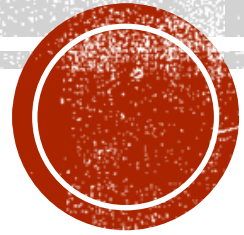
Indian Institute of Information Technology Allahabad

Data Structures and Algorithms

Breadth First Search (BFS)



Dr. Shiv Ram Dubey
Assistant Professor
Department of Information Technology
Indian Institute of Information Technology, Allahabad

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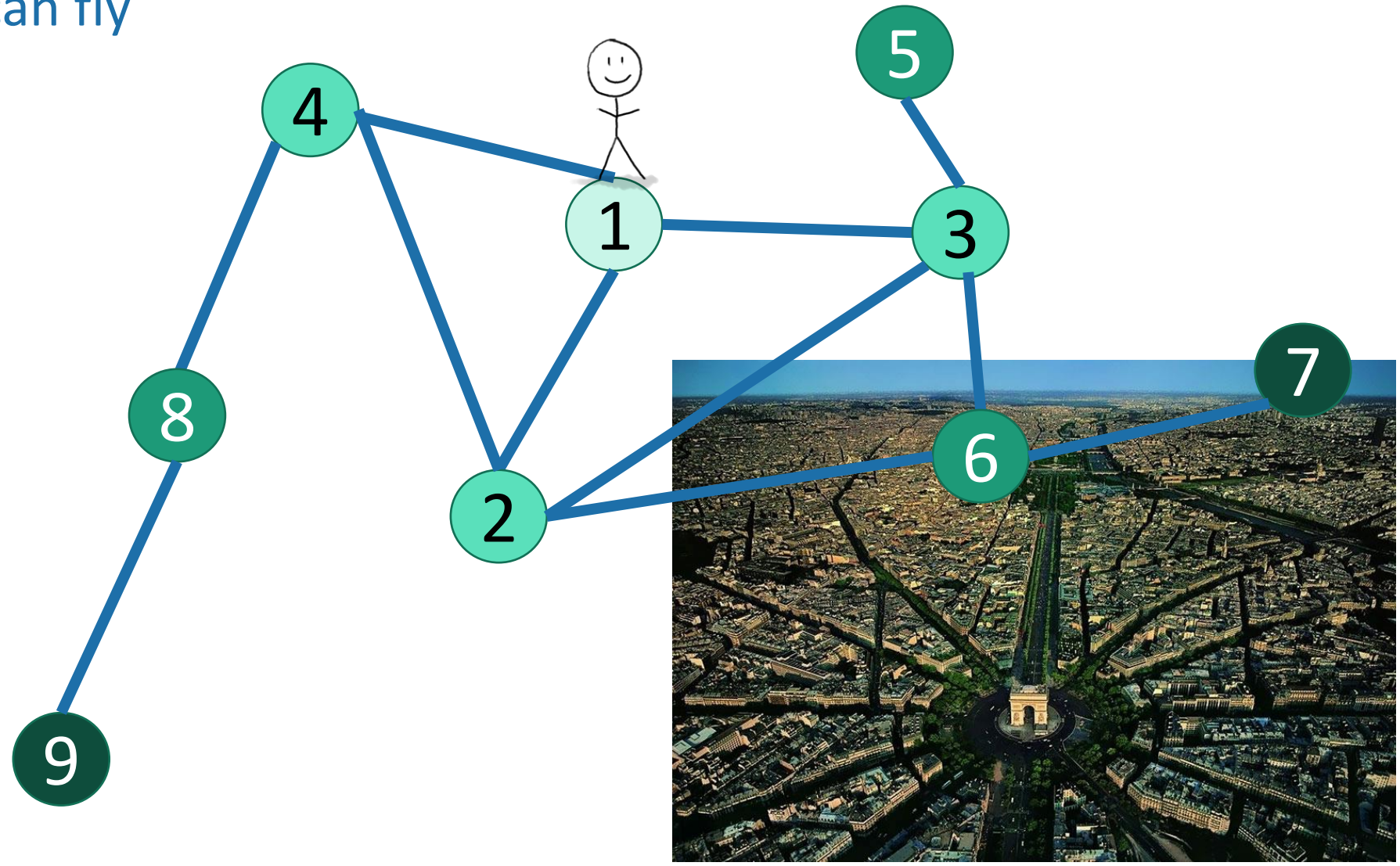
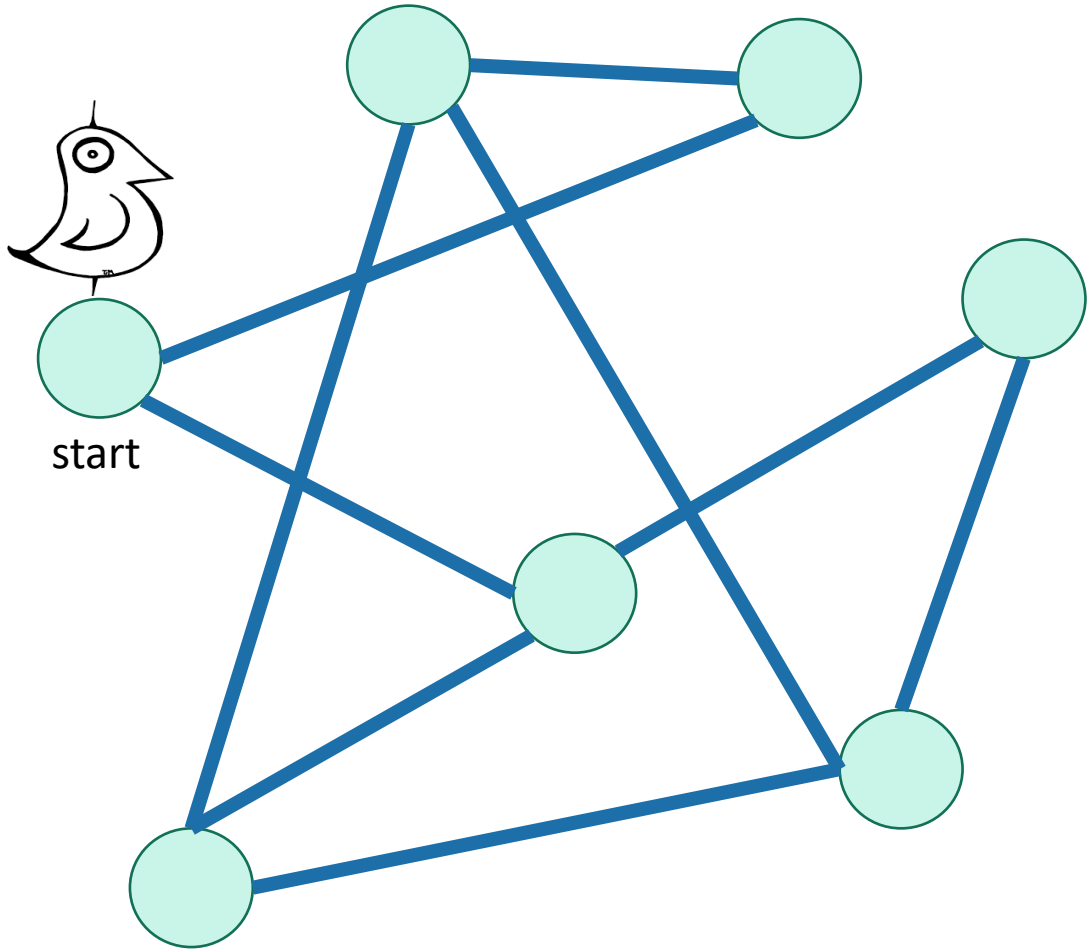
How do we explore a graph?

