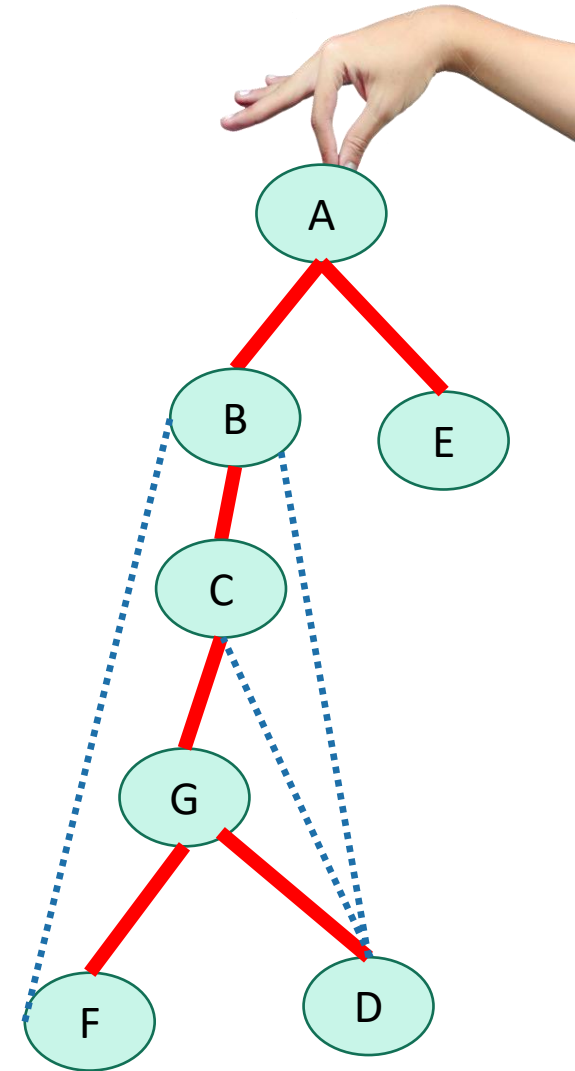
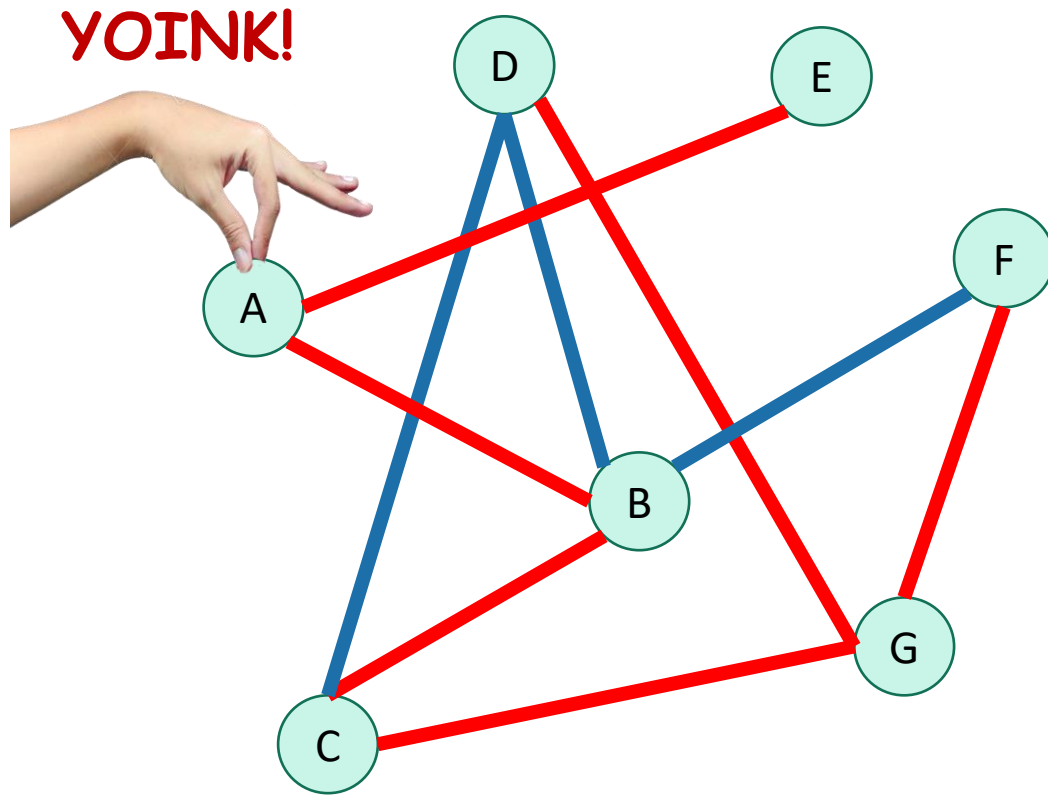
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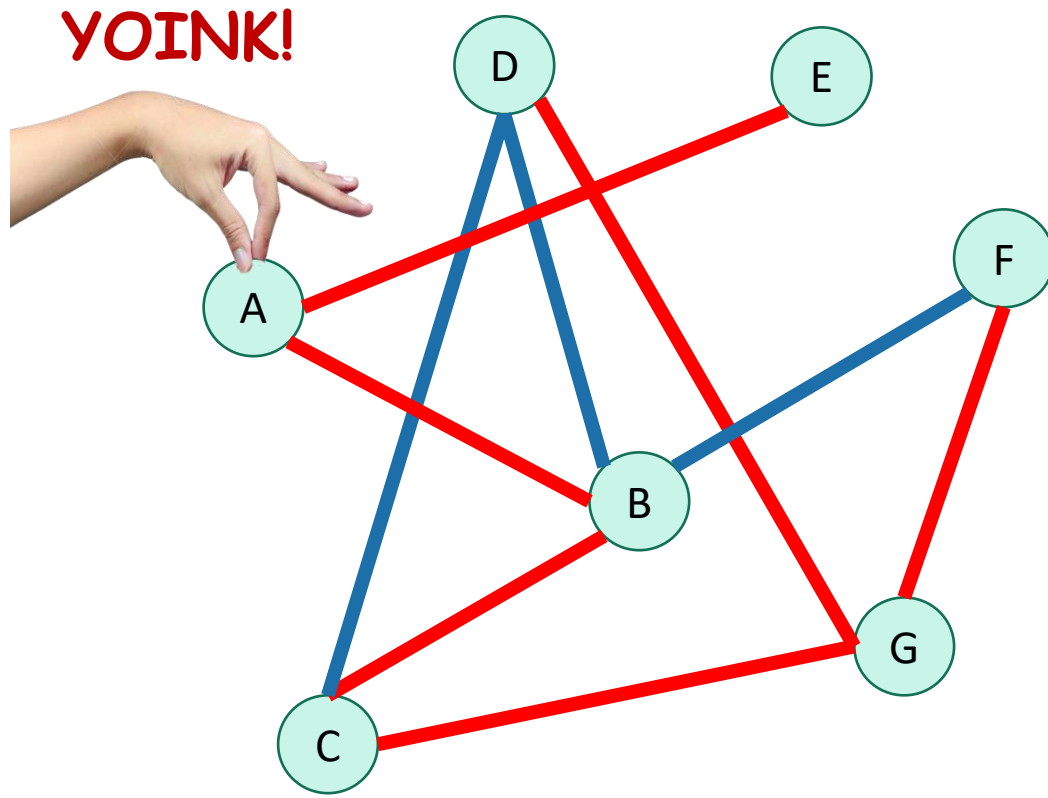
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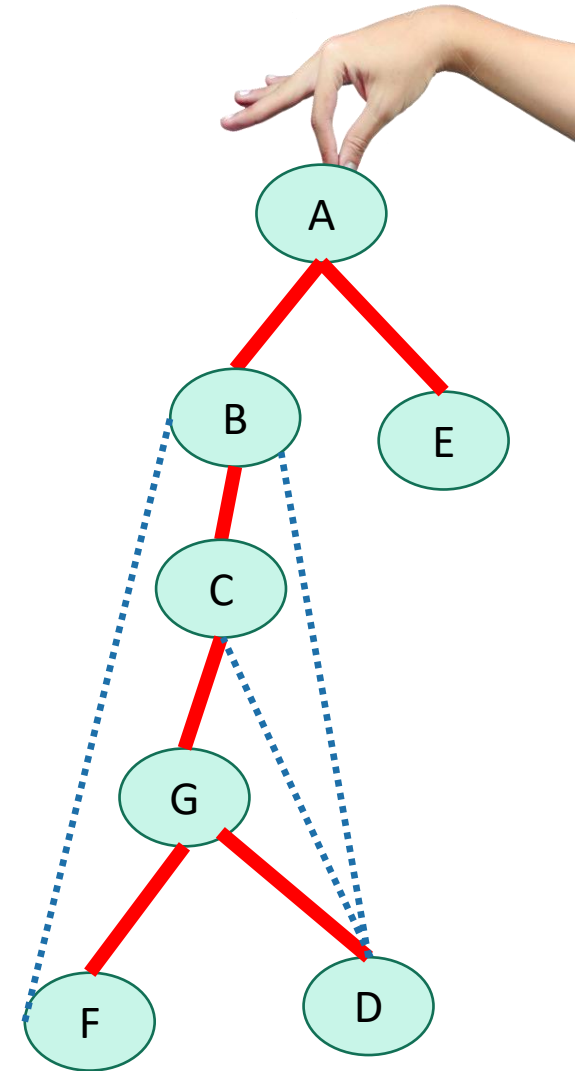


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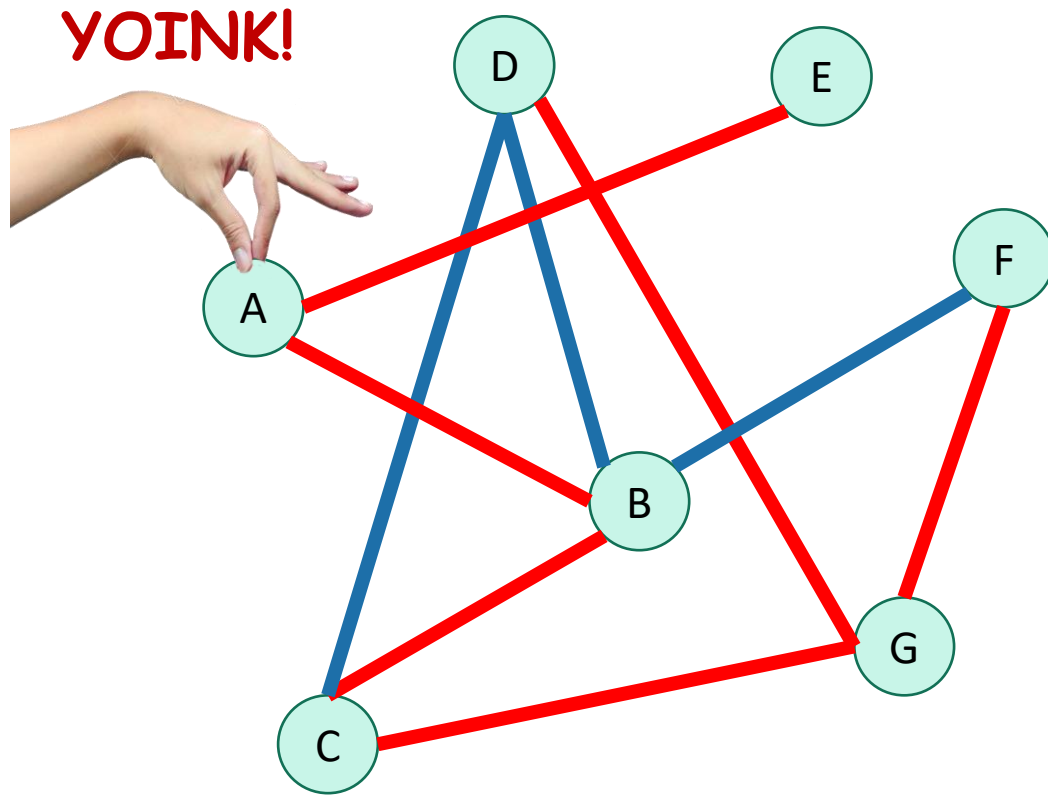


- First, we go as deep as we can.

