



**Indian Institute of Information Technology Allahabad**

# Data Structures and Algorithms

## Queues and Lists



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# Queues and Linked Lists

Queues

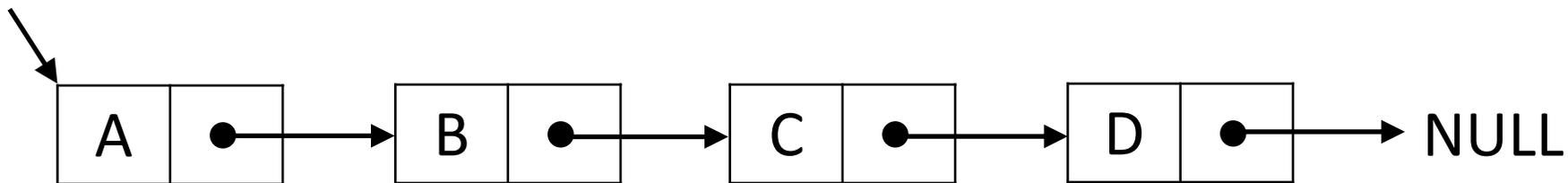
Linked Lists

Doubly Linked List

Double-Ended Queues

Circular List

head



# Queues

- A queue differs from a stack in that its insertion and removal routines follows the **first-in-first-out (FIFO)** principle.
- Elements may be inserted at any time, but only the element which has been in the queue the longest may be removed.
- Elements are inserted at the **rear (enqueued)** and removed from the **front (dequeued)**



# Queue Abstract Data Type (ADT)

The queue supports following fundamental methods:

- **New():ADT** – Creates an empty queue
- **Enqueue(S:ADT, o:element):ADT** – Inserts o at the rear of the queue
- **Dequeue(S:ADT):ADT** – Removes the element from the front of the queue, an error occurs when queue is empty, so need to take care.
- **Front(S:ADT):element** – Returns front element without removing it, an error occurs when queue is empty, so need to take care.

# Queue Abstract Data Type (ADT)

These support methods should also be defined:

- **Size(S:ADT):integer**
- **IsEmpty(S:ADT):Boolean**

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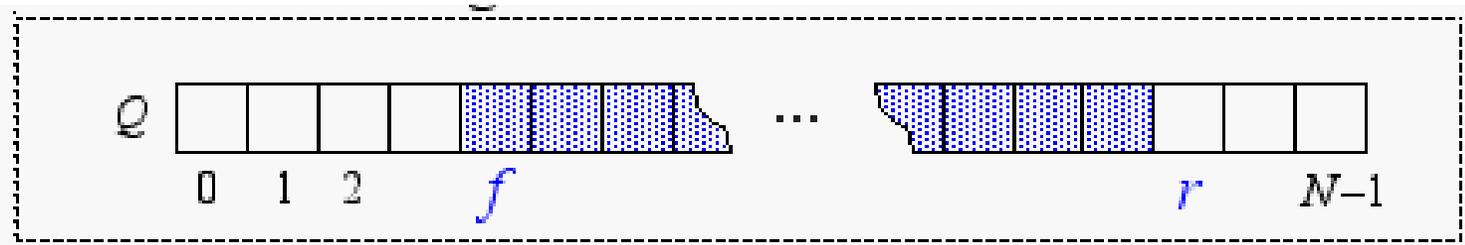
- **Size(S:ADT):integer**
- **IsEmpty(S:ADT):Boolean**

Axioms:

- **Front(Enqueue(New(), v)) = v**
- **Dequeue(Enqueue(New(), v)) = New()**
- **Front(Enqueue(Q, w)) = Front(Enqueue(Enqueue(Q, w), v))**
- **Dequeue(Enqueue(Enqueue(Q, w), v)) = Enqueue(Dequeue(Enqueue(Q, w), v))**

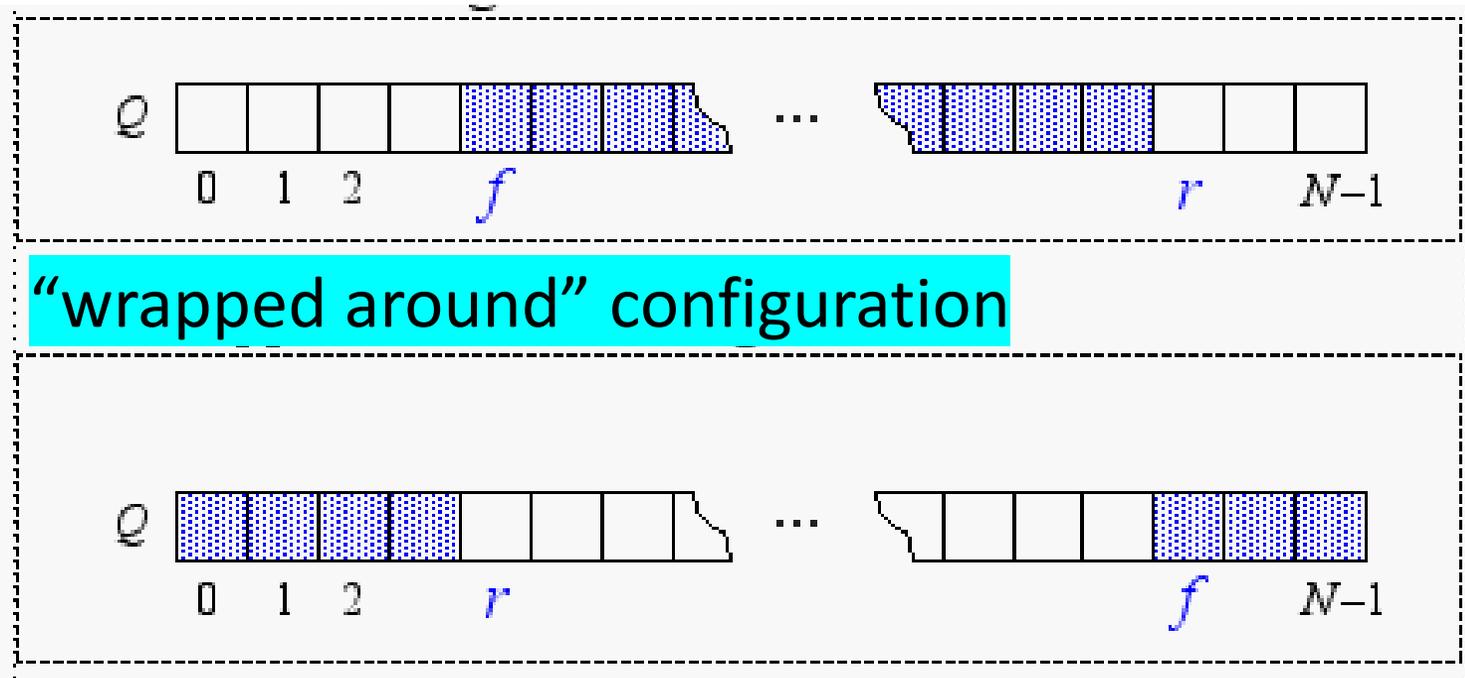
# An Array-Based Queue

- Create a queue using an array in a circular fashion
- A maximum size  $N$  is specified, e.g.  $N = 1,000$ .
- The queue consists of an  $N$ -element array  $Q$  and two integer variables:
  - $f$ , index of the front element (head – for dequeue)
  - $r$ , index of the element after the rear one (tail – for enqueue)