If we can fly



Breadth-First Search

Exploring the world with a bird's-eye view



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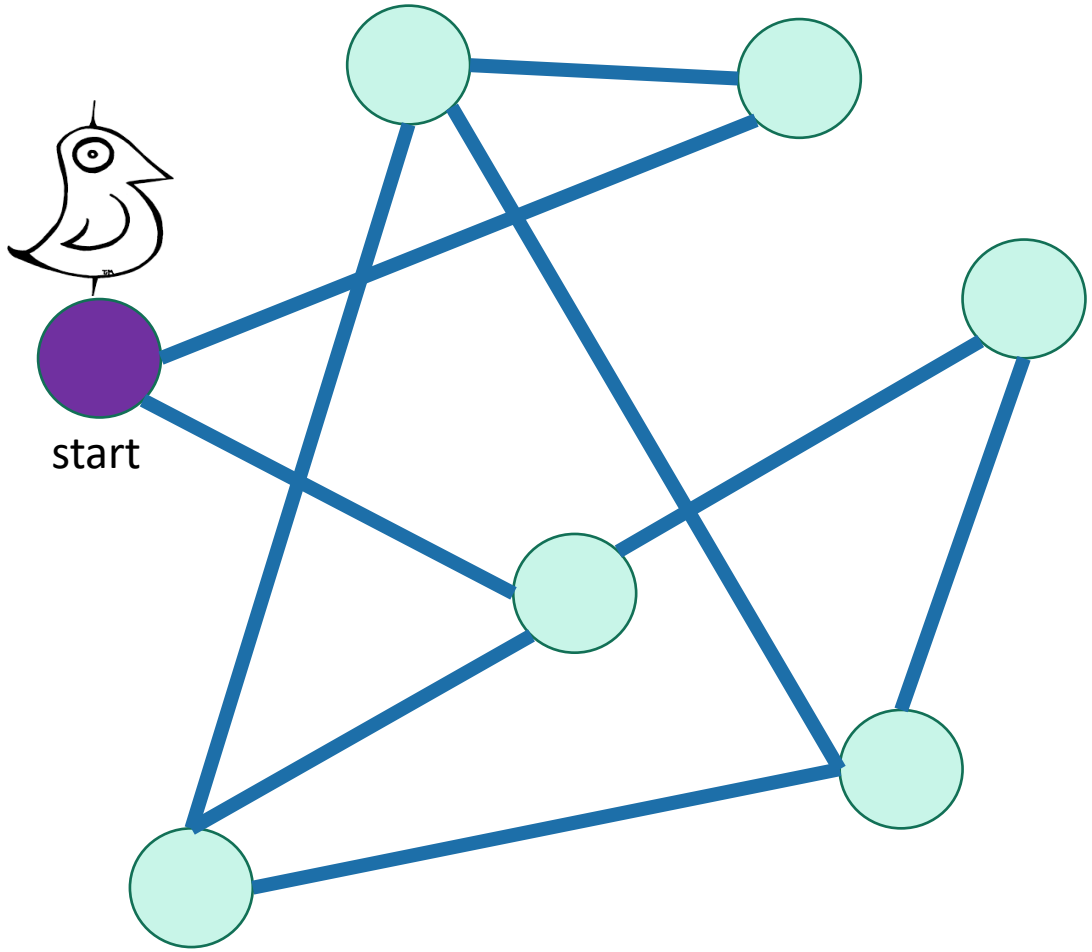

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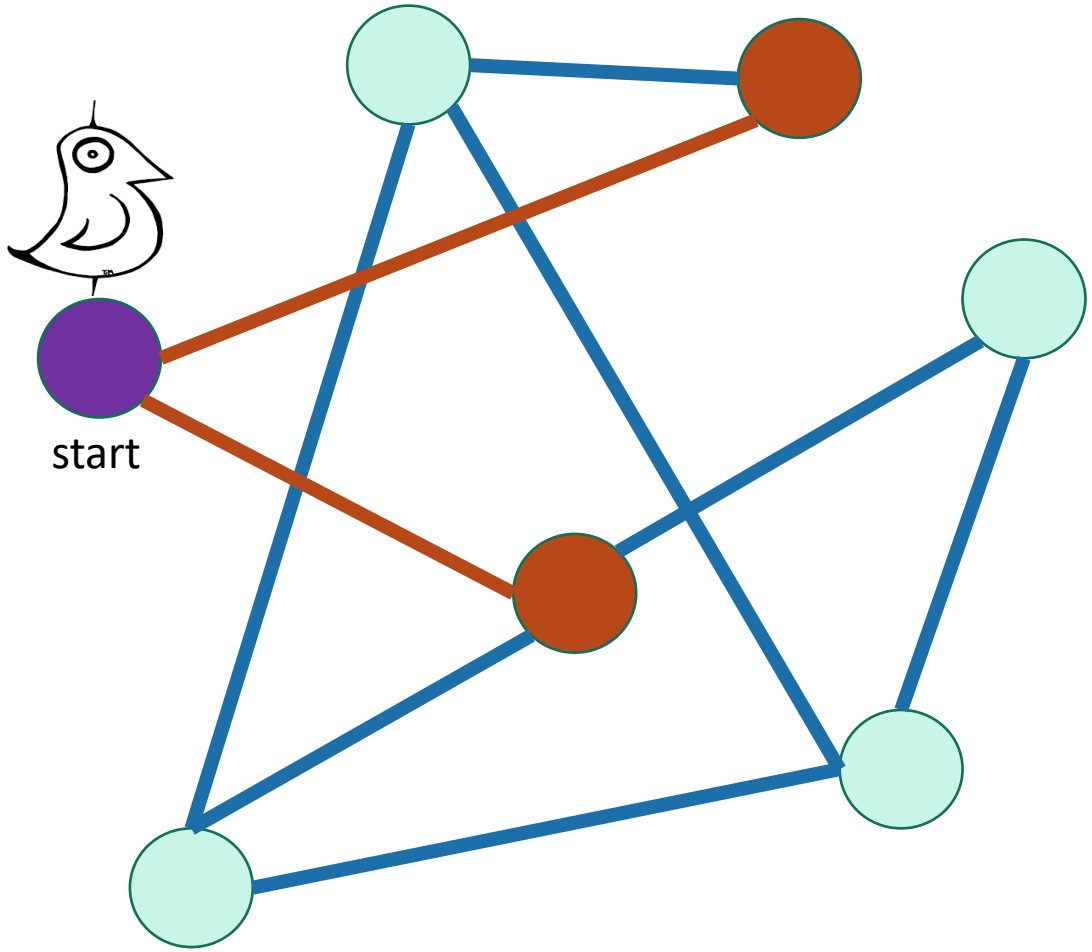

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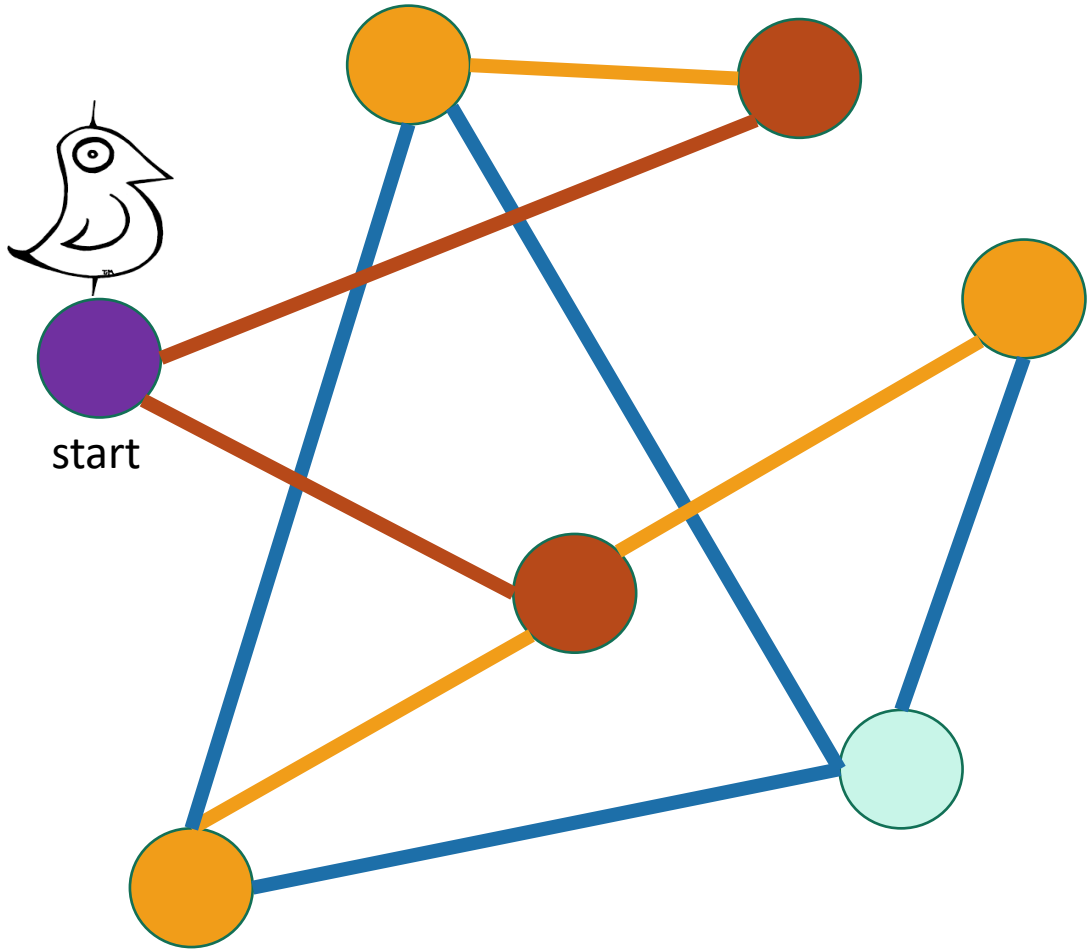