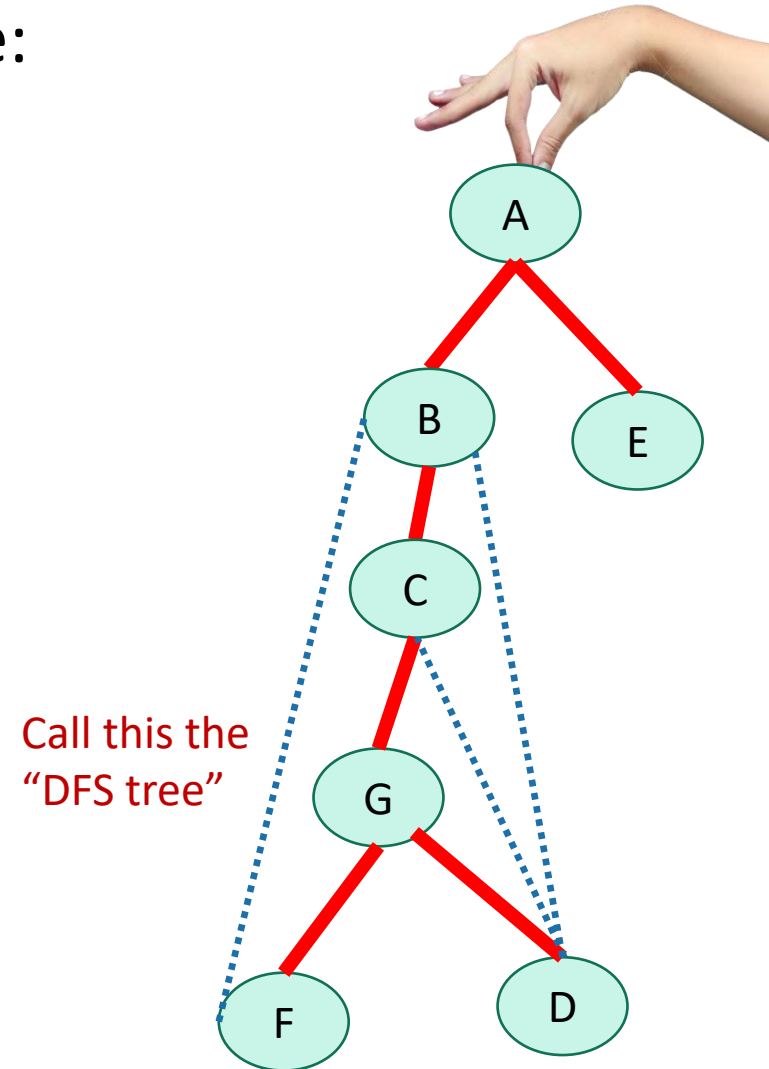


# Why is it called depth-first?

- We are implicitly building a tree:



- First, we go as deep as we can.



# Running time

To explore **just the connected component** we started in

- We look at each edge at most twice.
  - Once from each of its endpoints
- And basically we don't do anything else.
- So...



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$O(m)$

# Running time

To explore just the connected component we started in

- Assume we are using the linked-list format for  $G$ .
- Say  $C = (V', E')$  is a connected component.
- We visit each vertex in  $C$  exactly once.
  - Here, “visit” means “call DFS on”



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  - Do some book-keeping:  $O(1)$
  - Loop over  $w$ 's neighbors and check if they are visited (and then potentially make a recursive call):  $O(1)$  per neighbor or  $O(\deg(w))$  total.



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- Total time:
  - $\sum_{w \in V'} (O(\deg(w)) + O(1))$
  - $= O(|E'| + |V'|)$
  - $= O(|E'|)$



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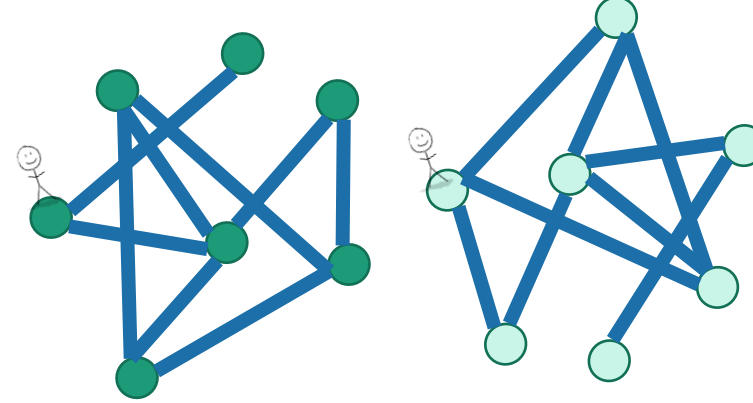
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  - $= O(|E'| + |V'|)$
  - $= O(|E'|)$  ←



In a connected graph,  
 $|V'| \leq |E'| + 1$ .

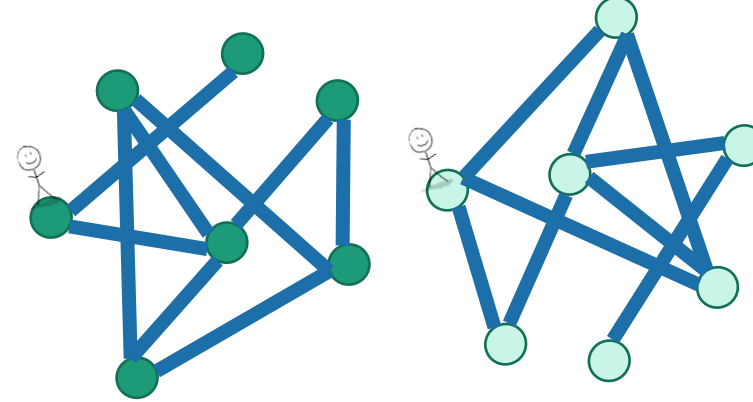
# Running time

To explore **the whole graph**



# Running time

To explore **the whole graph**

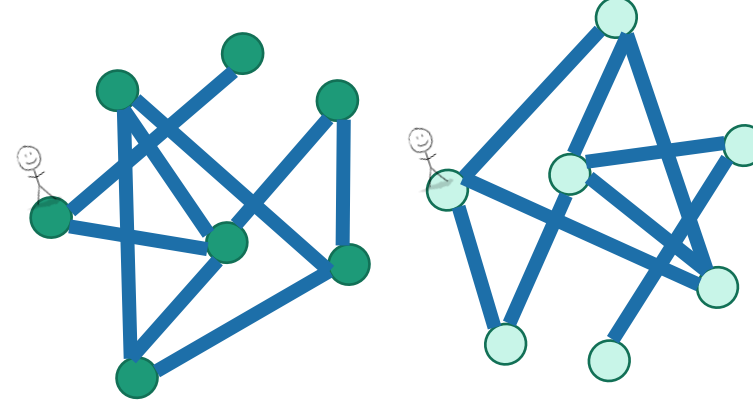


- Explore the connected components one-by-one.
- This takes time  $O(n + m)$ 
  - Same computation as before:

$$\sum_{w \in V} (O(\deg(w)) + O(1)) = O(|E| + |V|) = O(n + m)$$

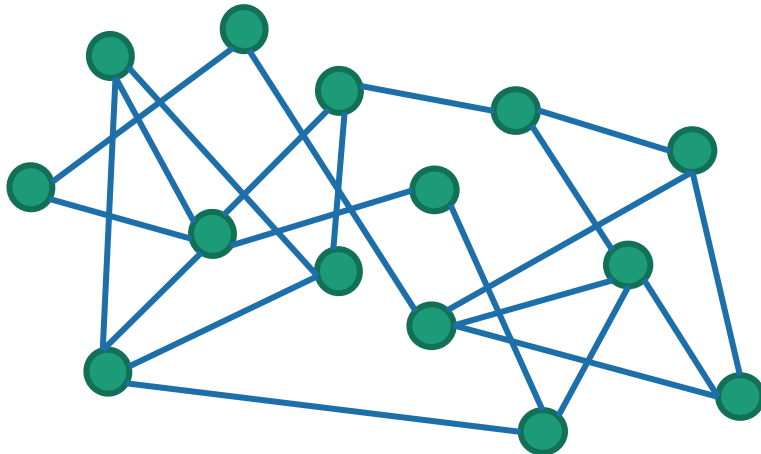
# Running time

To explore **the whole graph**



- Explore the connected components one-by-one.
- This takes time  $O(n + m)$ 
  - Same computation as before:

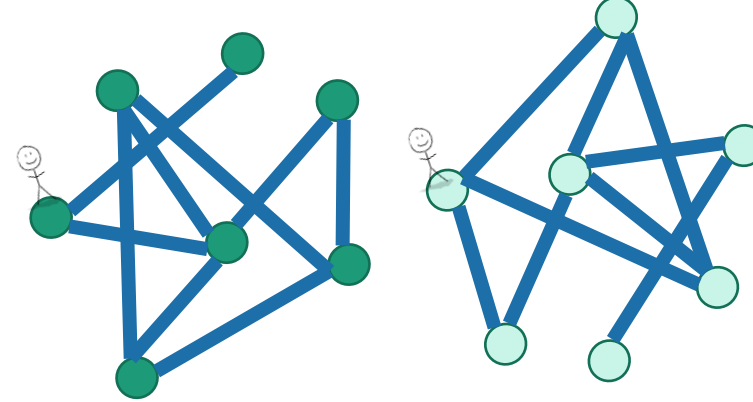
$$\sum_{w \in V} (O(\deg(w)) + O(1)) = O(|E| + |V|) = O(n + m)$$



Here the running time is  $O(m)$  like before

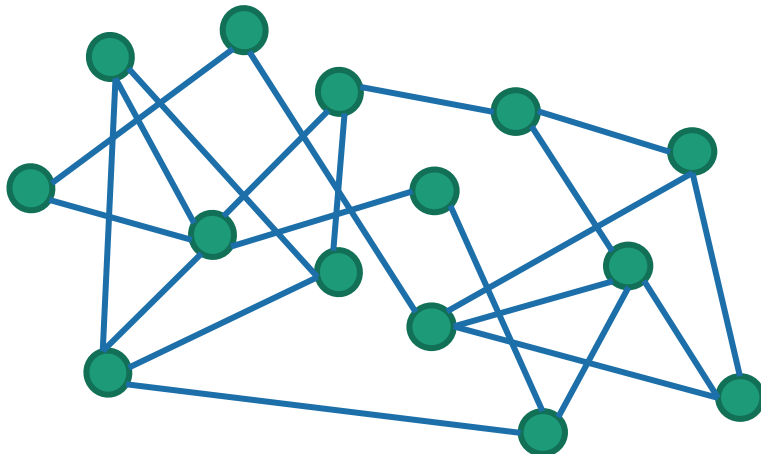
# Running time

To explore **the whole graph**



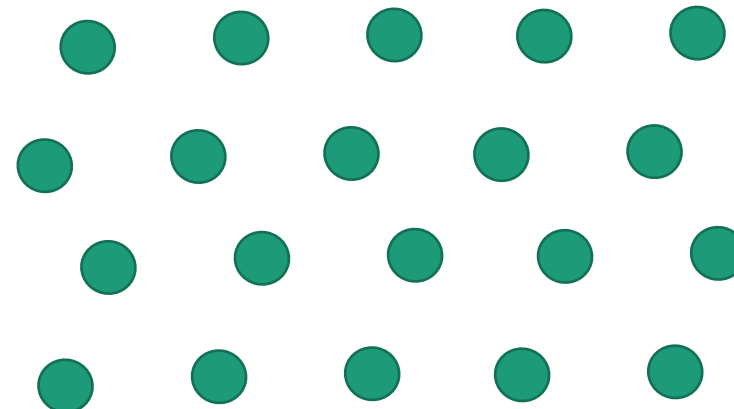
- Explore the connected components one-by-one.
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Here the running time is  $O(m)$  like before

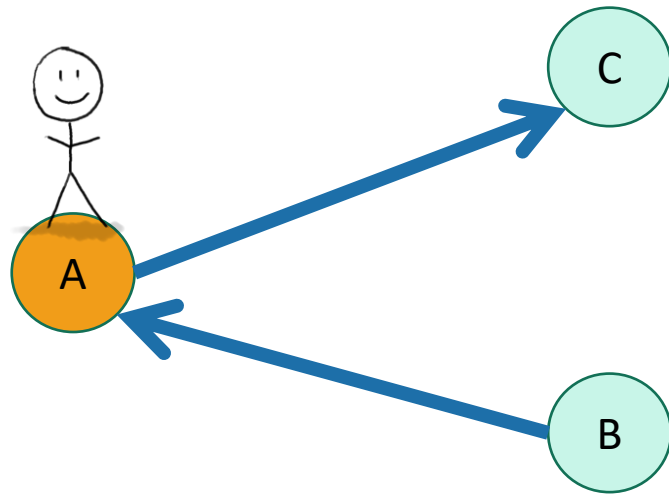
or



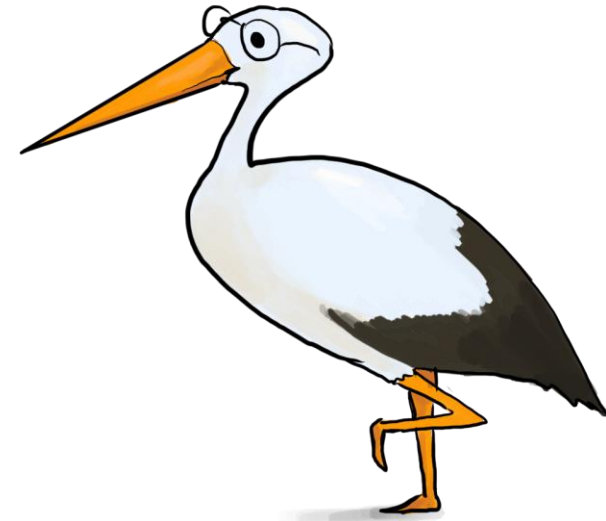
Here  $m=0$  but it still takes time  $O(n)$  to explore the graph.