# An Array-Based Queue

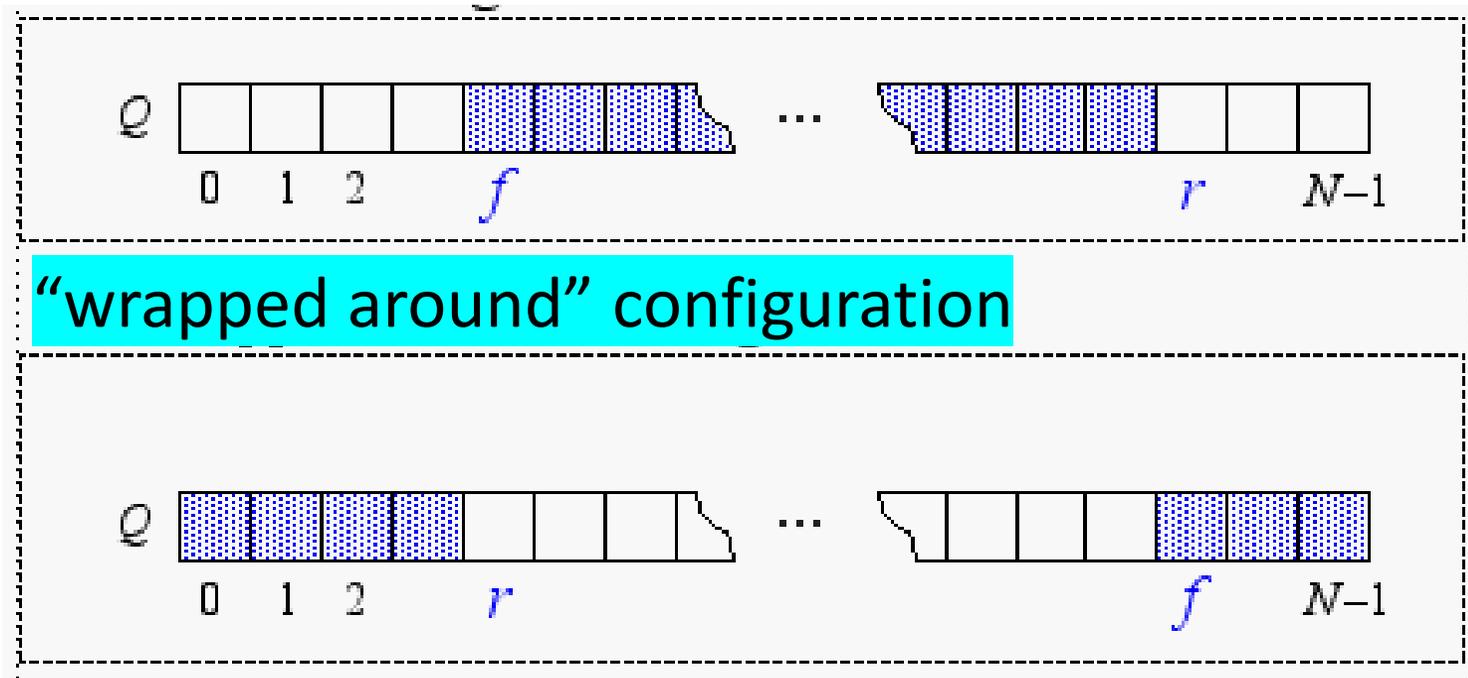
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# An Array-Based Queue