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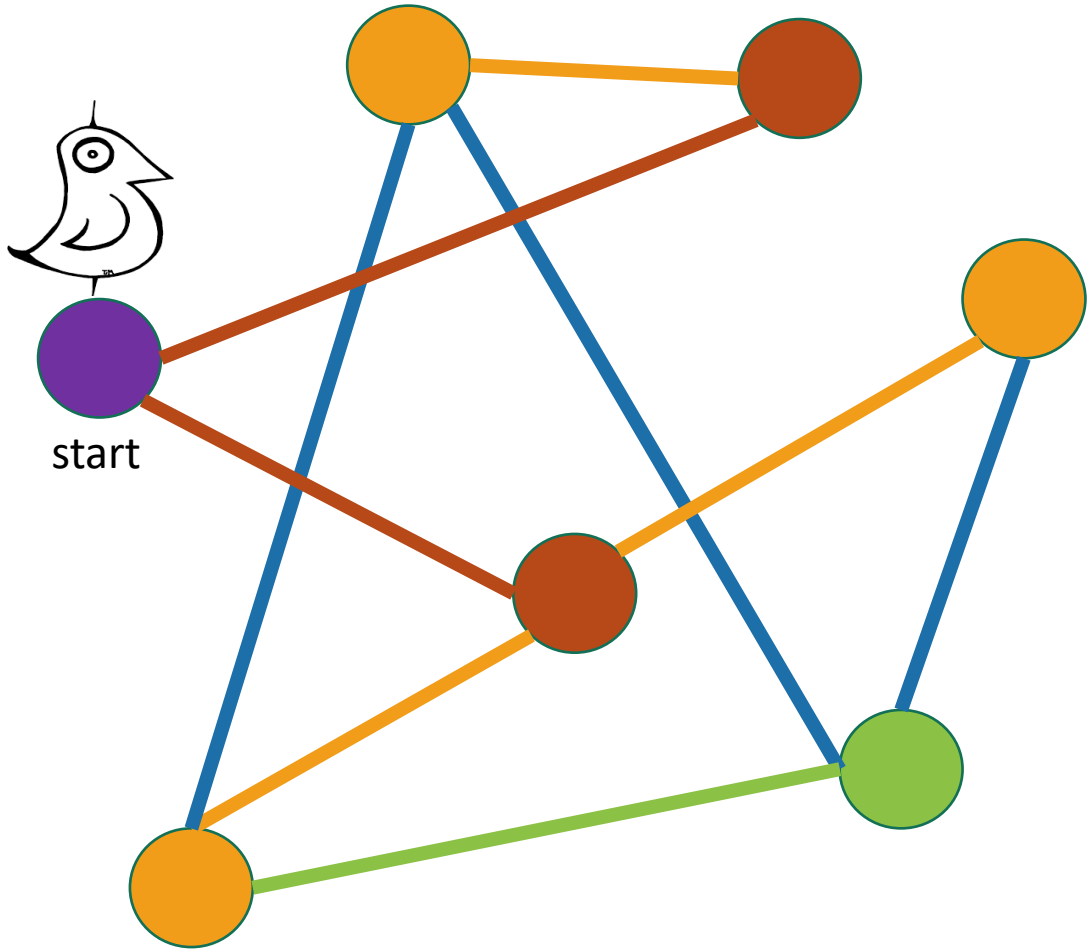

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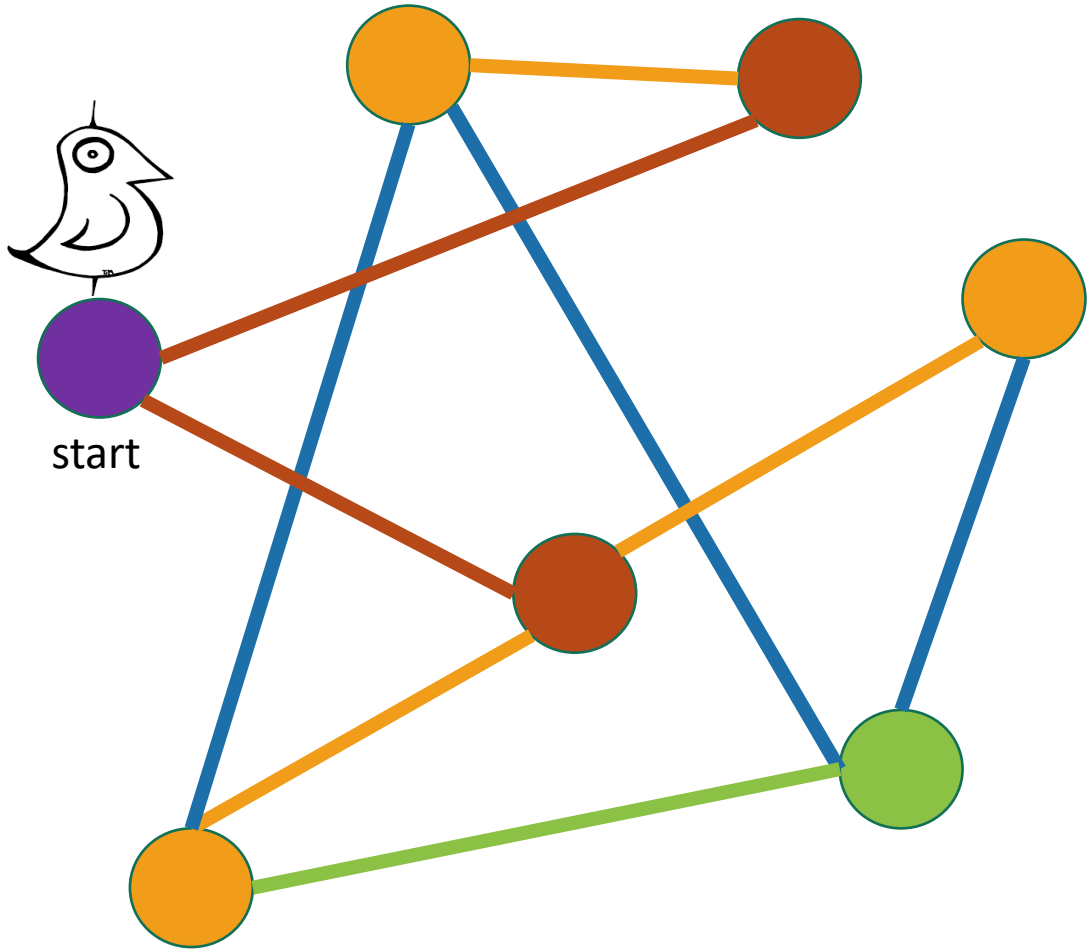

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World: explored!