# You check:

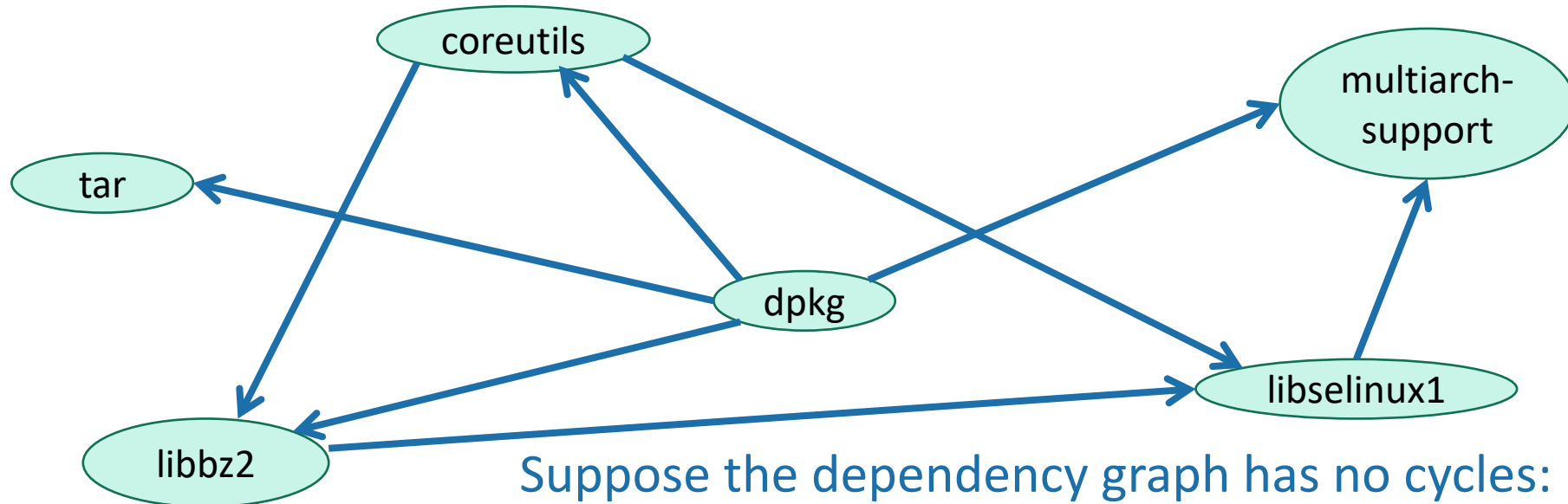
DFS works fine on directed graphs too!



Only walk to C, not to B.



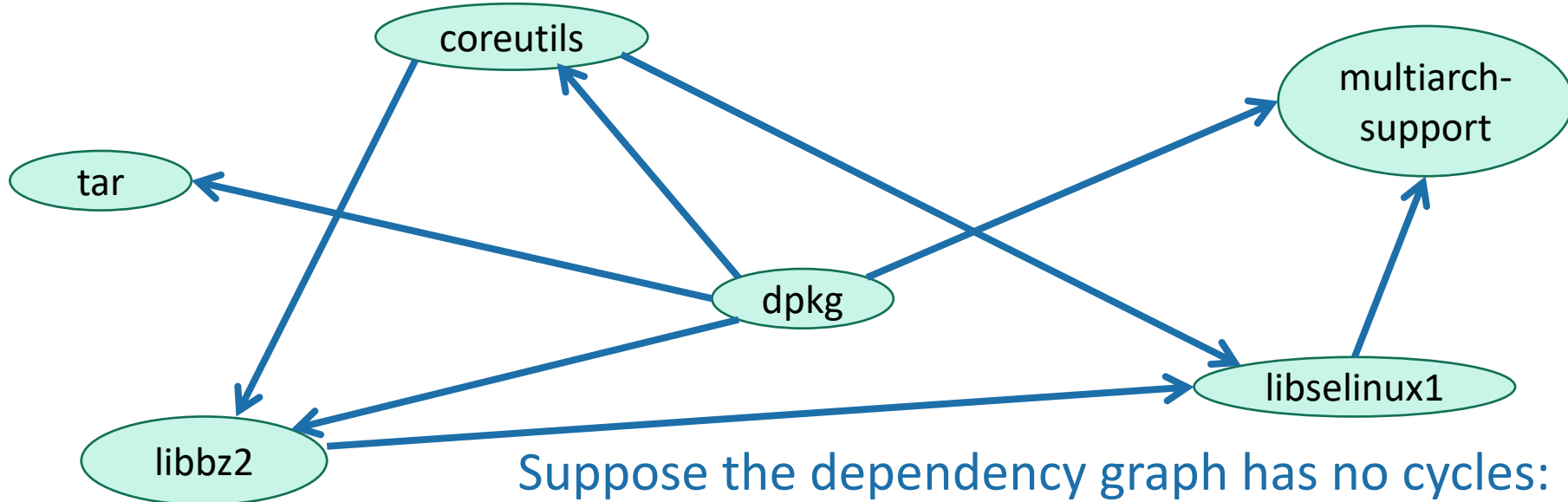
# Application of DFS: topological sorting



Suppose the dependency graph has no cycles:  
it is a **Directed Acyclic Graph (DAG)**

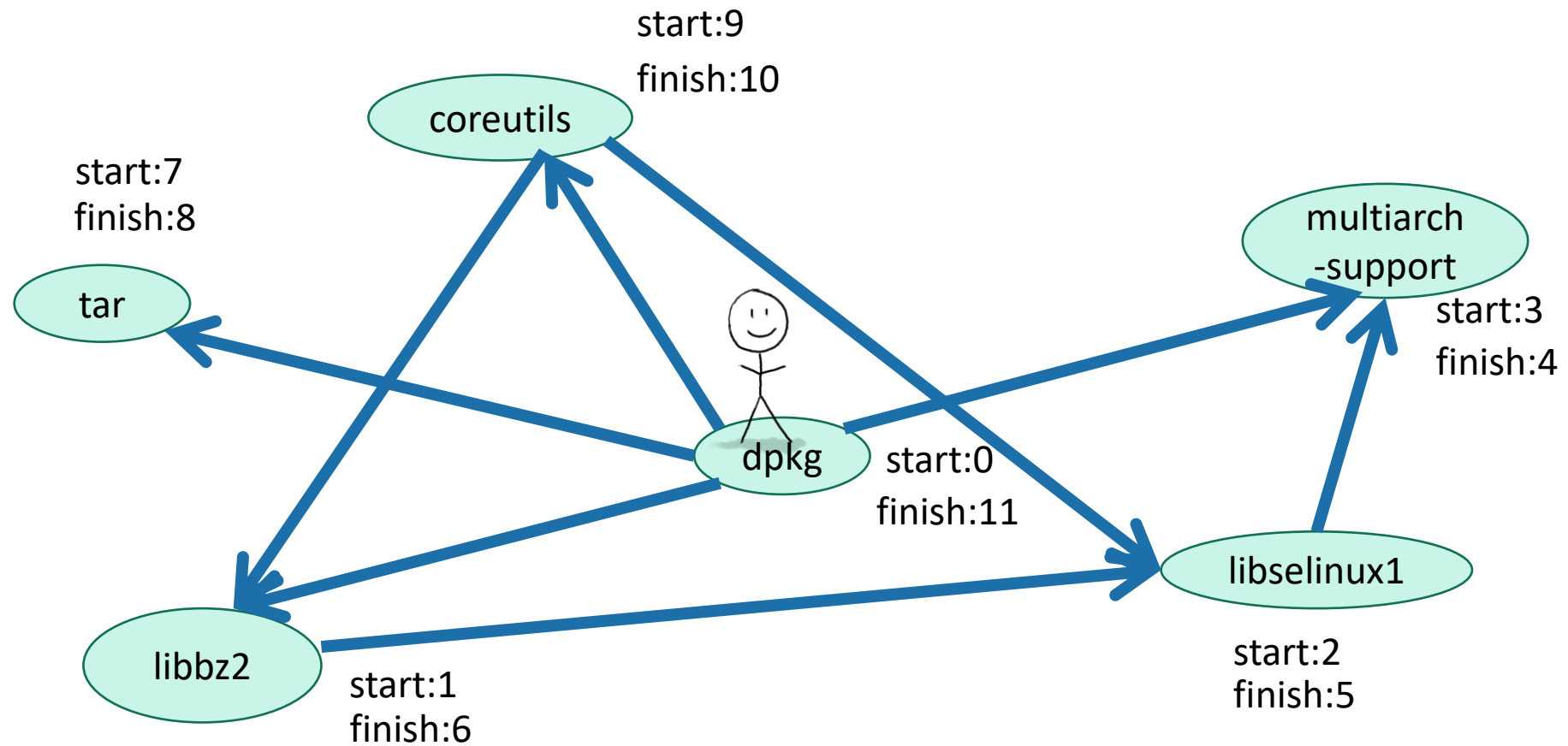
# Application of DFS: topological sorting

- Find an ordering of vertices so that all of the dependency requirements are met.
  - Aka, if  $v$  comes before  $w$  in the ordering, there is not an edge from  $w$  to  $v$ .



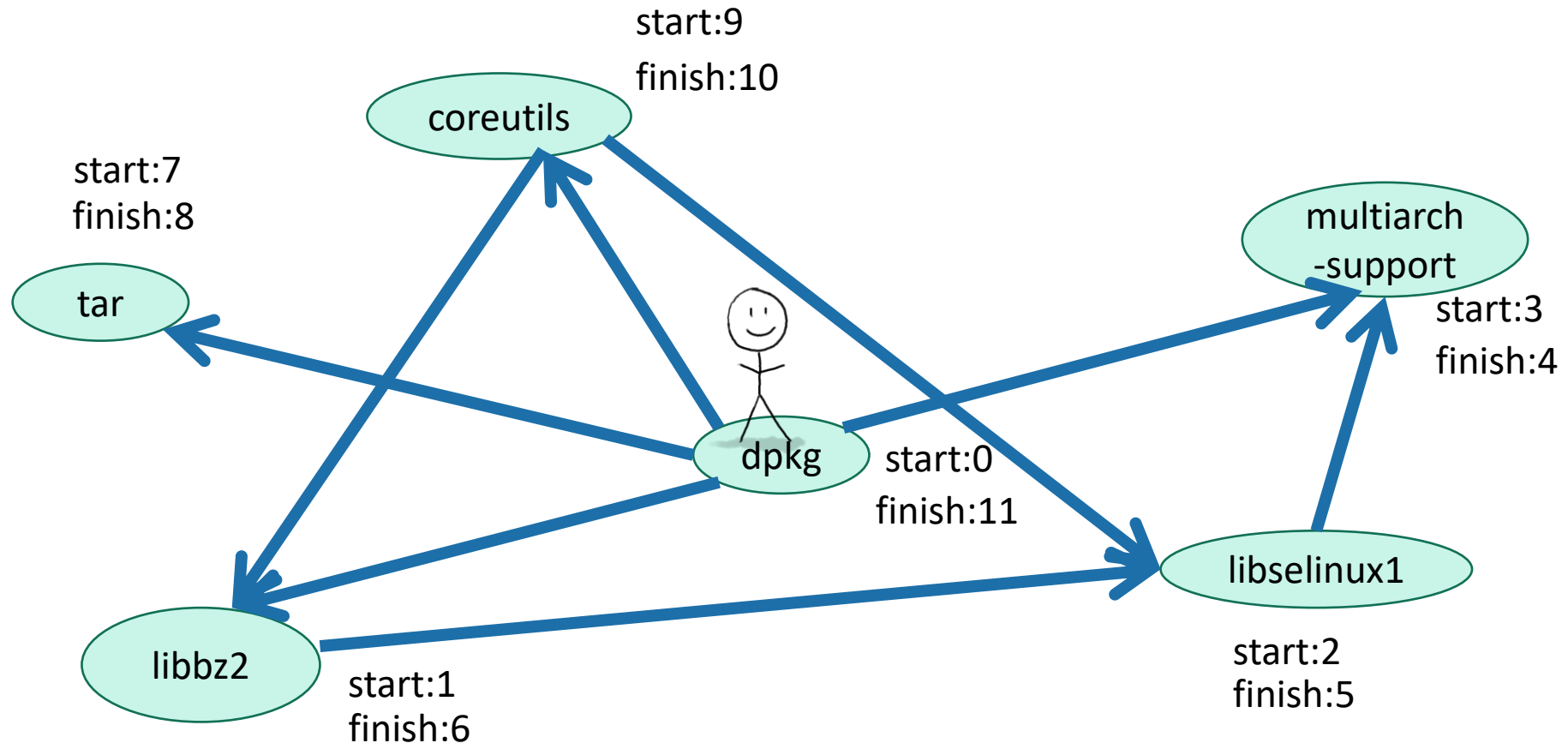
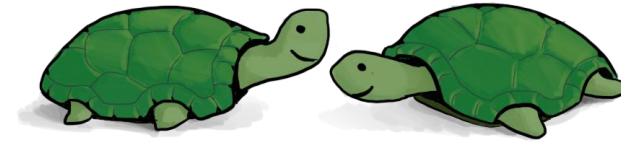
Suppose the dependency graph has no cycles:  
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# Let's do DFS