## Questions:

*What does  $f==r$  mean?*

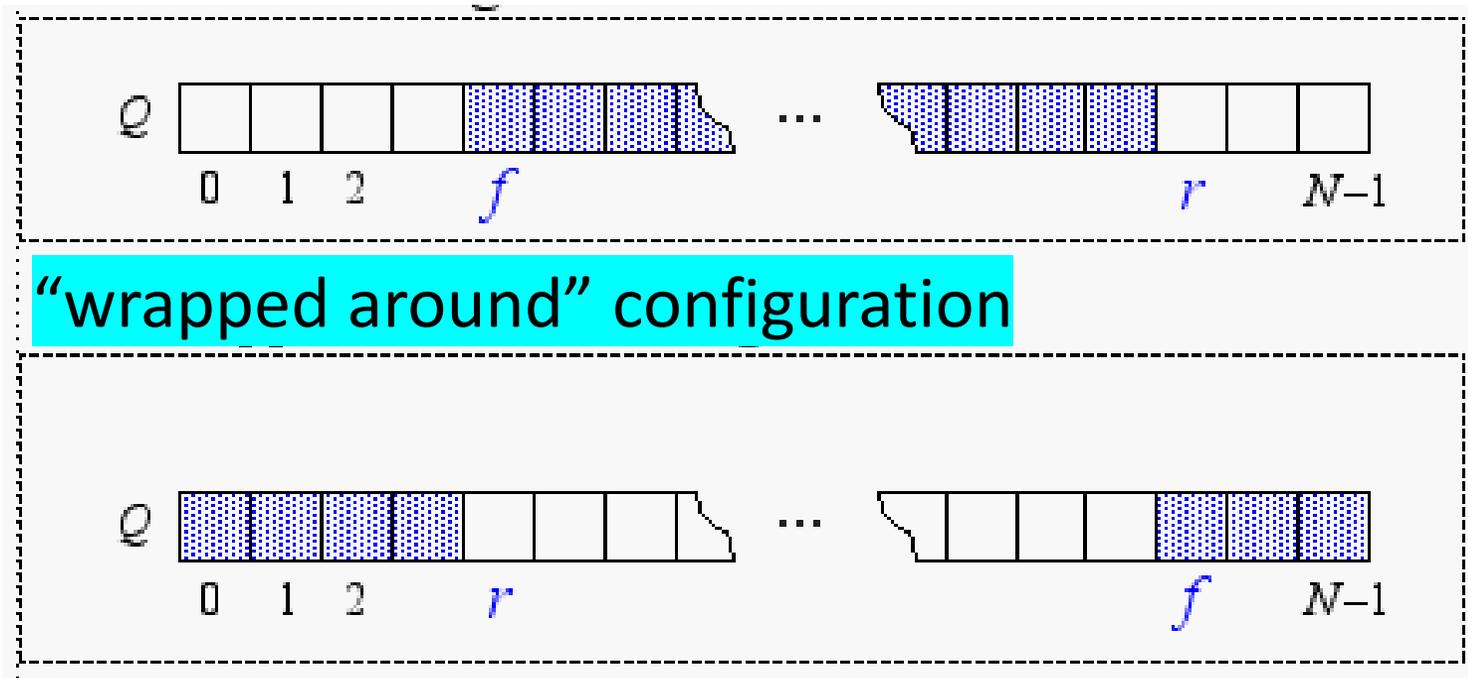


# An Array-Based Queue

## Questions:

*What does  $f==r$  mean?*

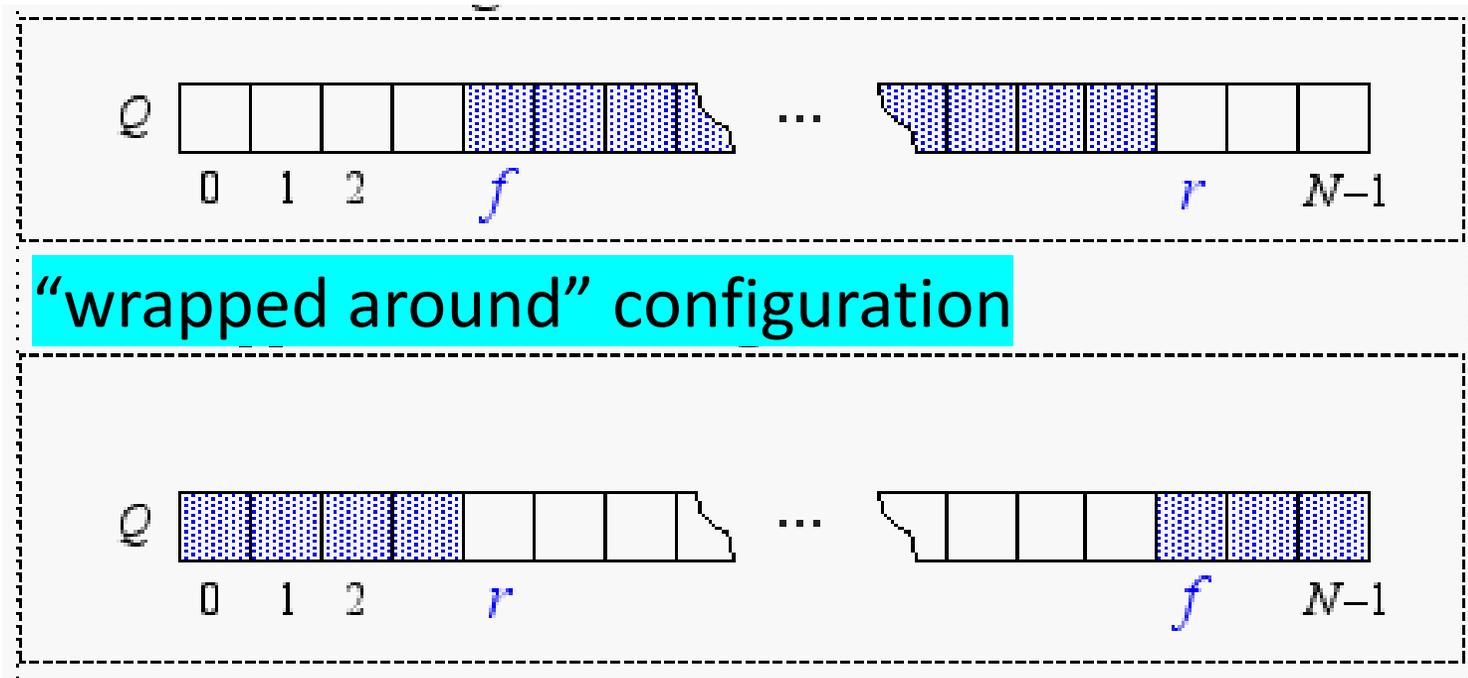
**Empty**



# An Array-Based Queue

## Questions:

*How do we compute the number of elements in the queue from  $f$  and  $r$ ?*

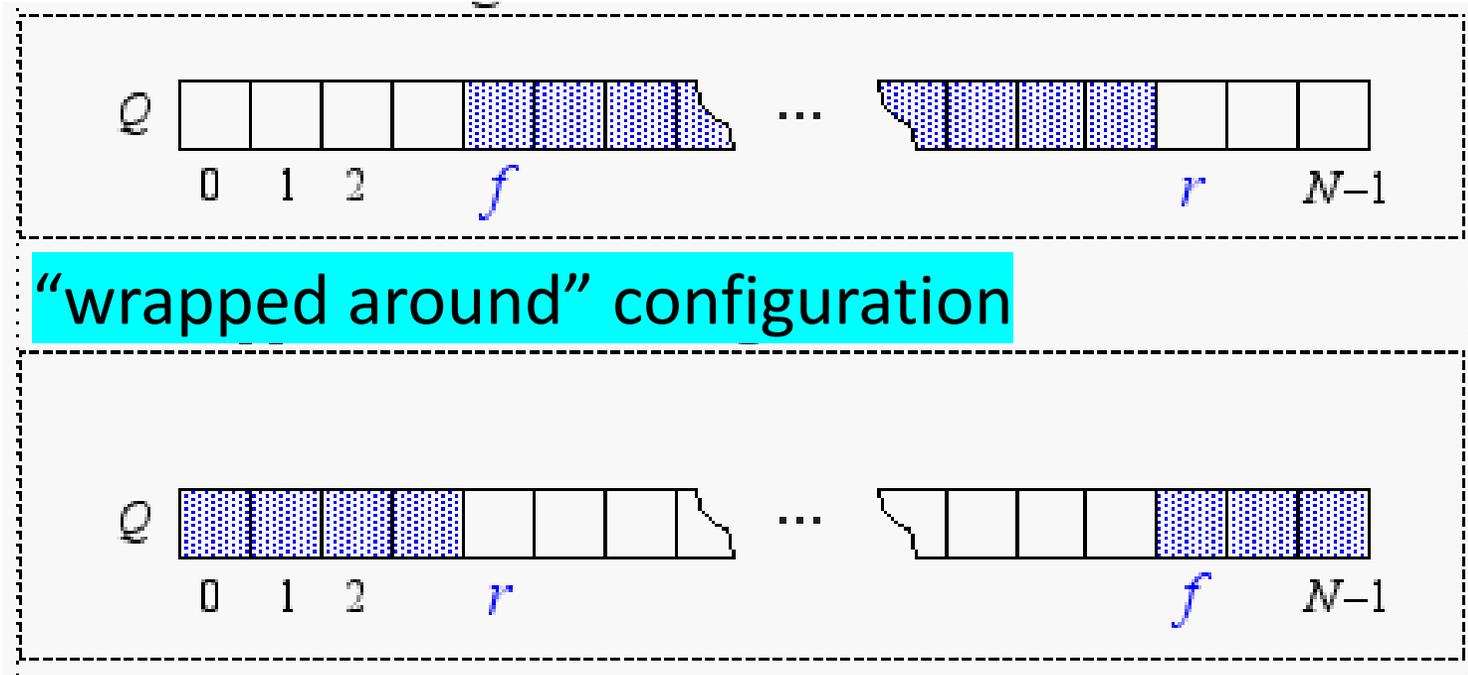


# An Array-Based Queue

## Questions:

*How do we compute the number of elements in the queue from  $f$  and  $r$ ?*

- *if  $r > f$ ,*  
     *$\#elements = r - f$*
- *if  $r < f$ ,*
  - *$\#elements = N - f + r$*
- *if  $r == f$ ,*
  - *$\#elements = 0$*