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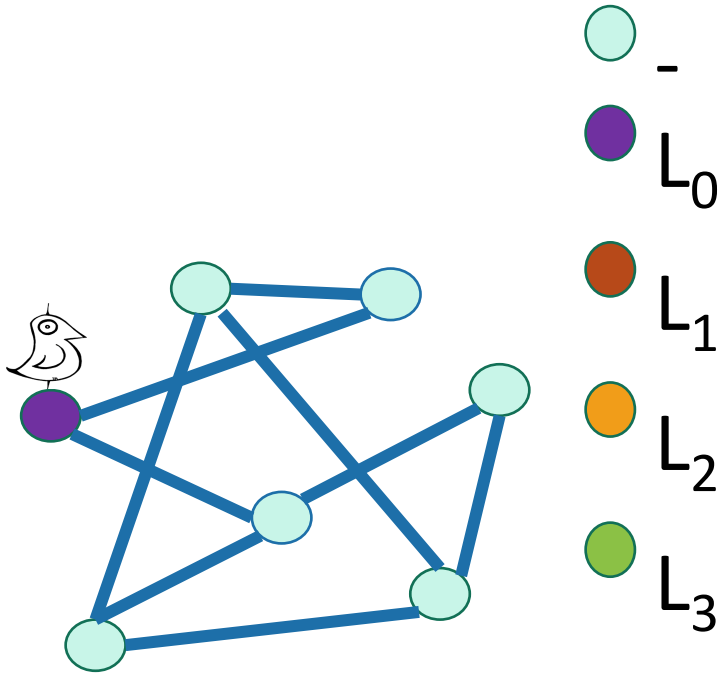
Breadth-First Search

Exploring the world with pseudocode



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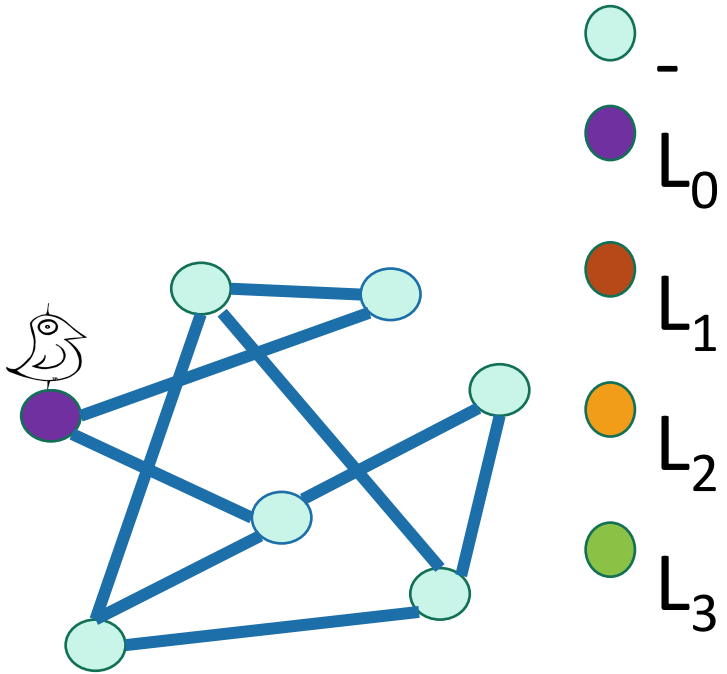

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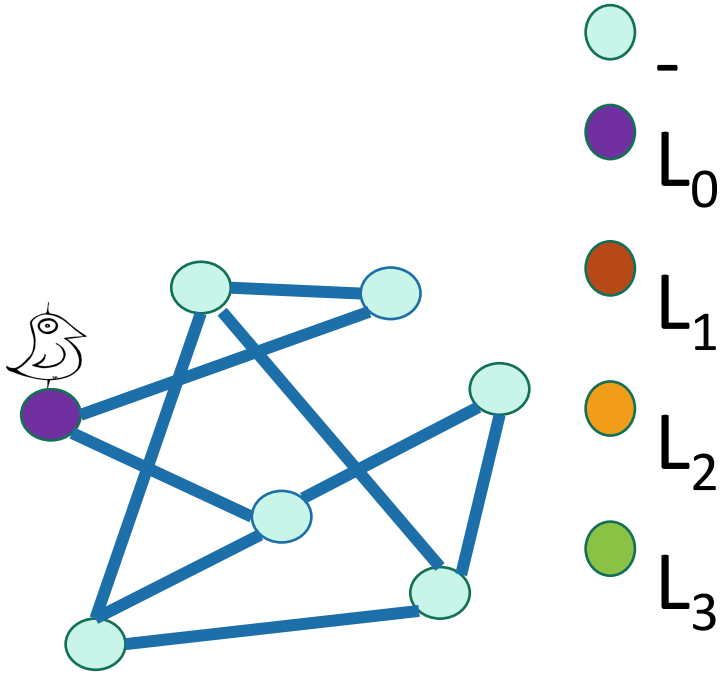

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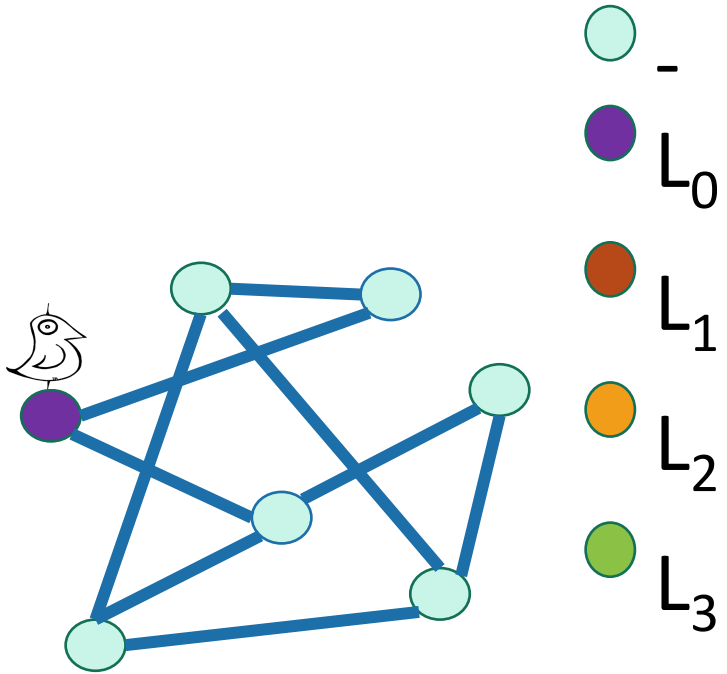

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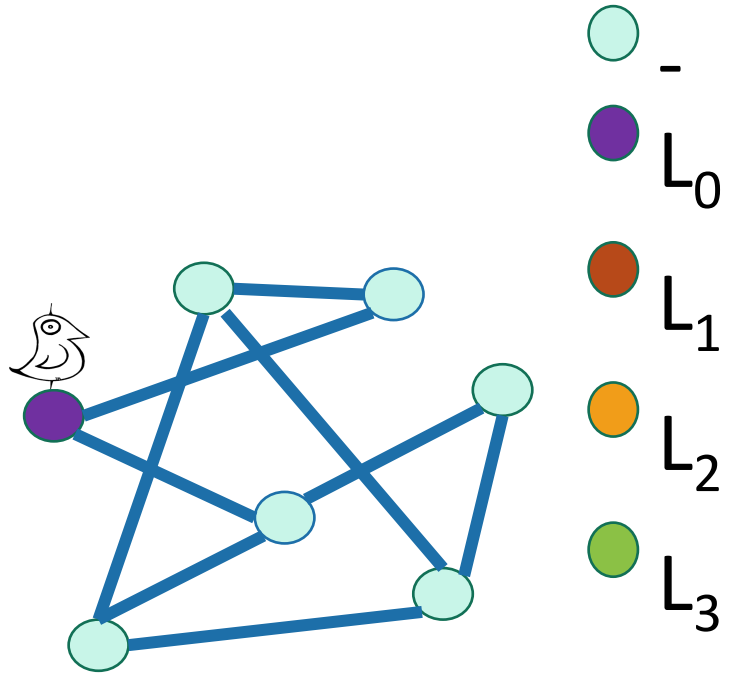
Go through all the nodes in L_i and add their unvisited neighbors to L_{i+1}