# Let's do DFS

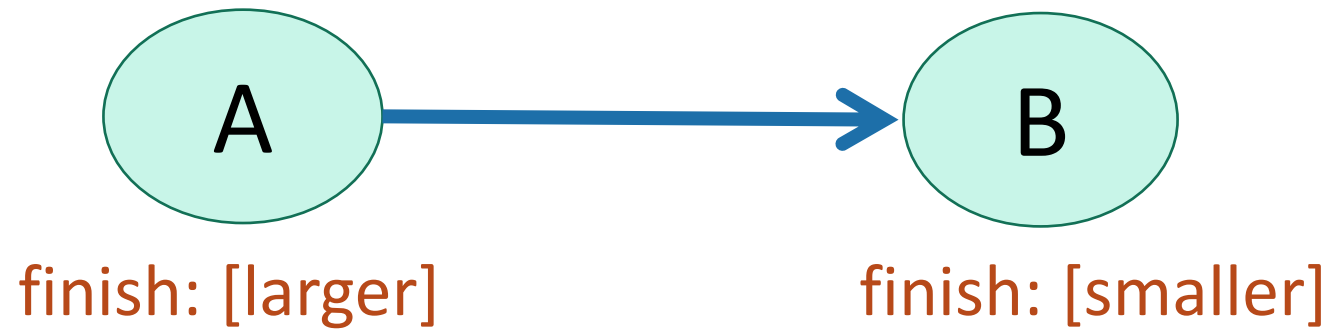
What do you notice about the finish times? Any ideas for how we should do topological sort?



Suppose the underlying  
graph has no cycles

# Finish times seem useful

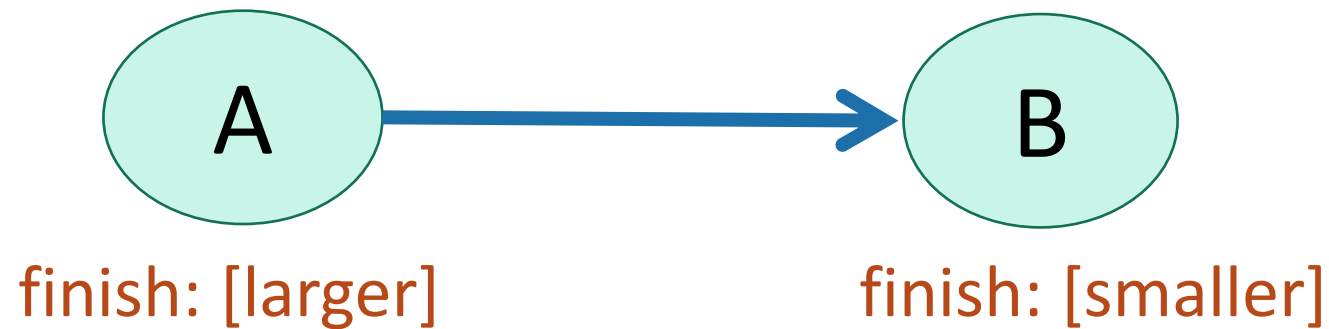
**Claim:** In general, we'll always have:



Suppose the underlying  
graph has no cycles

# Finish times seem useful

**Claim:** In general, we'll always have:



To understand why, let's go back to that DFS tree.



# A more general statement

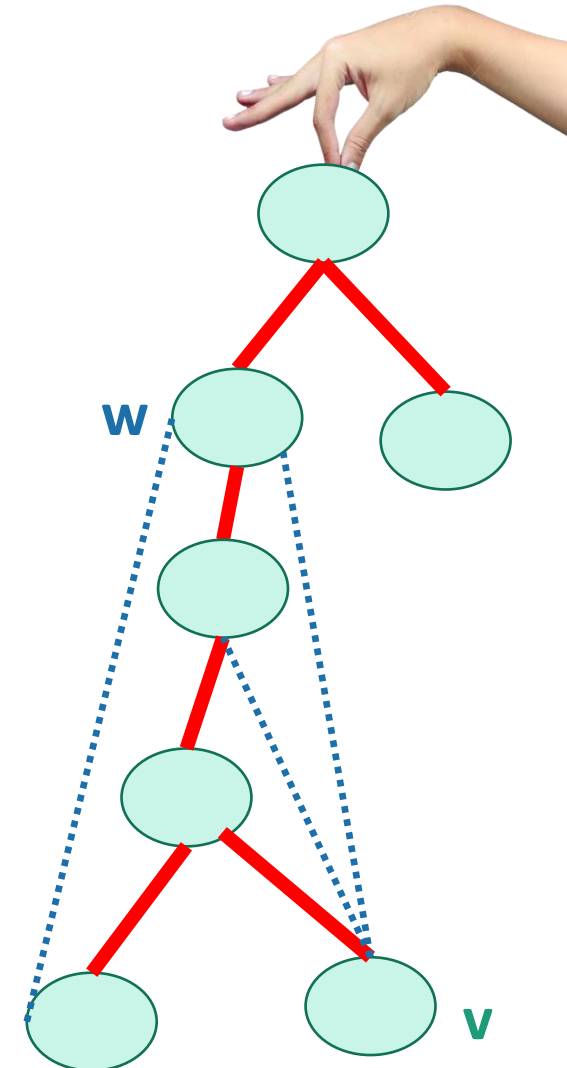
(this holds even if there are cycles)

This is called the “parentheses theorem”

- If  $v$  is a descendant of  $w$  in this tree:



(check this statement carefully!)



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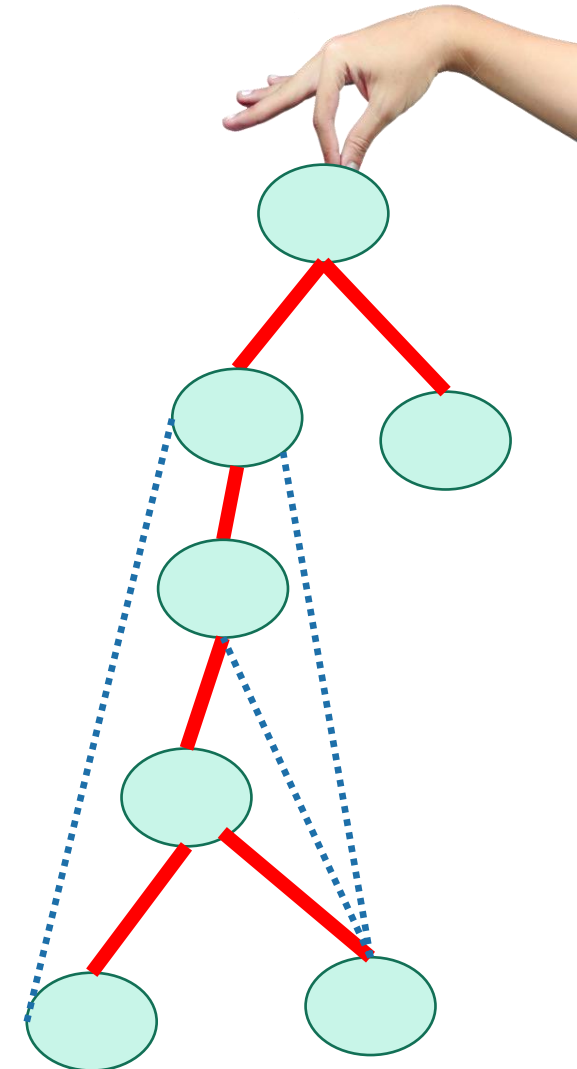
(check this statement carefully!)



- If  $v$  is a descendant of  $w$  in this tree:



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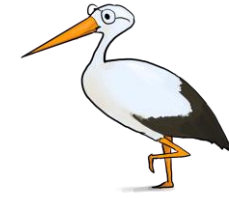


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- If  $v$  is a descendant of  $w$  in this tree:



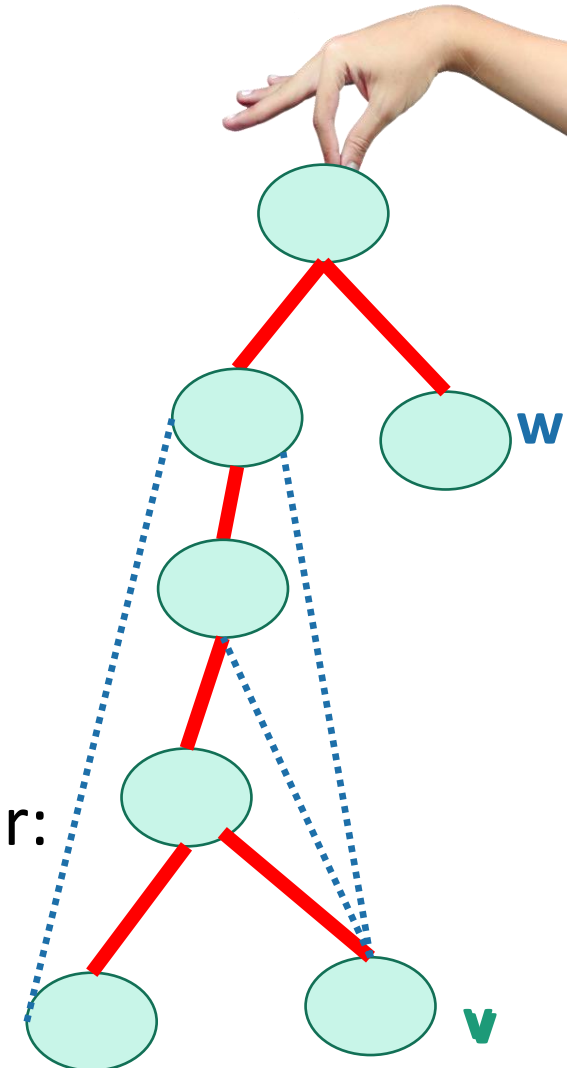
- If  $w$  is a descendant of  $v$  in this tree:



- If neither are descendants of each other:

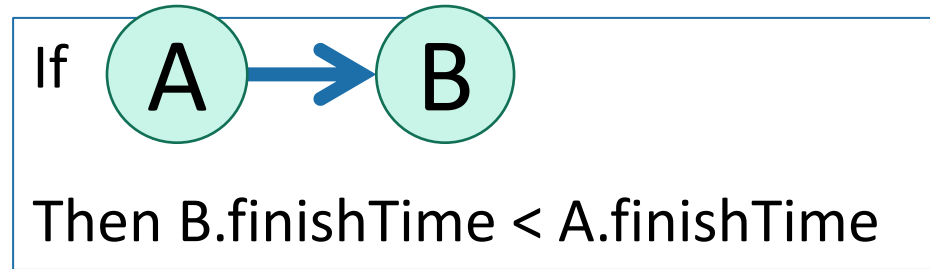


(or the other way around)