# An Array-Based Queue

## Questions:

*How do we compute the number of elements in the queue from  $f$  and  $r$ ?*

- *if  $r > f$ ,*

- *#elements =  $r - f + 1$*

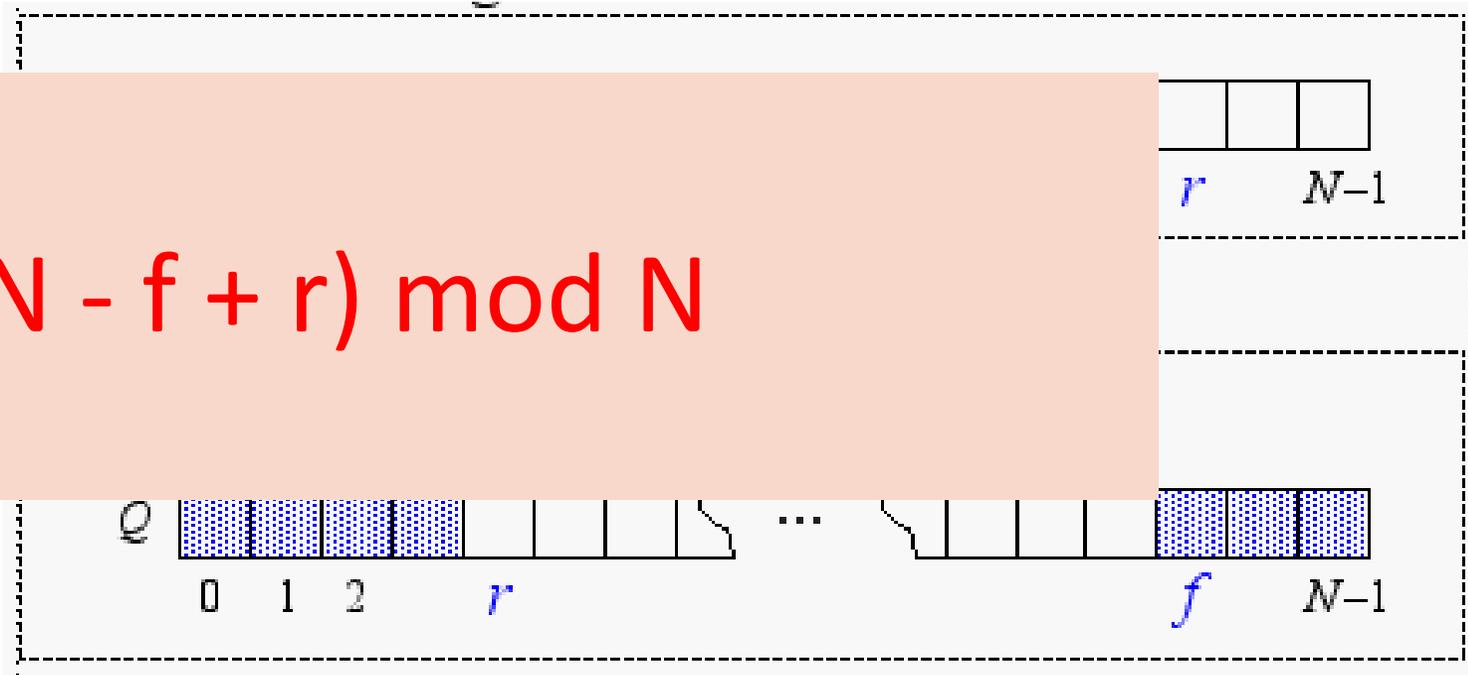
- *if  $r < f$ ,*

- *#elements =  $r + N - f + 1$*

- *if  $r = f$ ,*

- *#elements = 0*

*i.e.,  $(N - f + r) \bmod N$*



# An Array-Based Queue: Pseudo Code

Algorithm **Size()**:

```
return (N - f + r) mod N
```

Algorithm **isEmpty()**:

```
return (f == r)
```

Algorithm **Front()**:

```
if isEmpty() then  
    print "Queue is Empty"  
    return NULL  
return Q[f]
```

Algorithm **Dequeue()**:

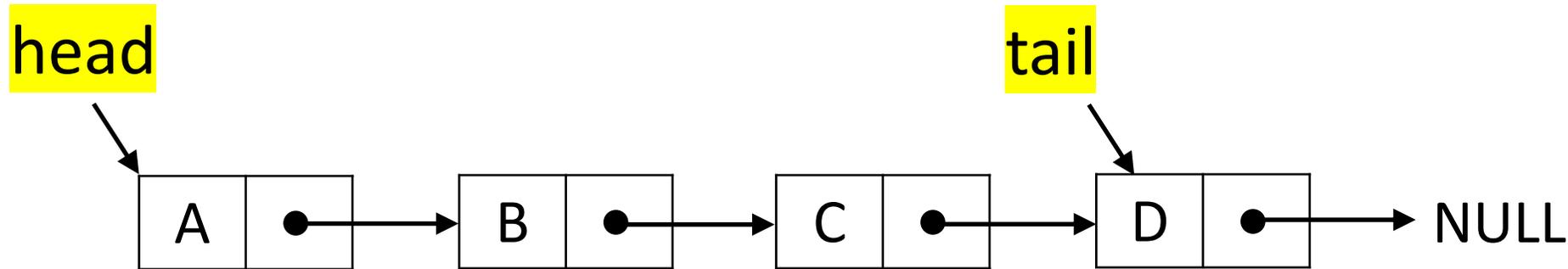
```
if isEmpty() then  
    print "Queue is Empty"; return NULL  
temp = Q[f]  
Q[f] = null  
f = (f + 1) mod N  
return temp
```

Algorithm **Enqueue(o)**:

```
if Size() == N - 1 then  
    print "Queue is Full"; return  
Q[r] = o  
r = (r + 1) mod N
```

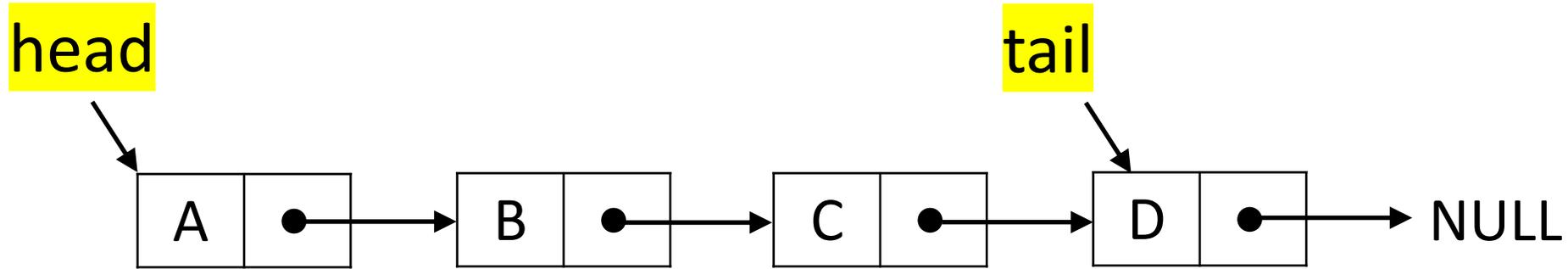
# Implementing Queue with a Singly Linked List

- Nodes (*data, pointers*) connected in a chain by links

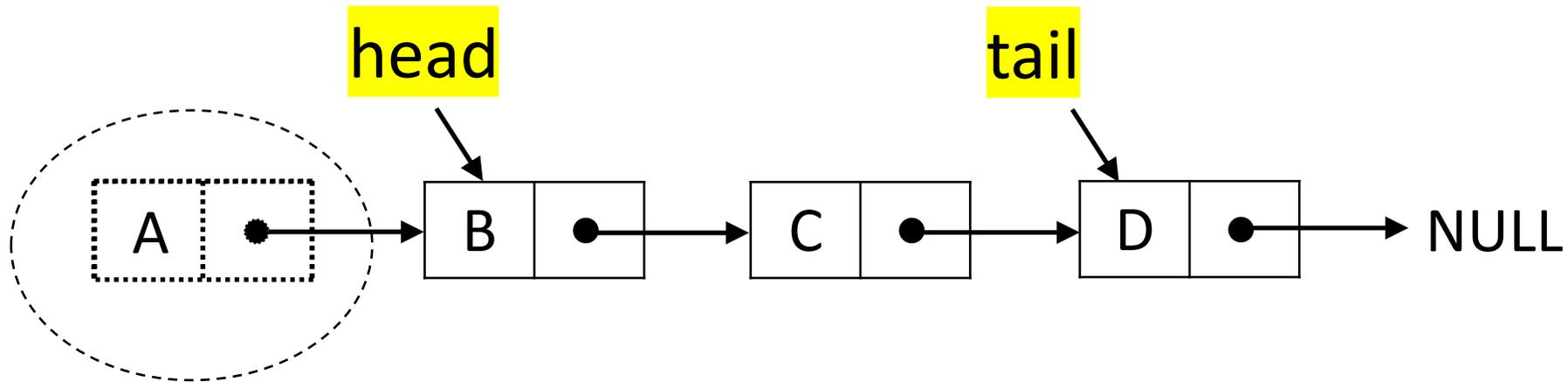


- The head of the list is the front of the queue, the tail of the list is the rear of the queue. ***Why not the opposite?***