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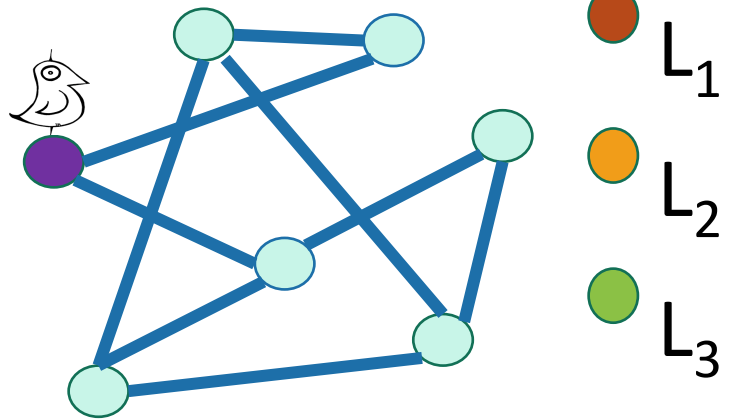

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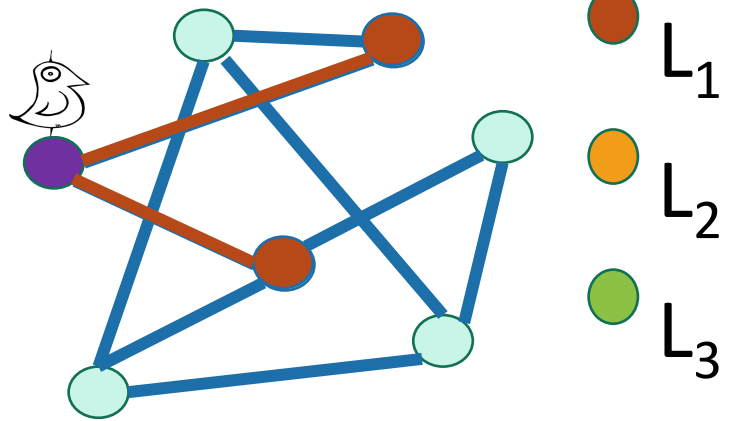
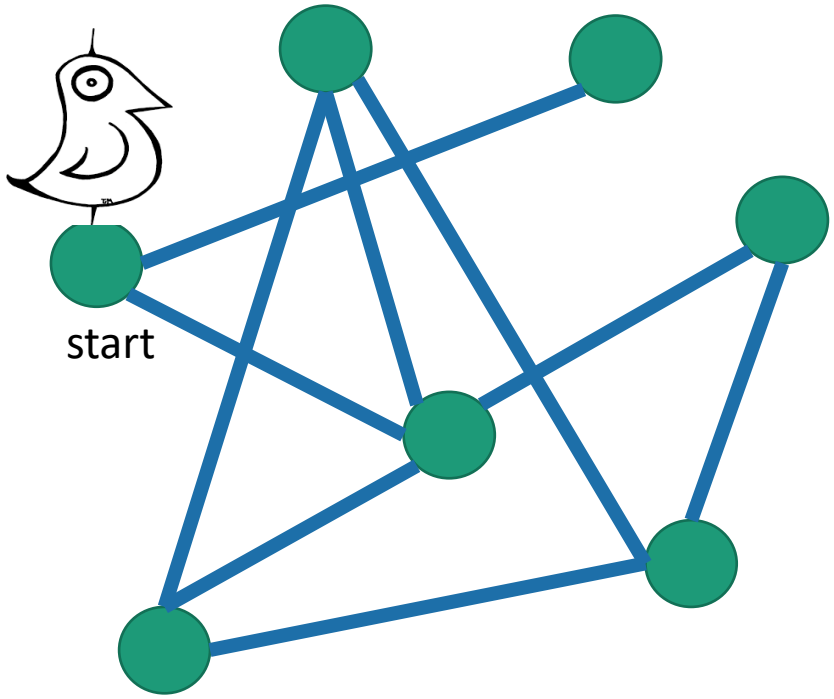
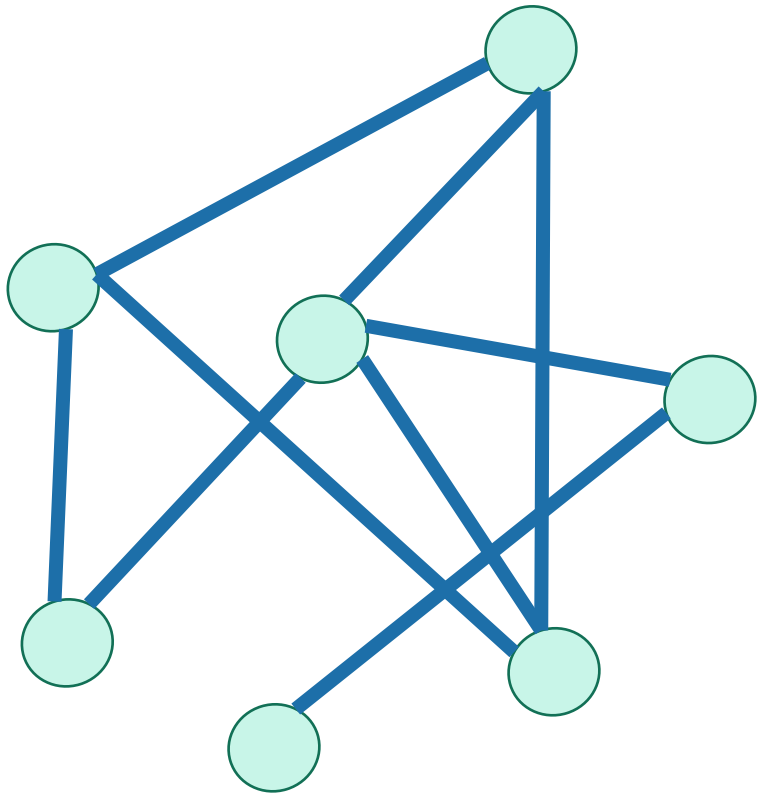


BFS also finds all the nodes reachable from the starting point





It is also a good way to find all the **connected components.**

Running time and extension to directed graphs

- To explore the whole graph, explore the connected components one-by-one.
  - Same argument as DFS: BFS running time is O(n + m)

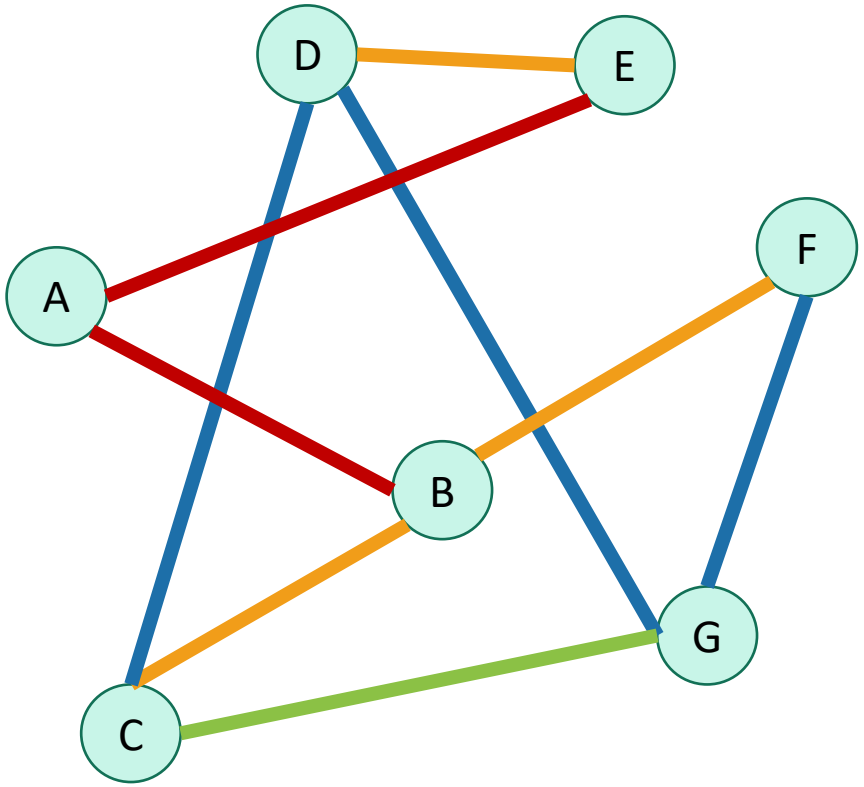
Running time and extension to directed graphs

- To explore the whole graph, explore the connected components one-by-one.
  - Same argument as DFS: BFS running time is O(n + m)
- Like DFS, BFS also works fine on directed graphs.