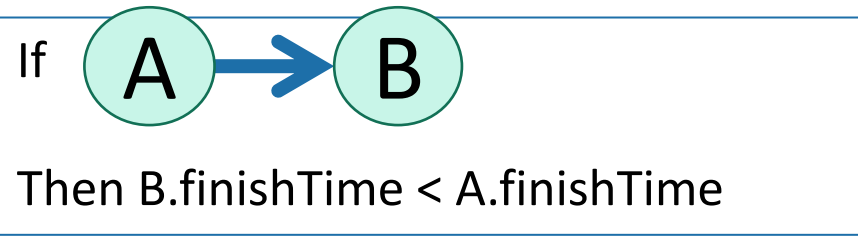


# Theorem

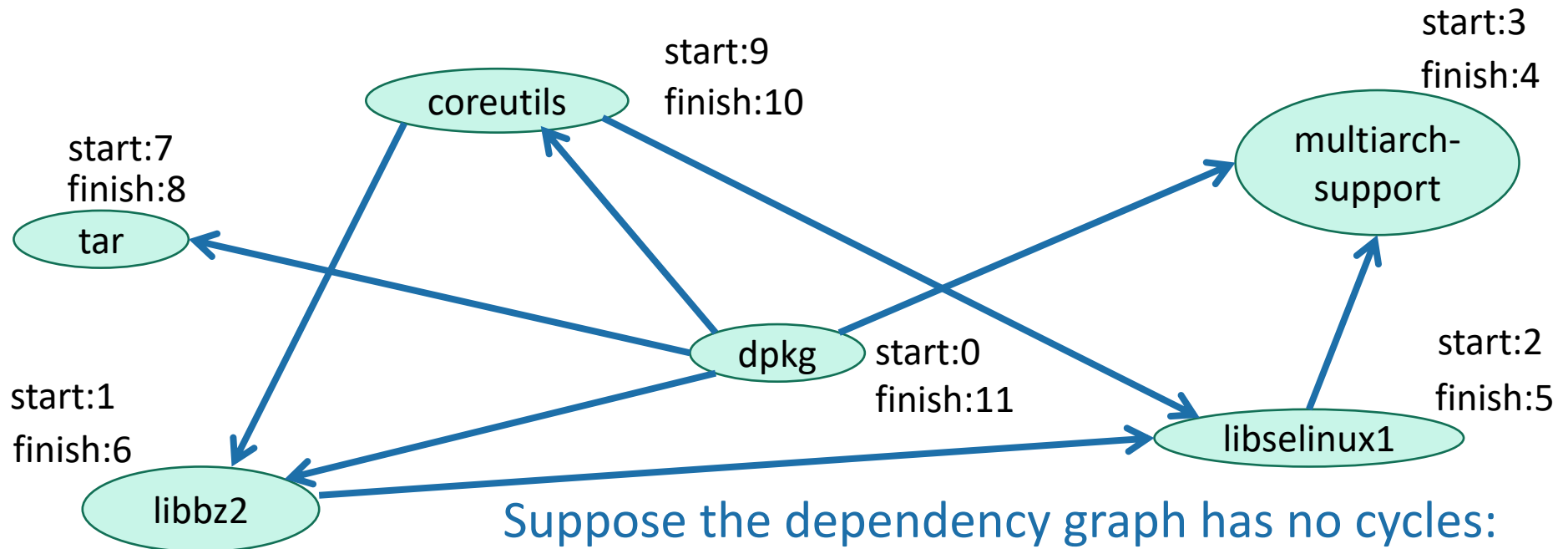
- If we run DFS on a directed acyclic graph,



# Back to topological sorting

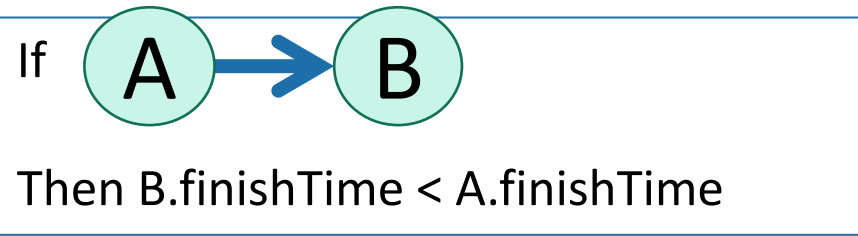


- In what order should I install packages?

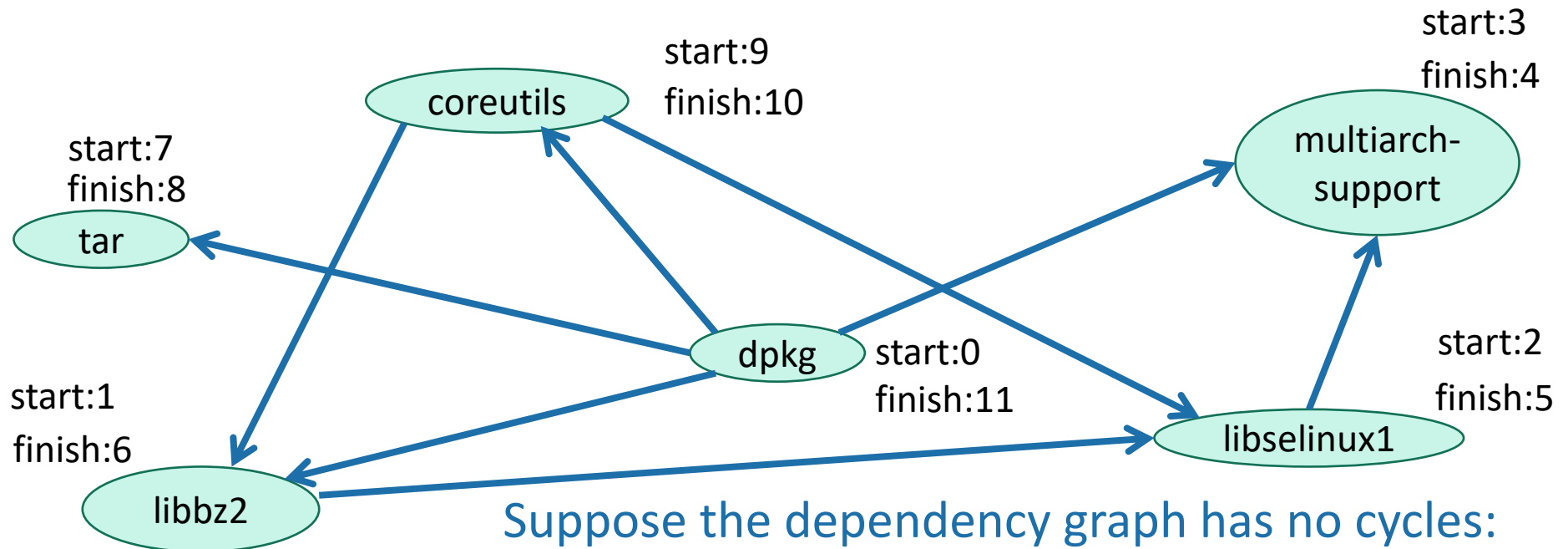


Suppose the dependency graph has no cycles:  
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# Back to topological sorting



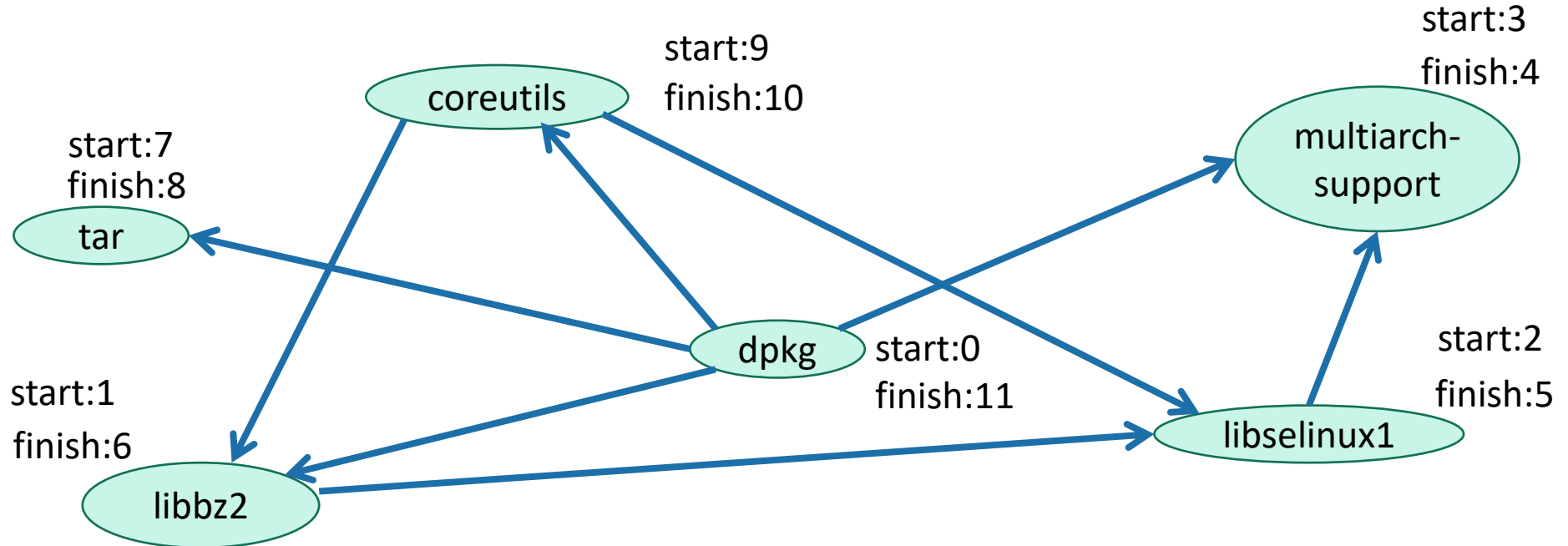
- In what order should I install packages?
- In reverse order of finishing time in DFS!



Suppose the dependency graph has no cycles:  
it is a **Directed Acyclic Graph (DAG)**

# Topological Sorting (on a DAG)

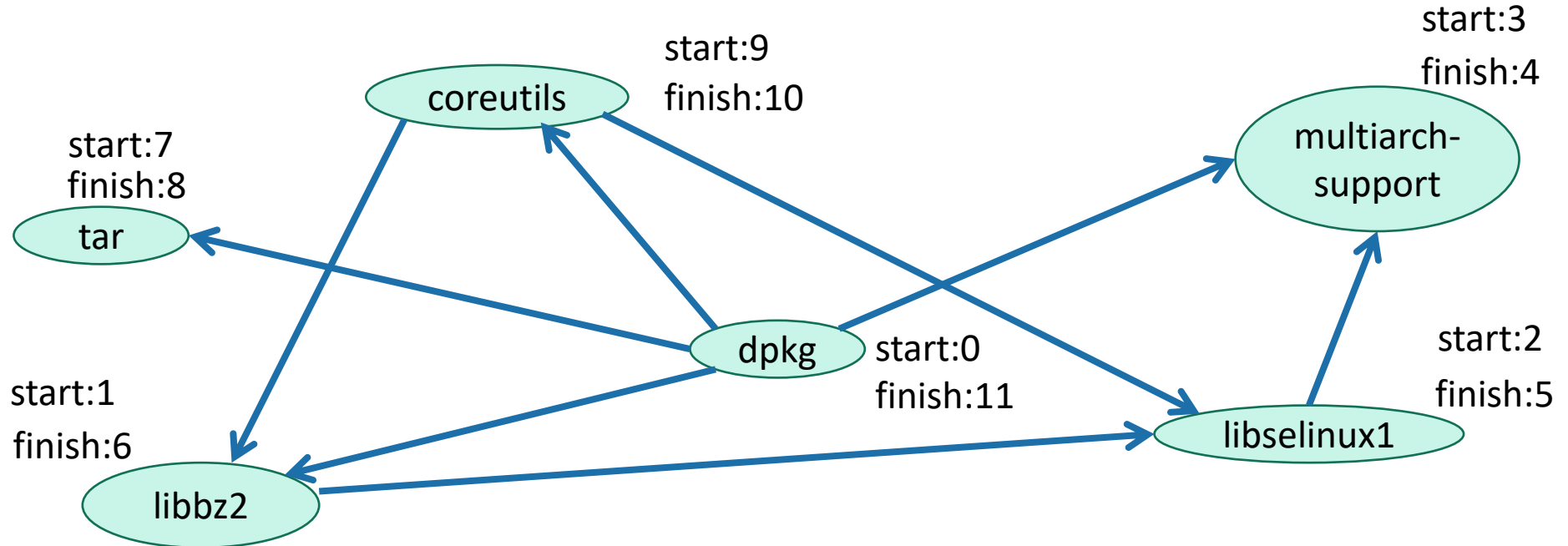
- Do DFS
- When you mark a vertex as **all done**, put it at the **beginning** of the list.