# Queue: Removing at the Head (Dequeue)



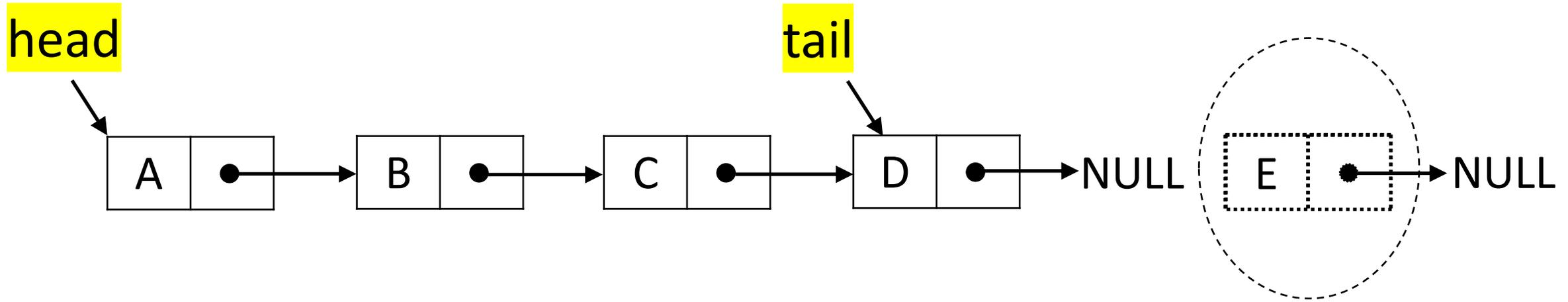
Advance head reference



Inserting at the head is just as easy

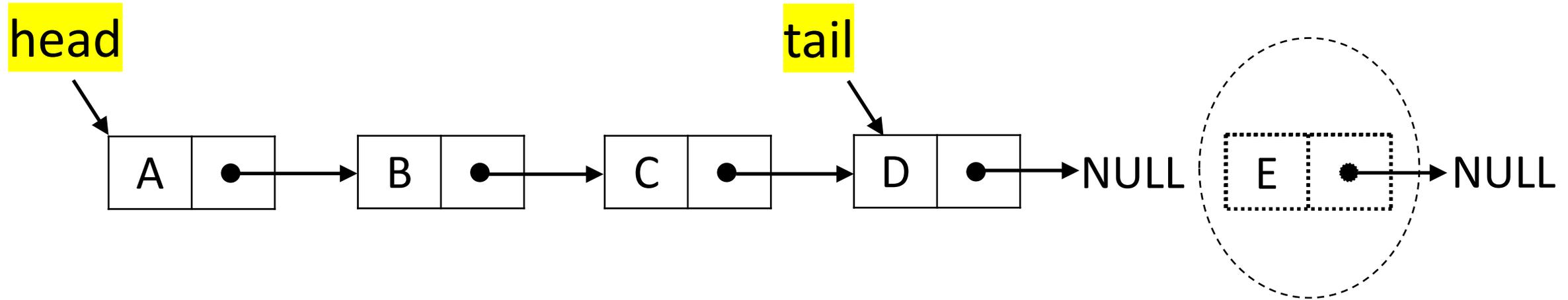
# Queue: Inserting at the Tail (Enqueue)

Create a new node

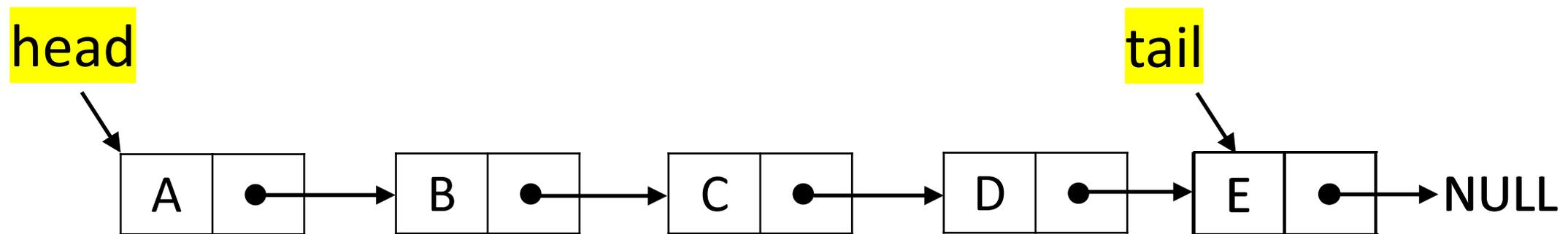


# Queue: Inserting at the Tail (Enqueue)

Create a new node



Chain it and move the tail reference



How about removing at the tail?

# Double-Ended Queue

- A **double-ended queue**, or **deque**, supports insertion and deletion from the front and back.
- The Deque supports following fundamental methods:
  - **insertFirst(S:ADT, o:element):ADT** - Inserts e at the beginning of deque.
  - **insertLast(S:ADT, o:element):ADT** - Inserts e at the end of deque.
  - **removeFirst(S:ADT):ADT** - Removes the first element.
  - **removeLast(S:ADT):ADT** - Removes the last element.
  - **first(S:ADT):element** - Return the first element.
  - **last(S:ADT):element** - Return the last element.

# Implementing Deques

## With Singly Linked Lists

- Not a good idea
  - As deletion at tail is costly

# Implementing Deques

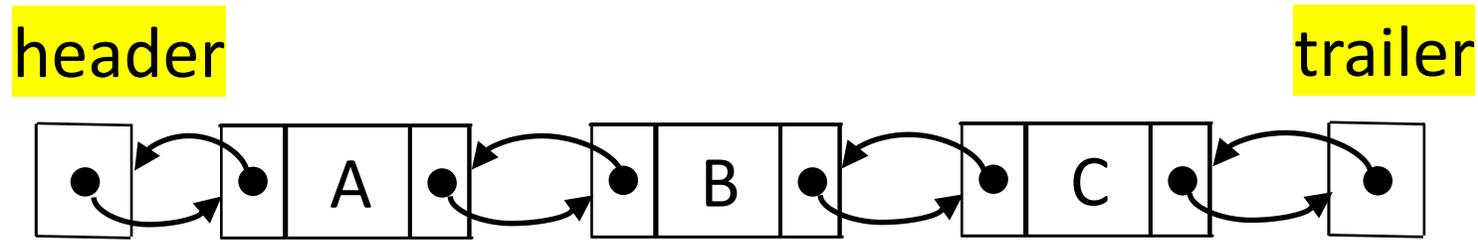
## With Singly Linked Lists

- Not a good idea
  - As deletion at tail is costly

Solution: Use Doubly Linked List

# Implementing Deques with Doubly Linked Lists

- Deletions at the tail of a singly linked list cannot be done in constant time.
- To implement a deque, we use a **doubly linked** list with special header and trailer nodes

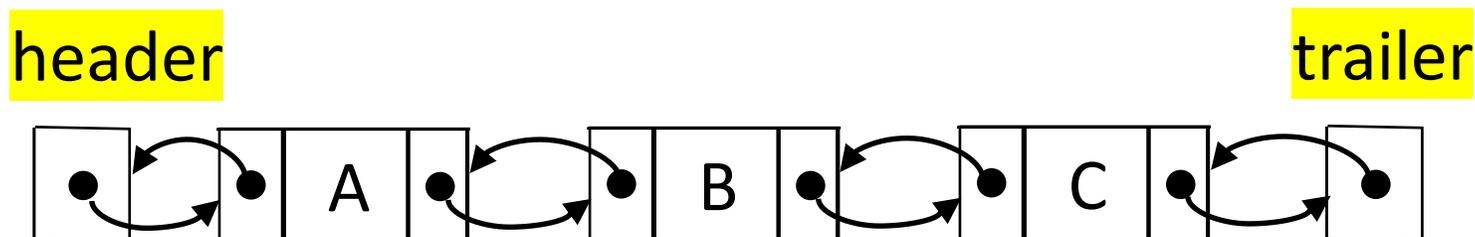


- A node of a doubly linked list has a **next** and a **prev** link.
- By using a doubly linked list, all the methods of a deque run in  $O(1)$  time.

# Implementing Deques with Doubly Linked Lists

- When implementing a doubly linked list, we add two special nodes to the ends of the lists: the **header** and **trailer** nodes.
  - The **header** node goes before the first list element. It has a valid next link but a null prev link.
  - The **trailer** node goes after the last element. It has a valid prev reference but a null next reference.