Verify these!

Why is it called breadth-first?

- We are implicitly building a tree:



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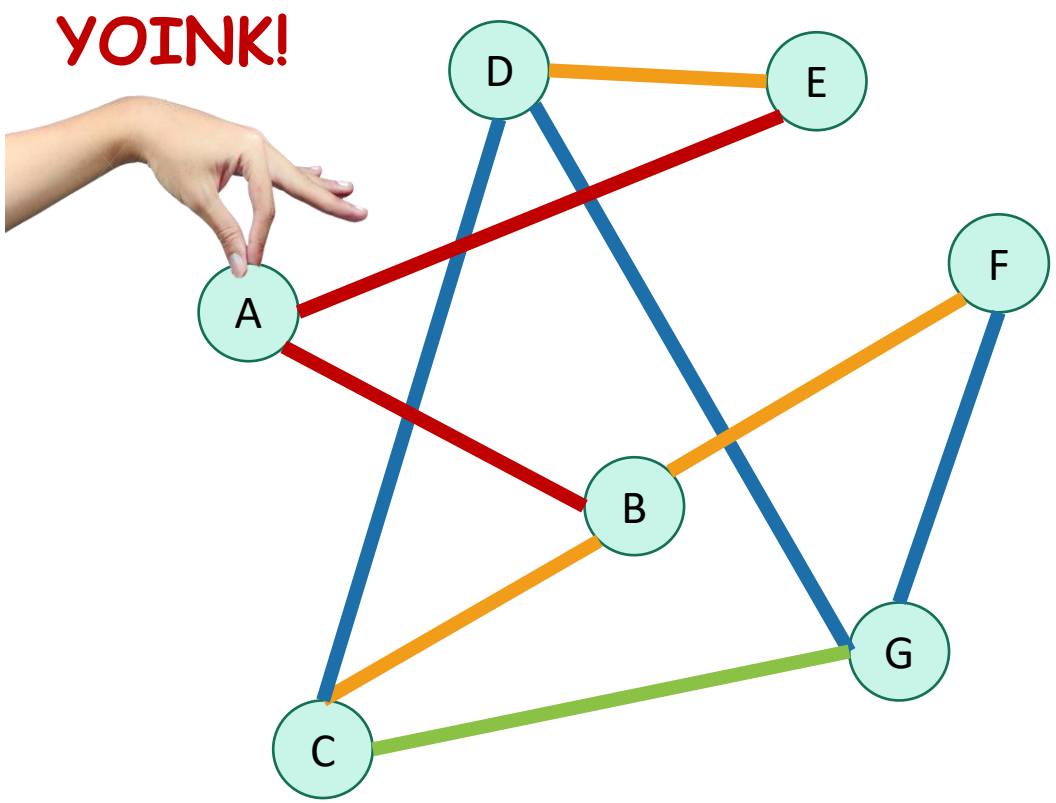

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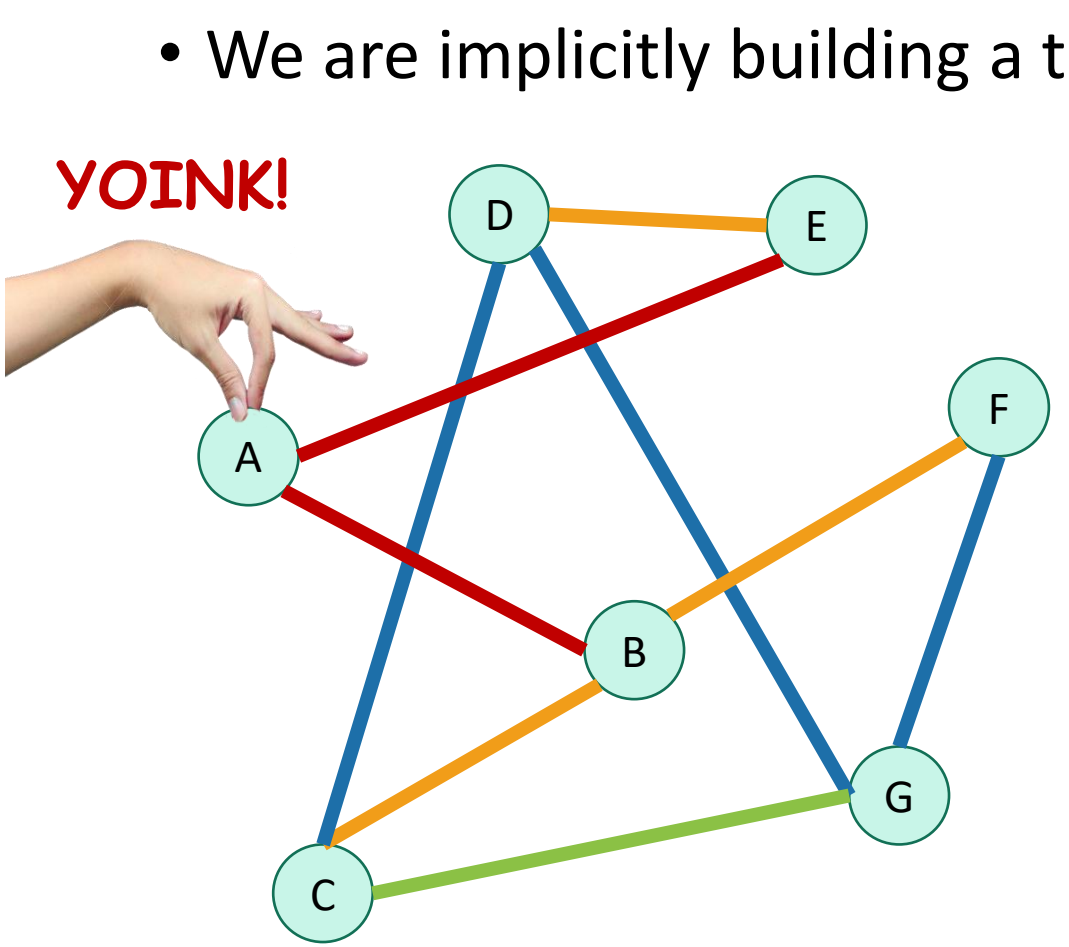