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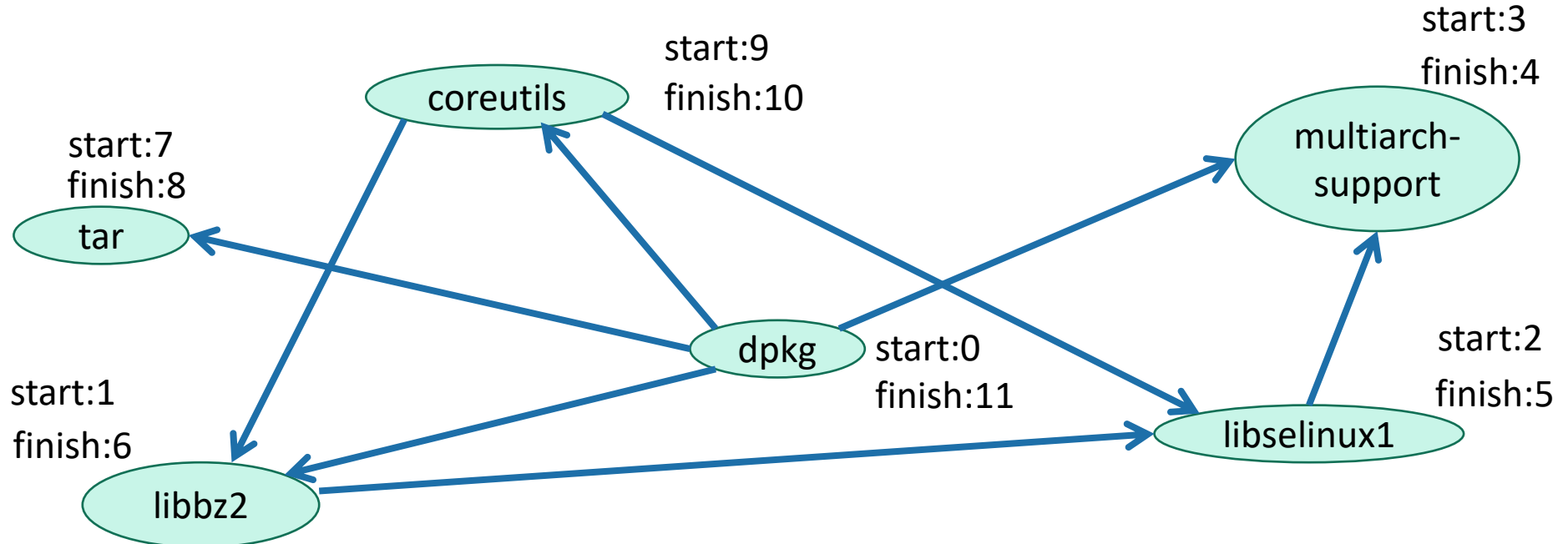
- `multiarch_support`



# Topological Sorting (on a DAG)

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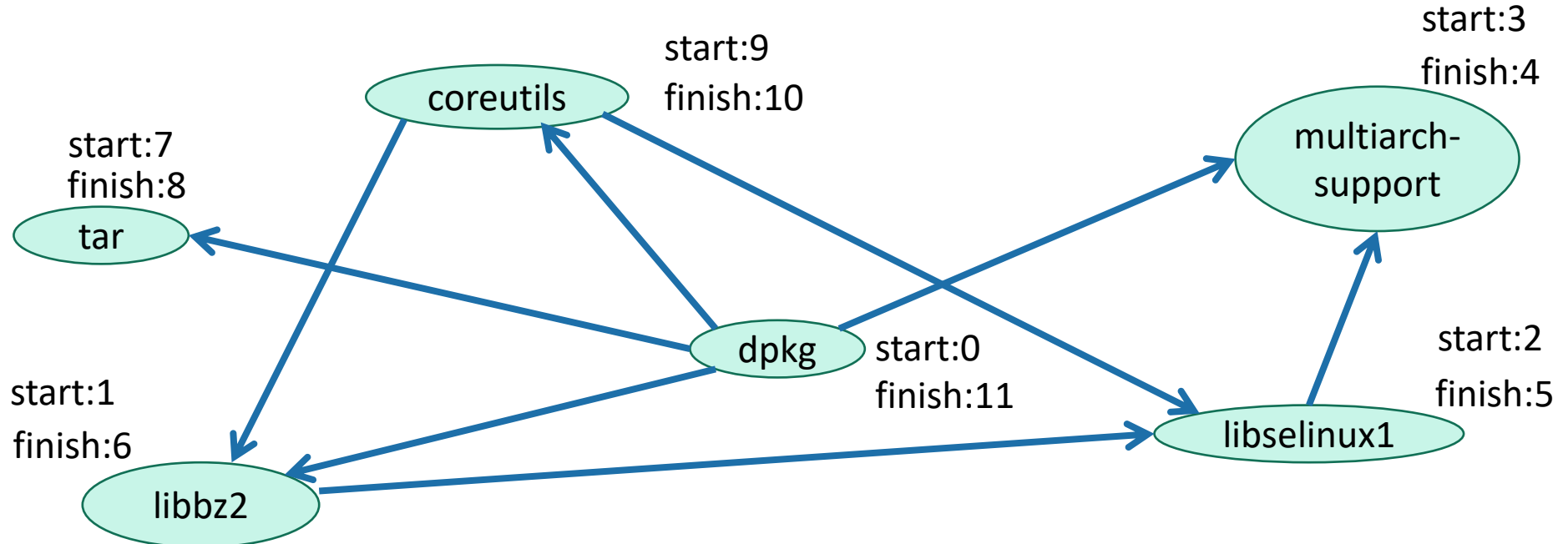
- libselinux1
- multiarch\_support



# Topological Sorting (on a DAG)

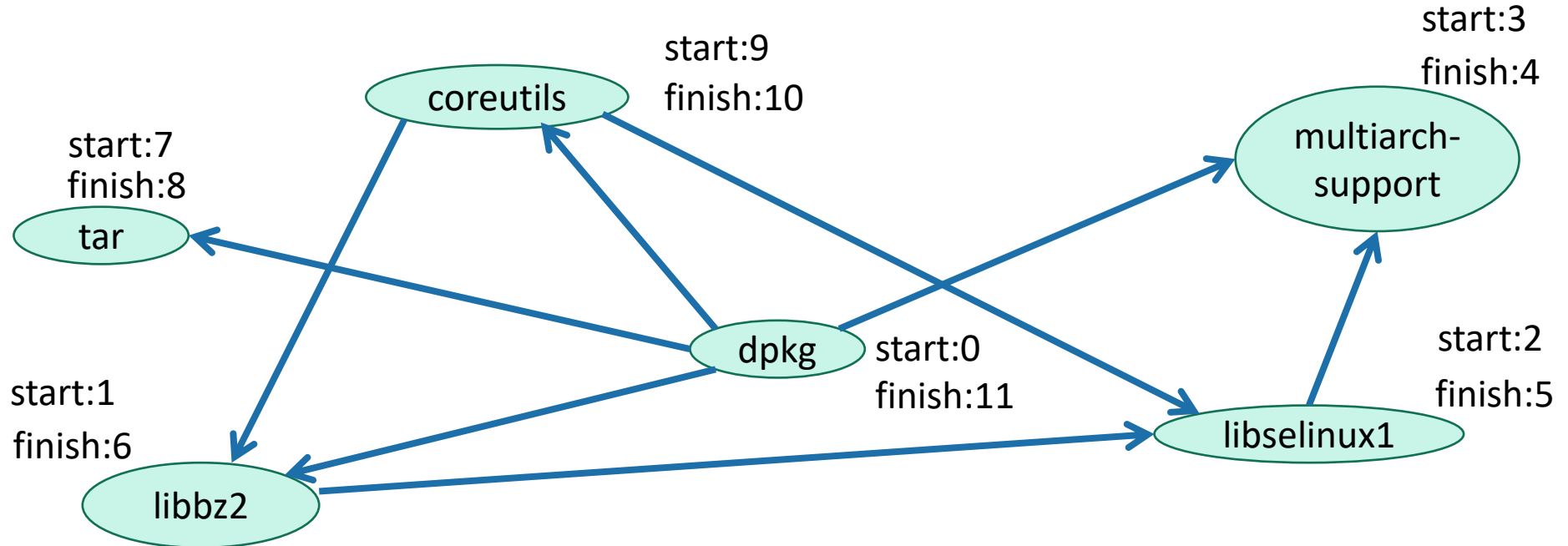
- Do DFS
- When you mark a vertex as **all done**, put it at the **beginning** of the list.

- libbz2
- libselinux1
- multiarch\_support



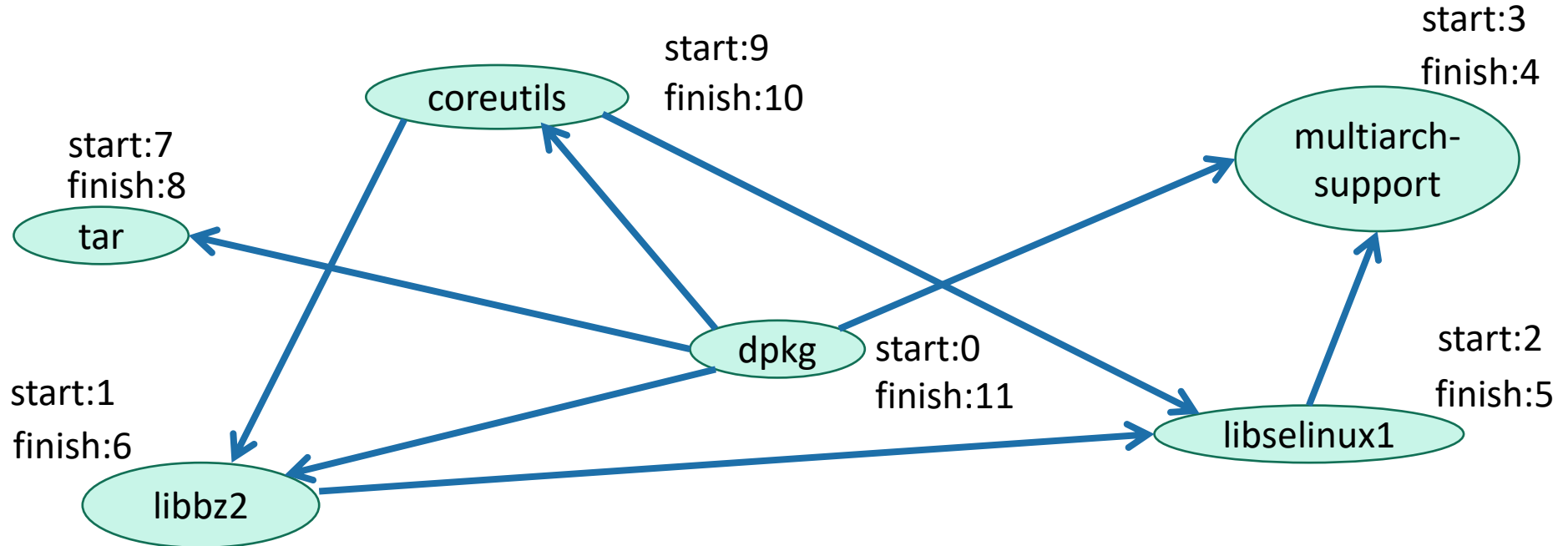
# Topological Sorting (on a DAG)

- Do DFS
  - When you mark a vertex as **all done**, put it at the **beginning** of the list.
- tar
  - libbz2
  - libselinux1
  - multiarch\_support



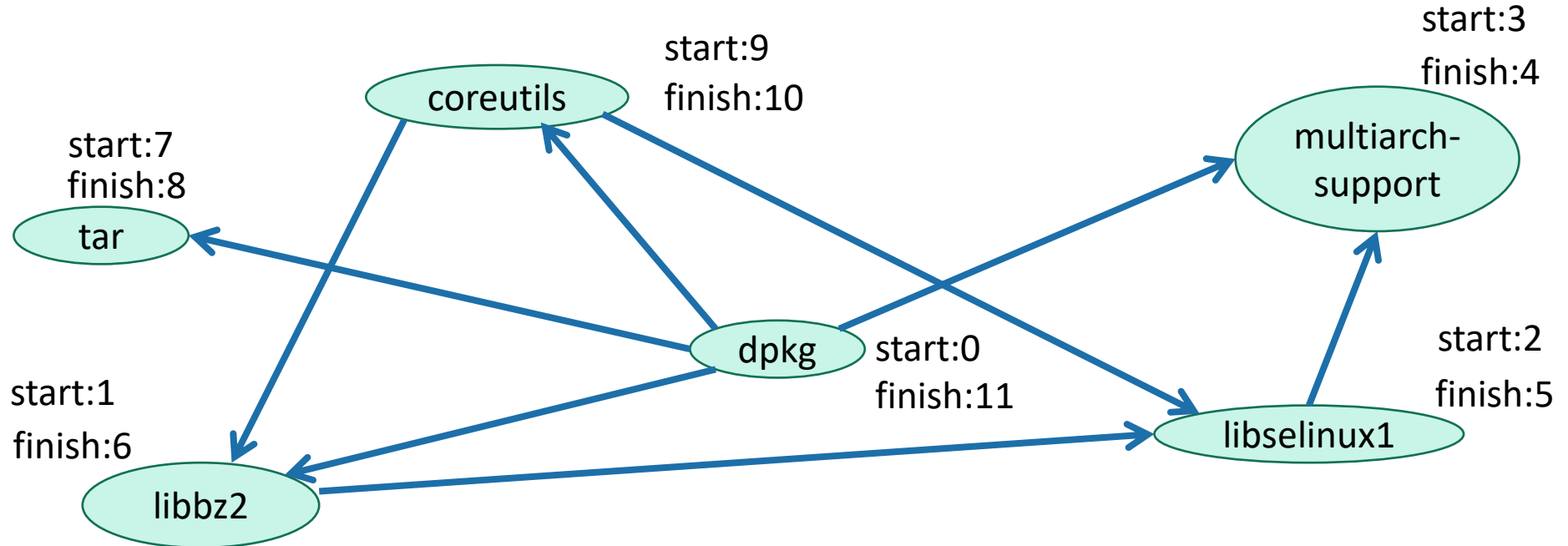
# Topological Sorting (on a DAG)

- Do DFS
  - When you mark a vertex as **all done**, put it at the **beginning** of the list.
- coreutils
  - tar
  - libbz2
  - libselinux1
  - multiarch\_support