**NOTE: the header and trailer nodes are sentinel or “dummy” nodes because they do not store elements. Here’s a diagram of our doubly linked list:**



# Implementing Deques with Doubly Linked Lists

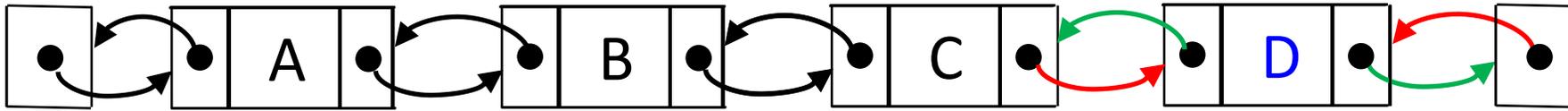
Here's a visualization of the code for `removeLast()`.

header

second to last

last

trailer

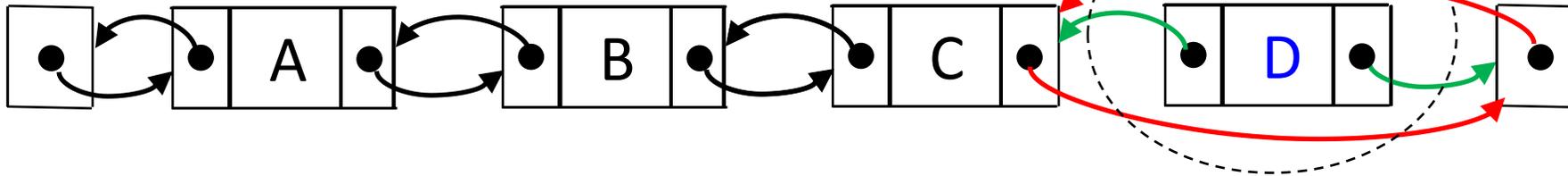


header

second to last

last

trailer



# Implementing Deques with Doubly Linked Lists

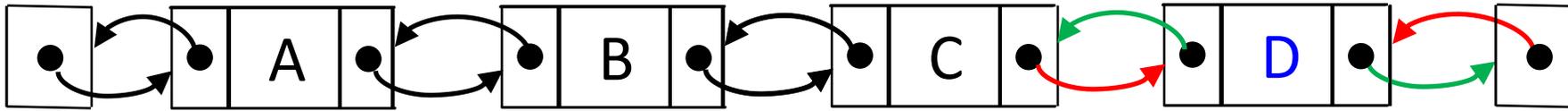
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header

second to last

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trailer

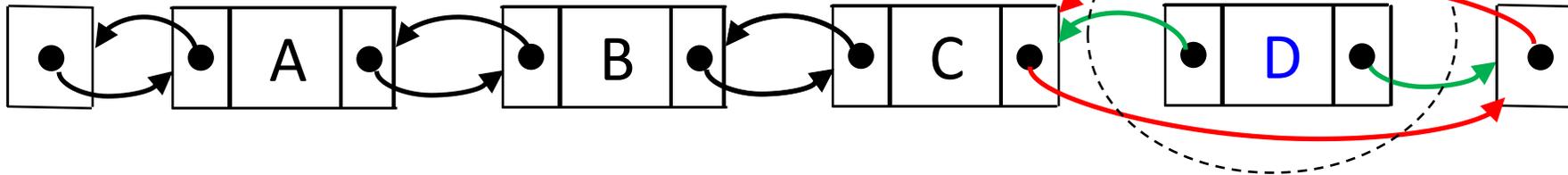


header

second to last

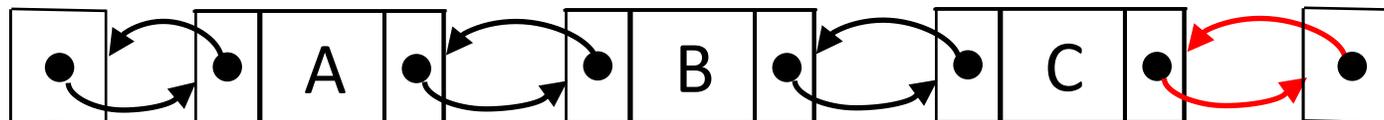
last

trailer



header

trailer



# Implementing Stacks and Queues with Deques

Implementing ADTs using implementations of other ADTs as building blocks

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Implementing ADTs using implementations of other ADTs as building blocks

## Stacks with Deques:

Stack Method	Deque Implementation
size()	size()
isEmpty()	isEmpty()
top()	last()
push(e)	insertLast(e)
pop()	removeLast()

# Implementing Stacks and Queues with Deques

Implementing ADTs using implementations of other ADTs as building blocks

## Stacks with Deques:

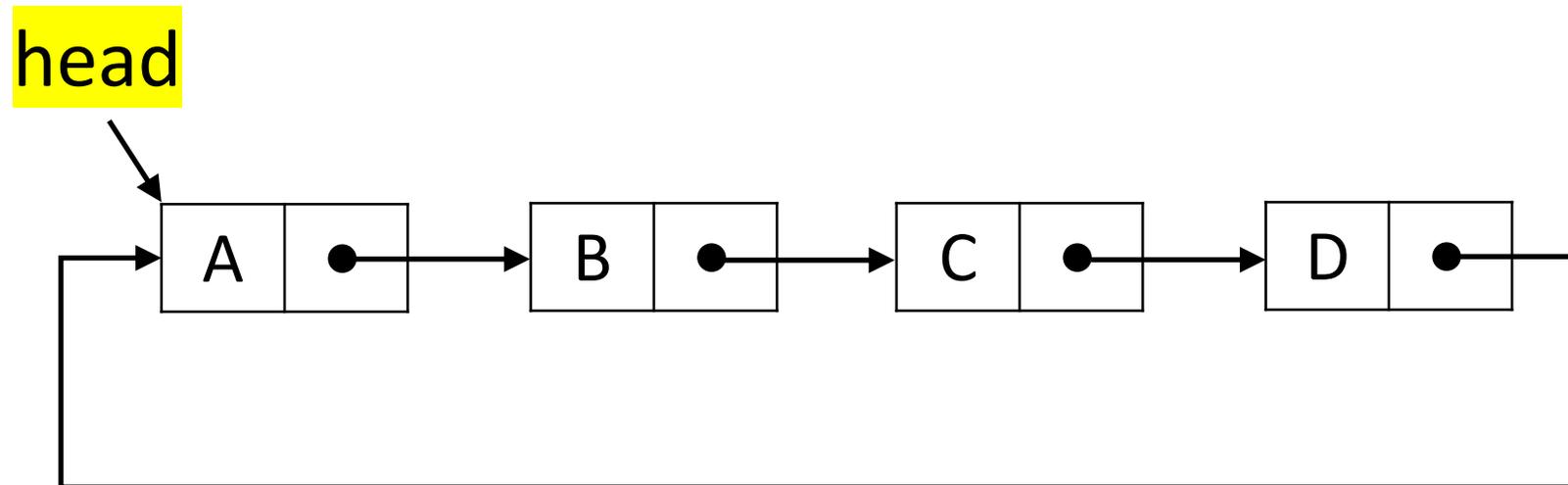
Stack Method	Deque Implementation
size()	size()
isEmpty()	isEmpty()
top()	last()
push(e)	insertLast(e)
pop()	removeLast()