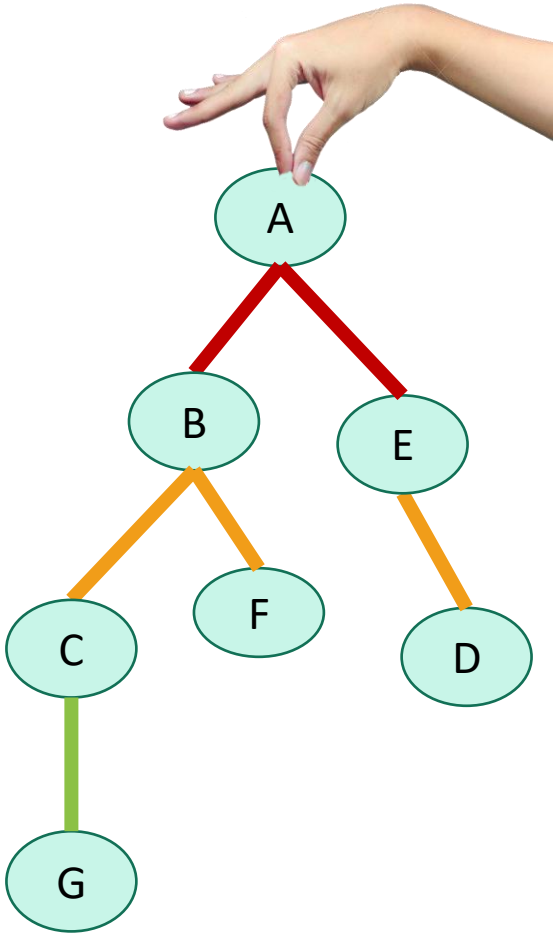

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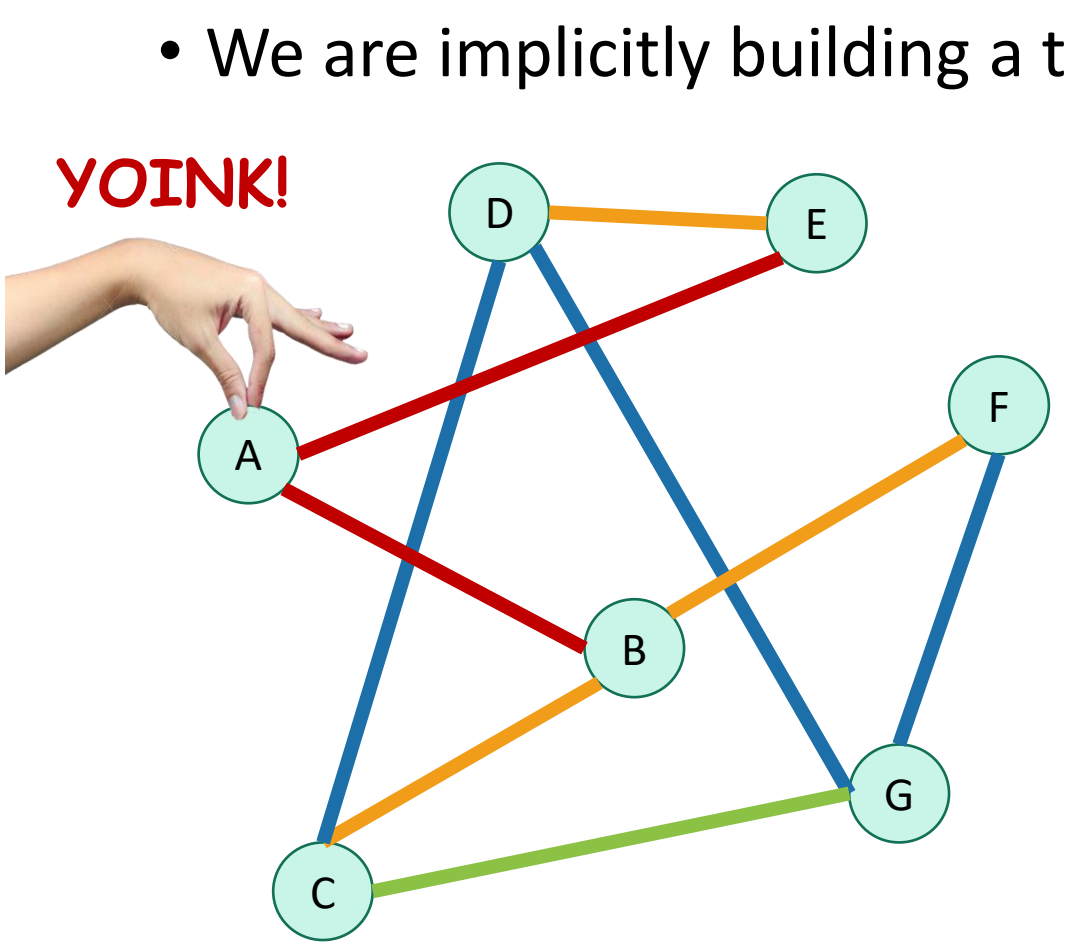

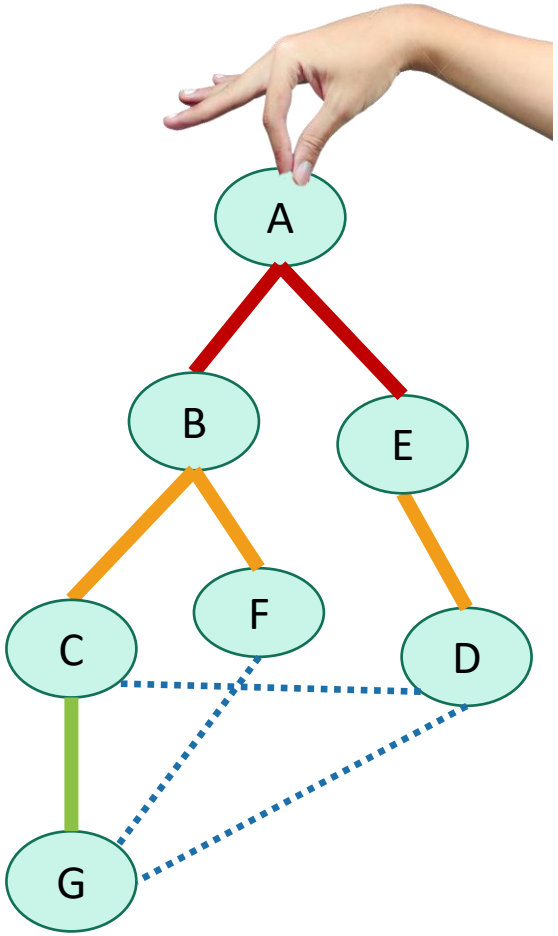

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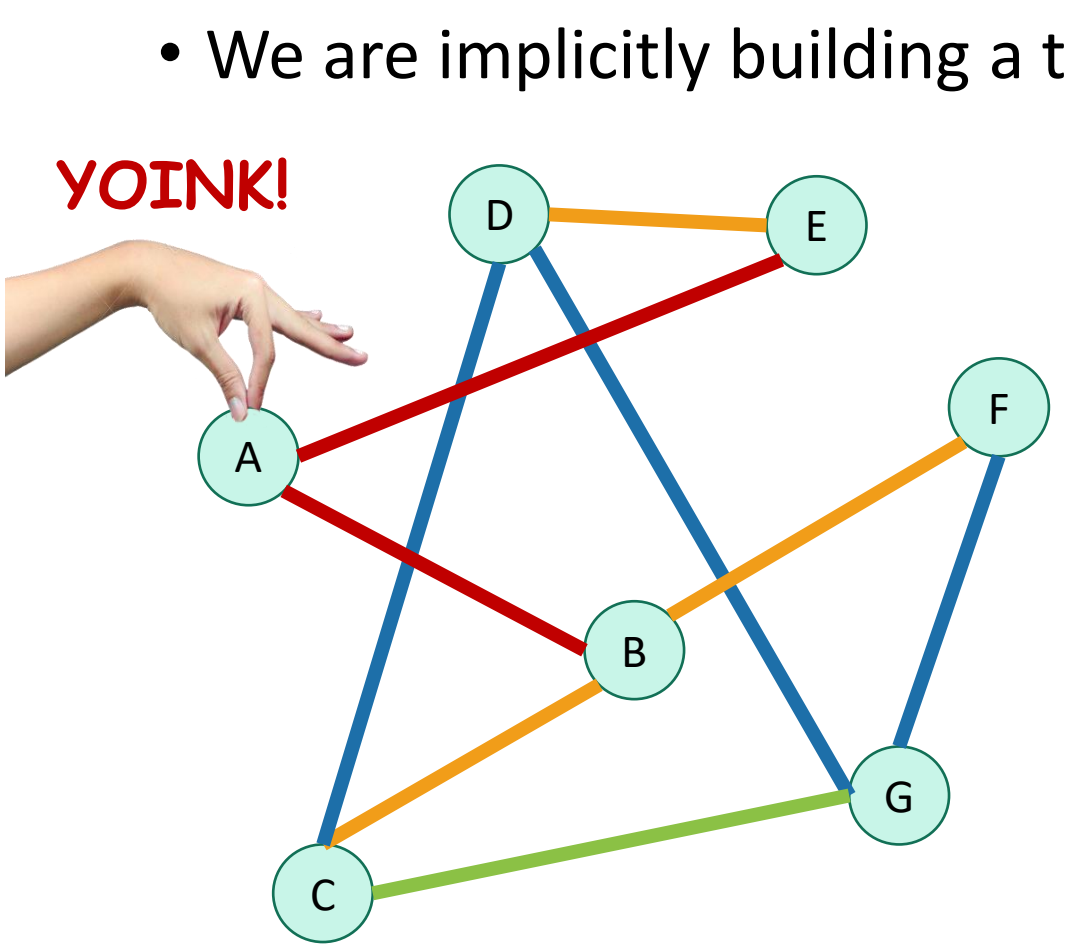