# Topological Sorting (on a DAG)

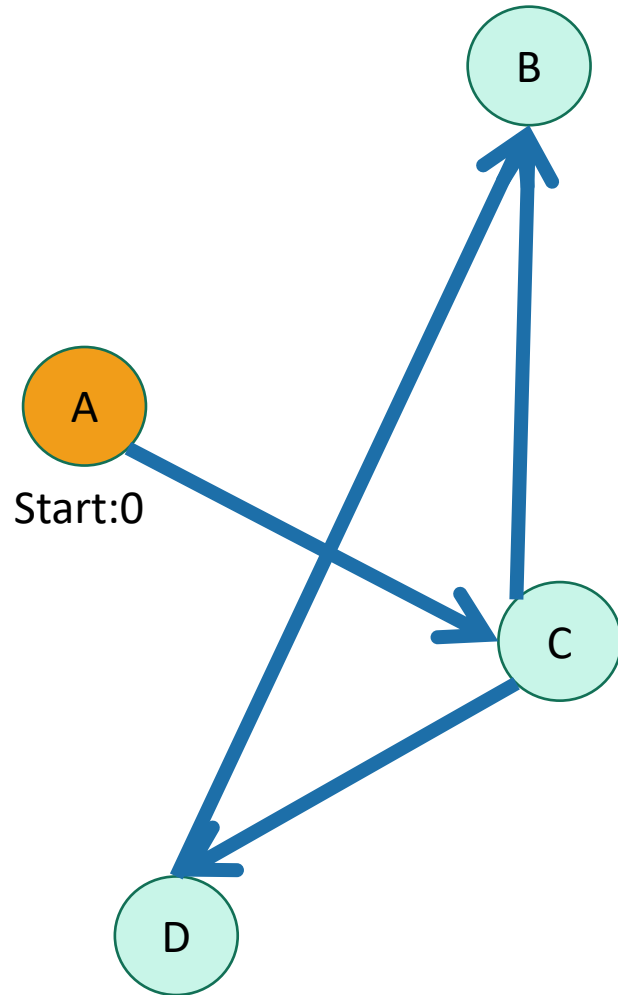
- Do DFS
  - When you mark a vertex as **all done**, put it at the **beginning** of the list.
- dpkg
  - coreutils
  - tar
  - libbz2
  - libselinux1
  - multiarch\_support



# What did we just learn?

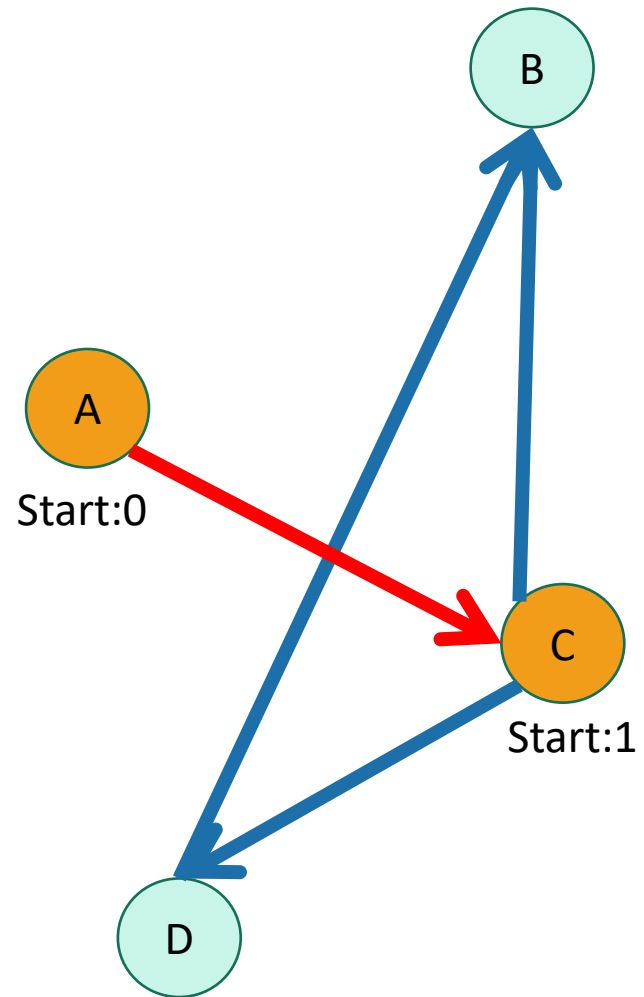
- DFS can help you solve the **topological sorting problem**
  - That's the fancy name for the problem of finding an ordering that respects all the dependencies
- Thinking about the DFS tree is helpful.

# Example:



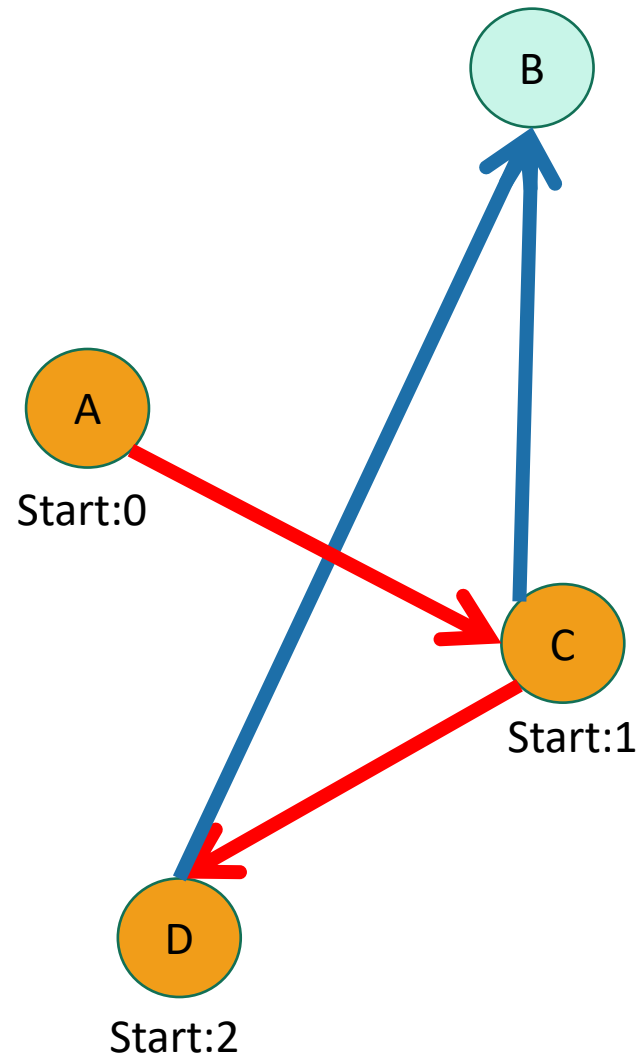
- Unvisited
- In progress
- All done

# Example



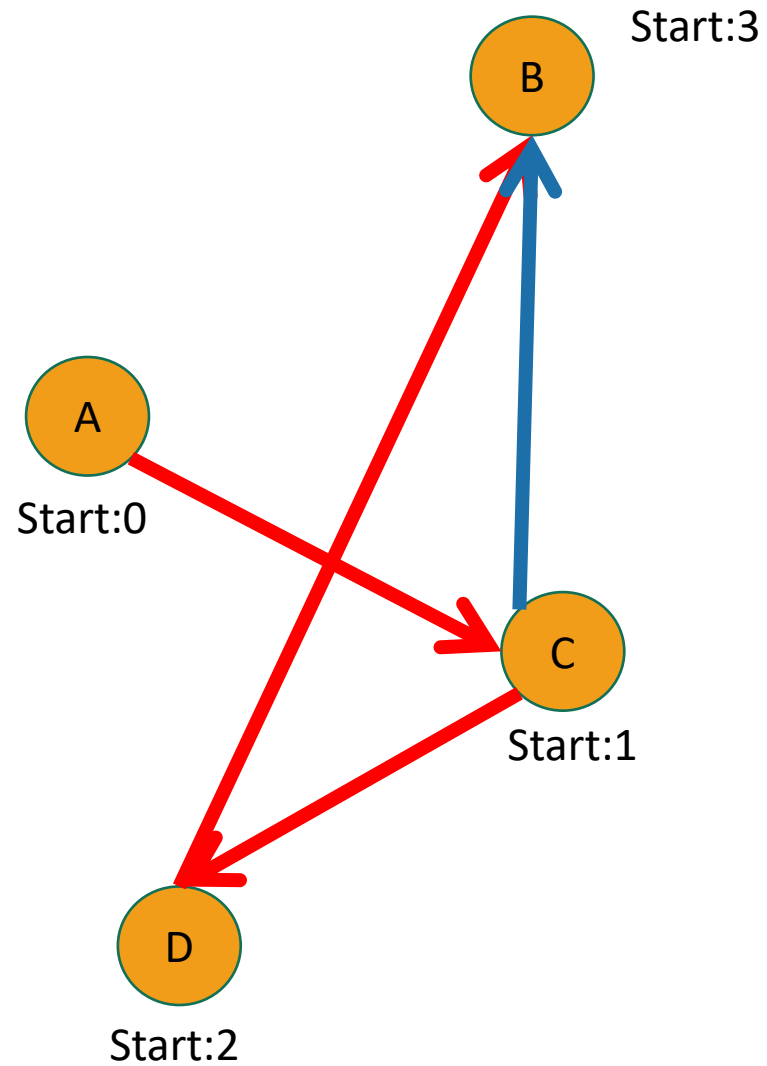
-  Unvisited
-  In progress
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# Example



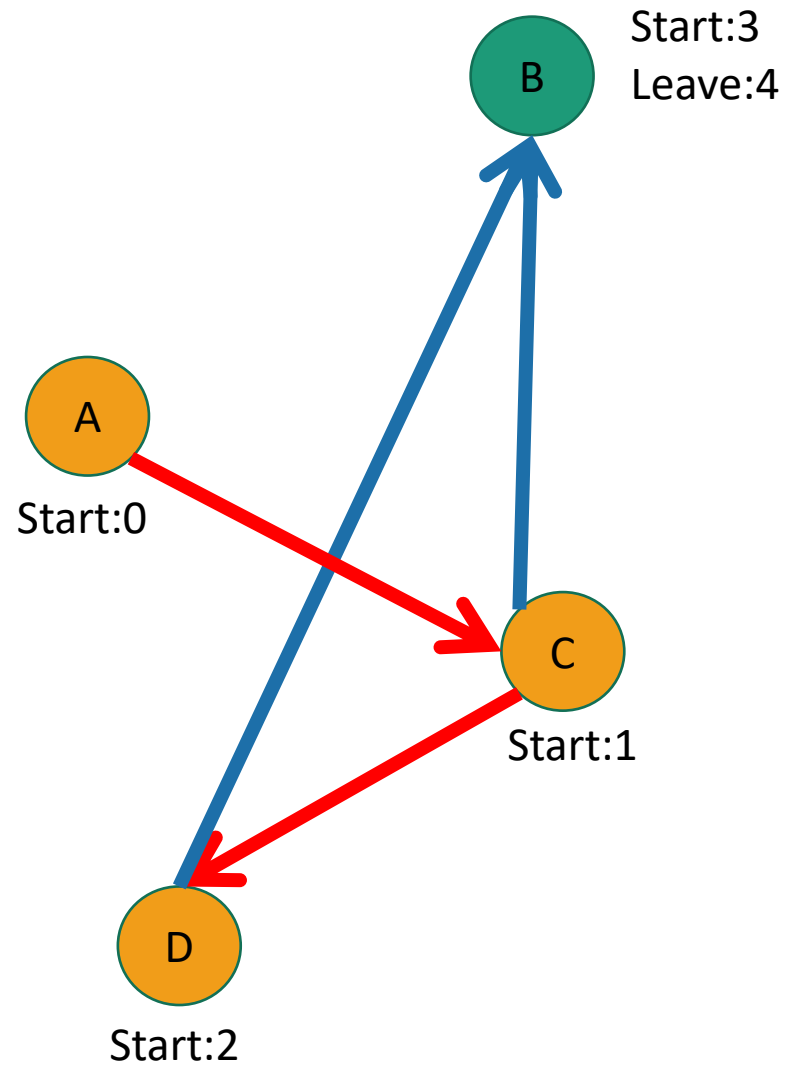
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# Example