## Queues with Deques:

Queue Method	Deque Implementation
size()	size()
isEmpty()	isEmpty()
front()	first()
enqueue(e)	insertLast(e)
dequeue()	removeFirst()

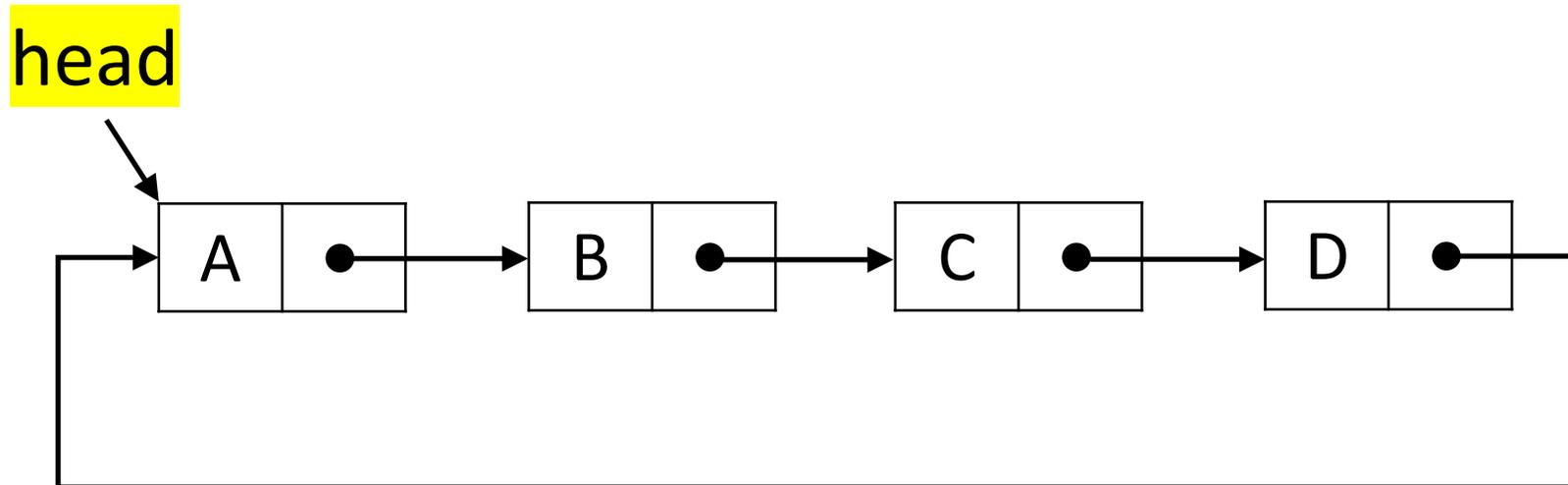
# Circular Lists

- No end and no beginning of the list, only one pointer as an **entry point**
- *Circular doubly linked list with a sentinel* is an elegant implementation of a stack or a queue



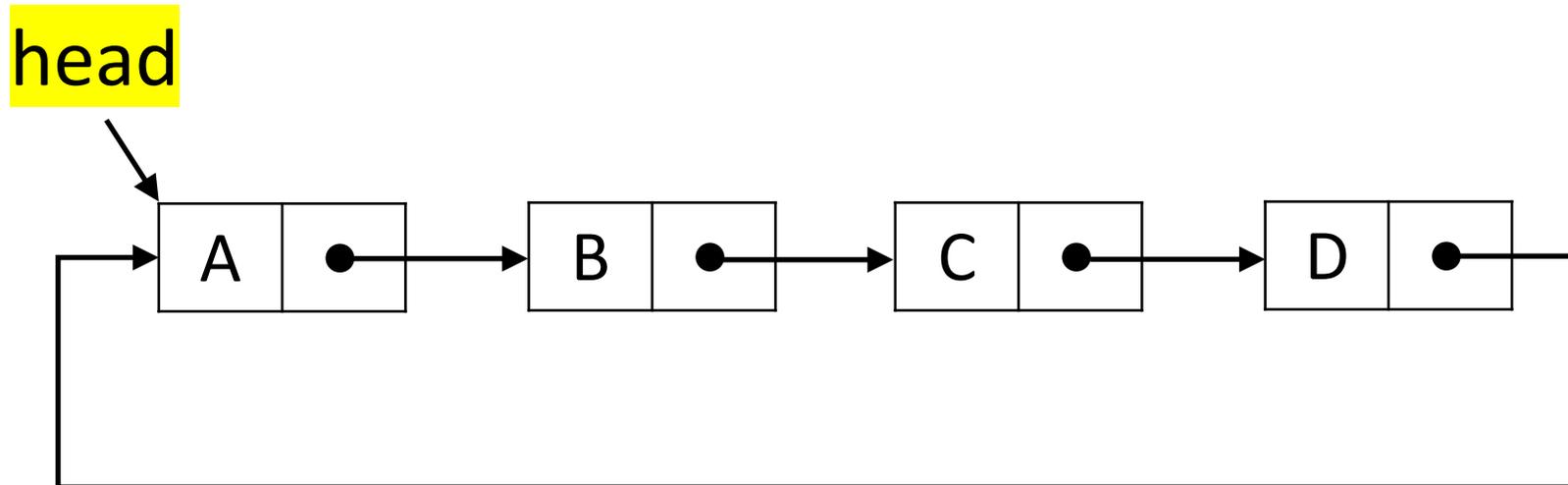
# Circular Lists

- **Insertion a node F at head:** create a new node, insert between A and B, copy A to this new node and replace A of head node with F.



# Circular Lists

- **Deleting the head node:** copy node B to node A and delete original node B.



# Acknowledgement

- IIT Delhi

Thank You