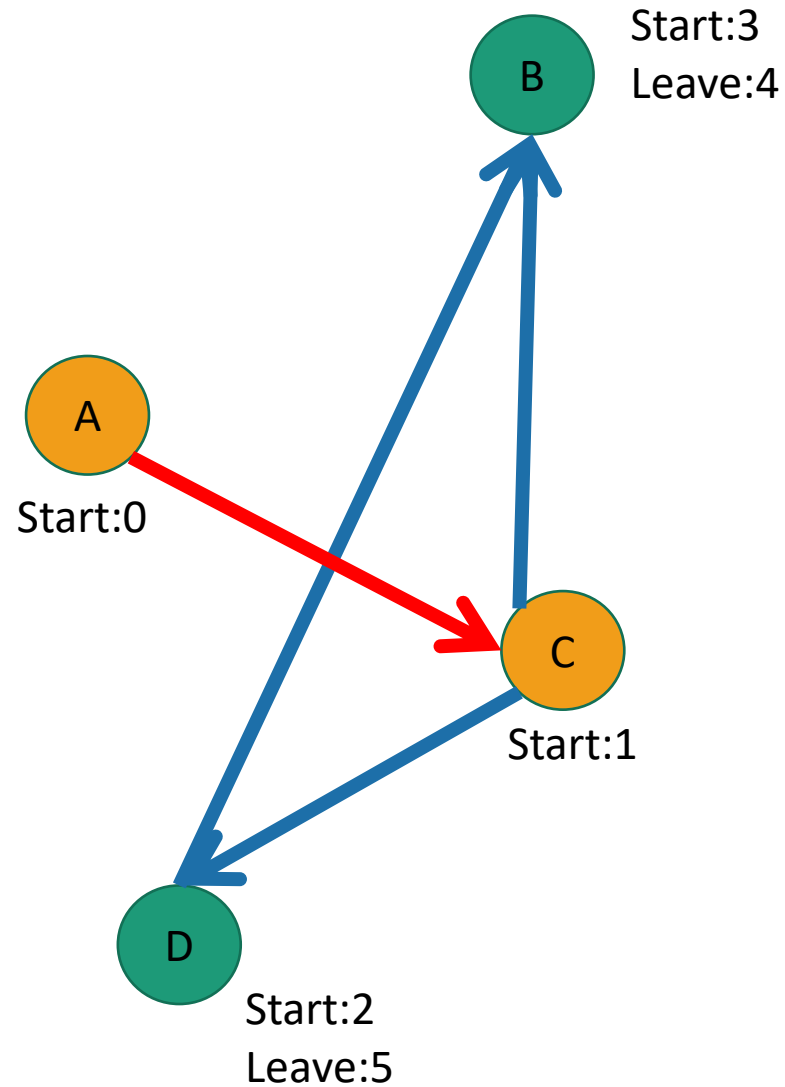


-  Unvisited
-  In progress
-  All done

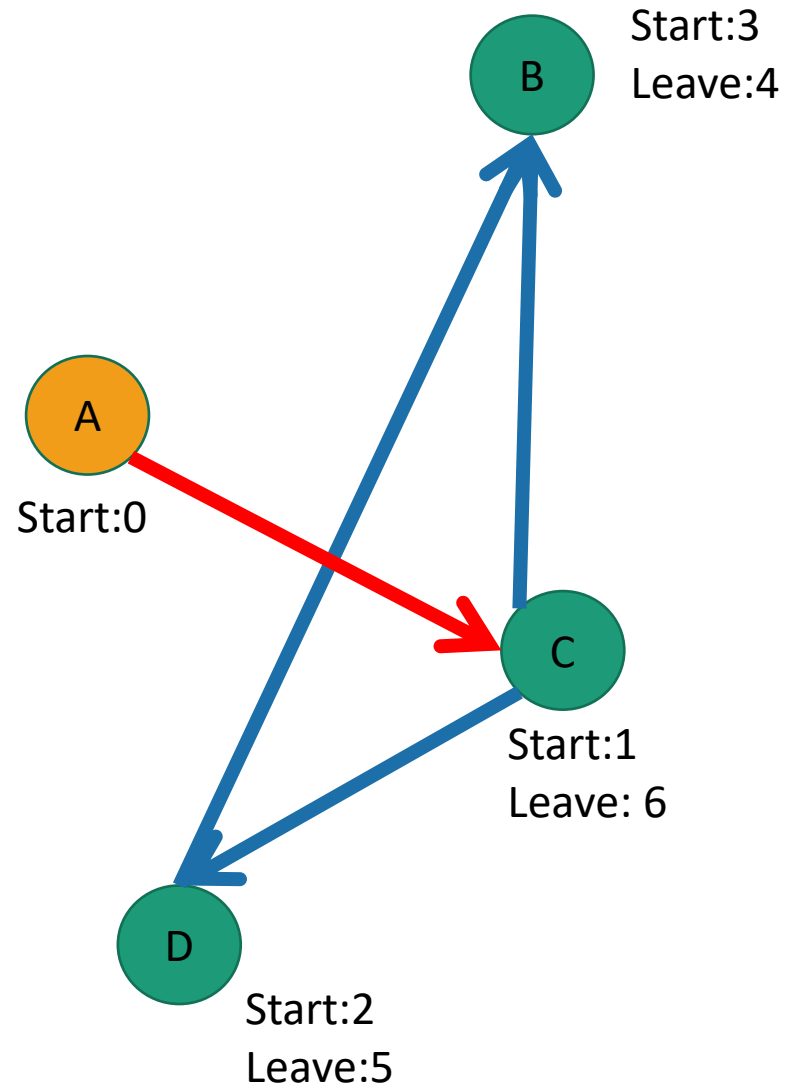
# Example



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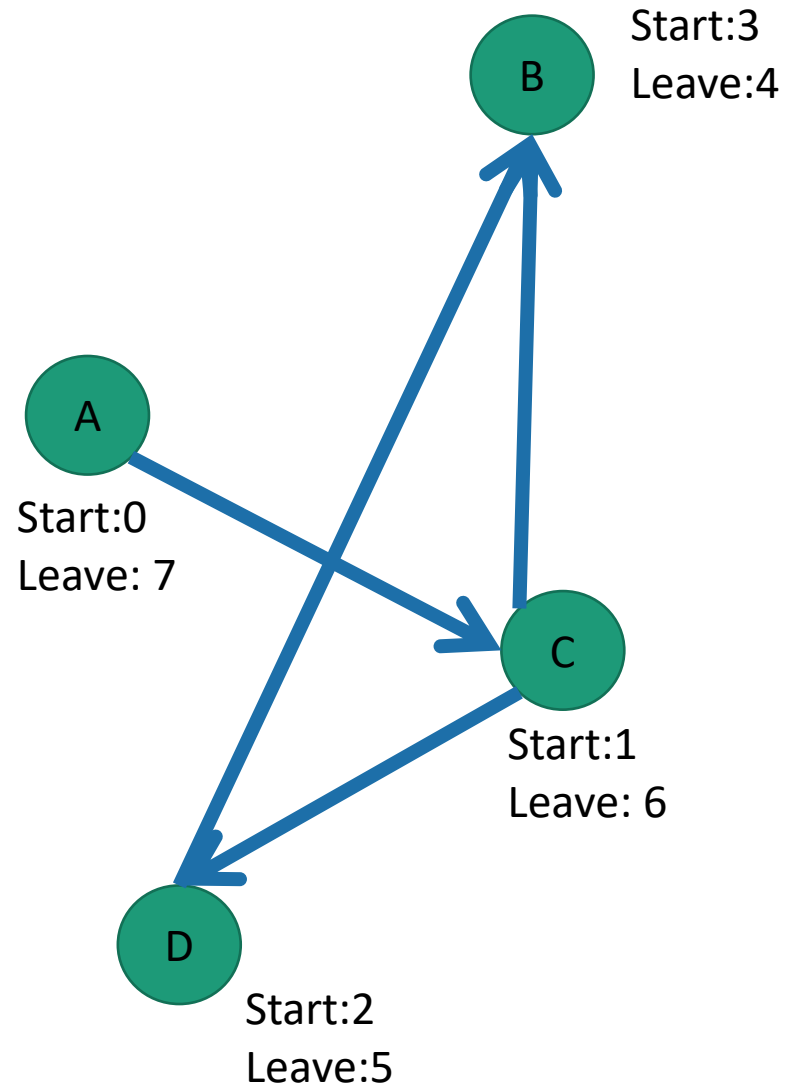
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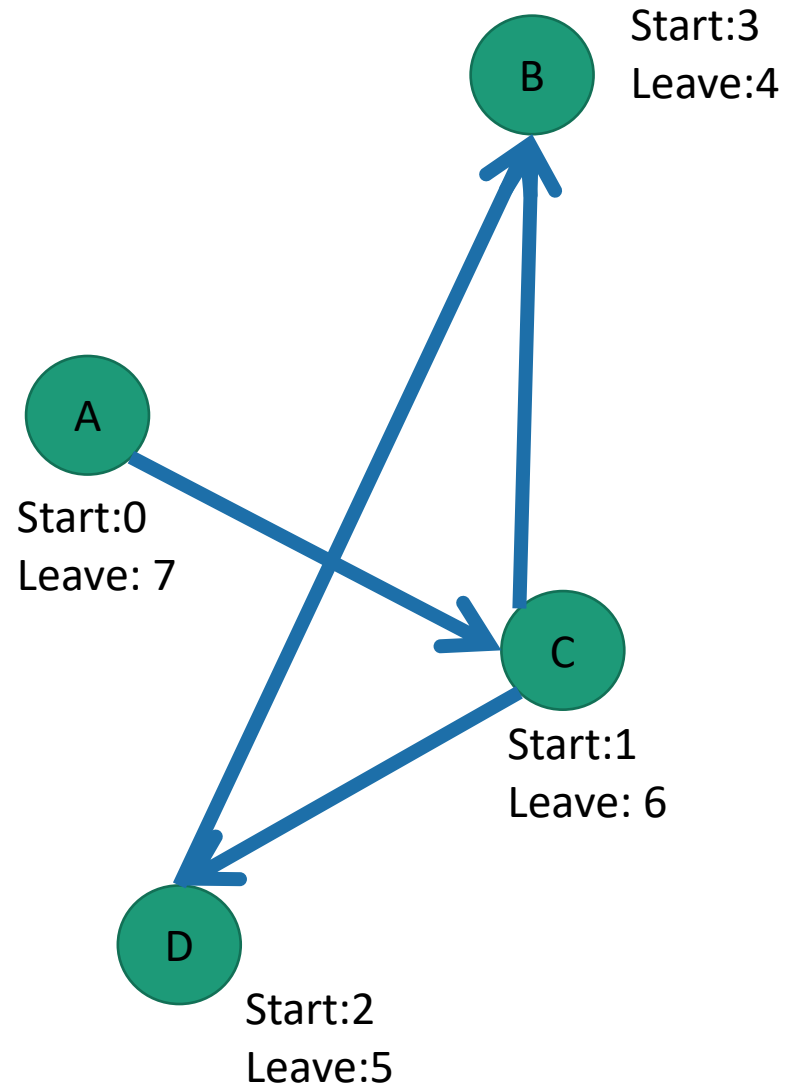
- Unvisited
- In progress
- All done



# Example



# Example

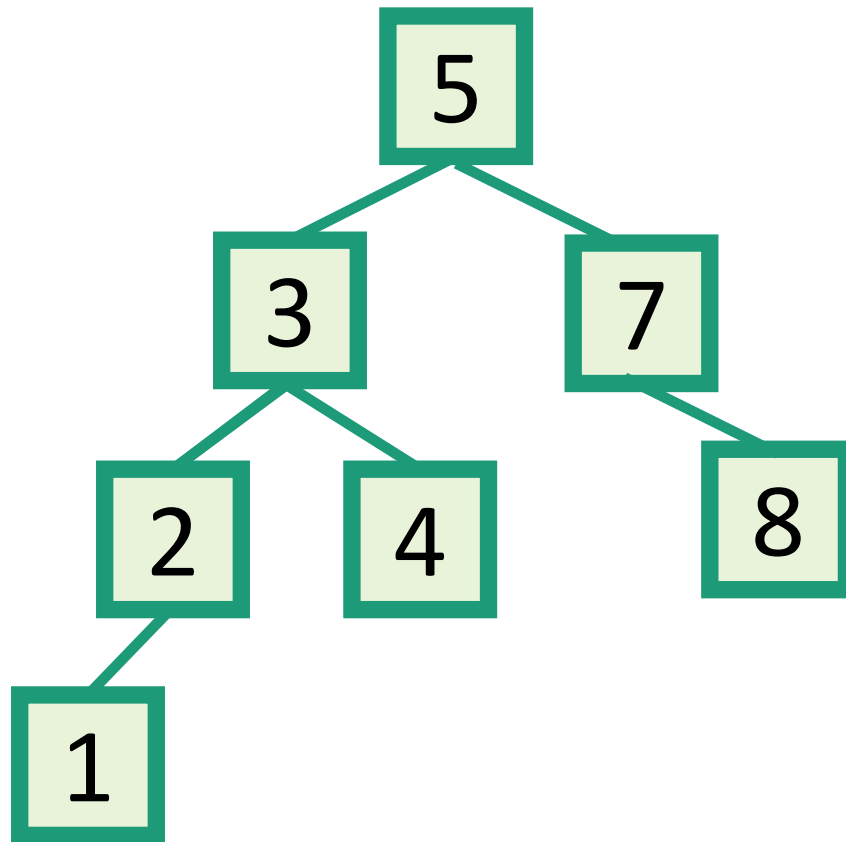


Do them in this order:



# Another use of DFS that we've already seen

- In-order enumeration of binary search trees



Do DFS and print a node's label when you are done with the left child and before you begin the right child.

# Acknowledgement

- Stanford University

Thank You