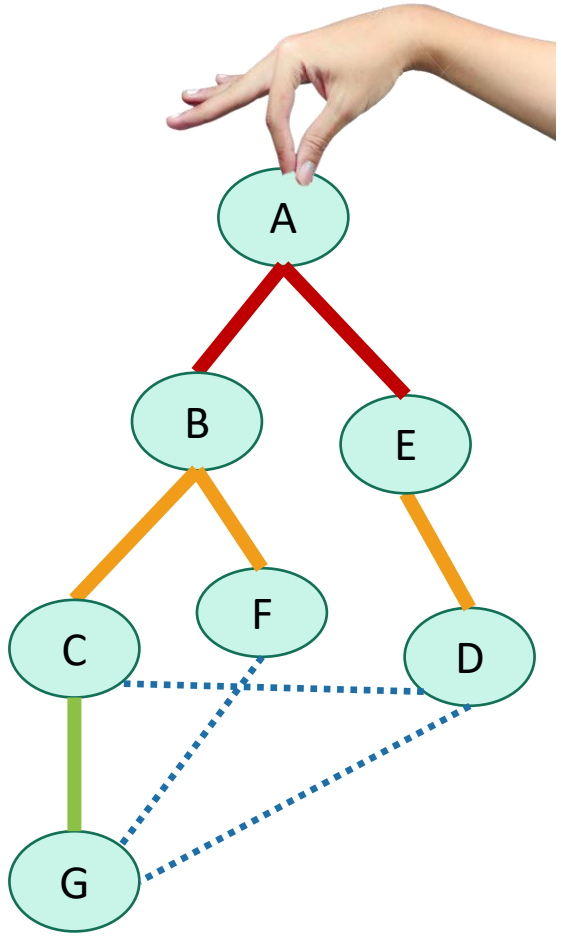

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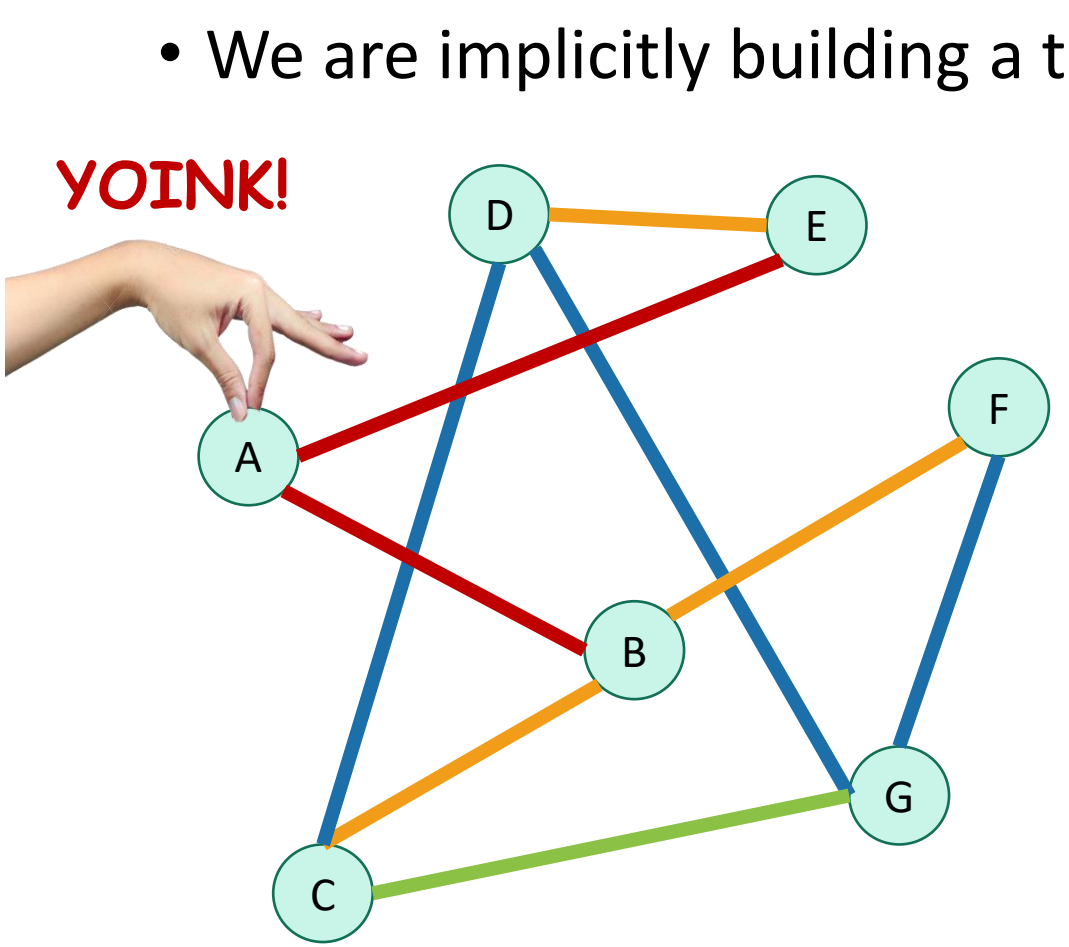

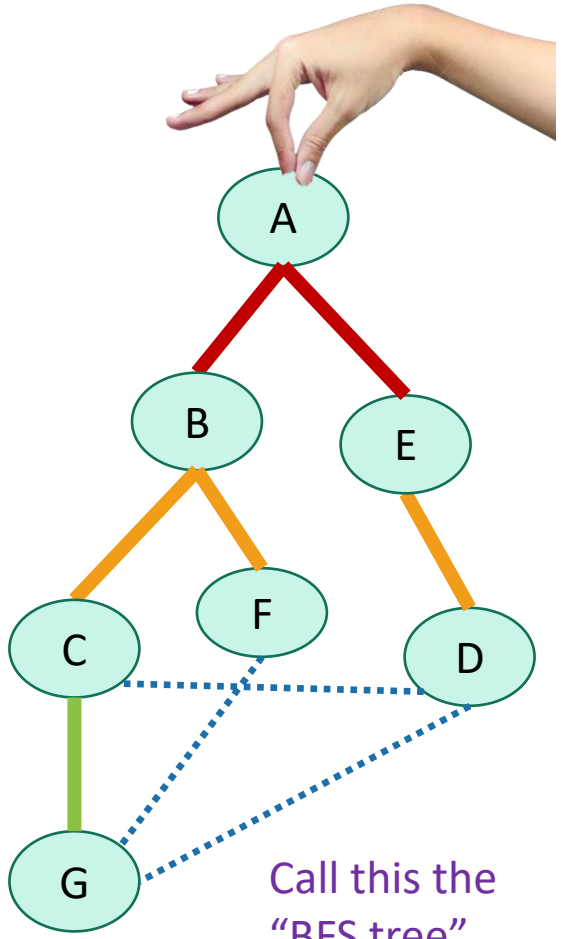

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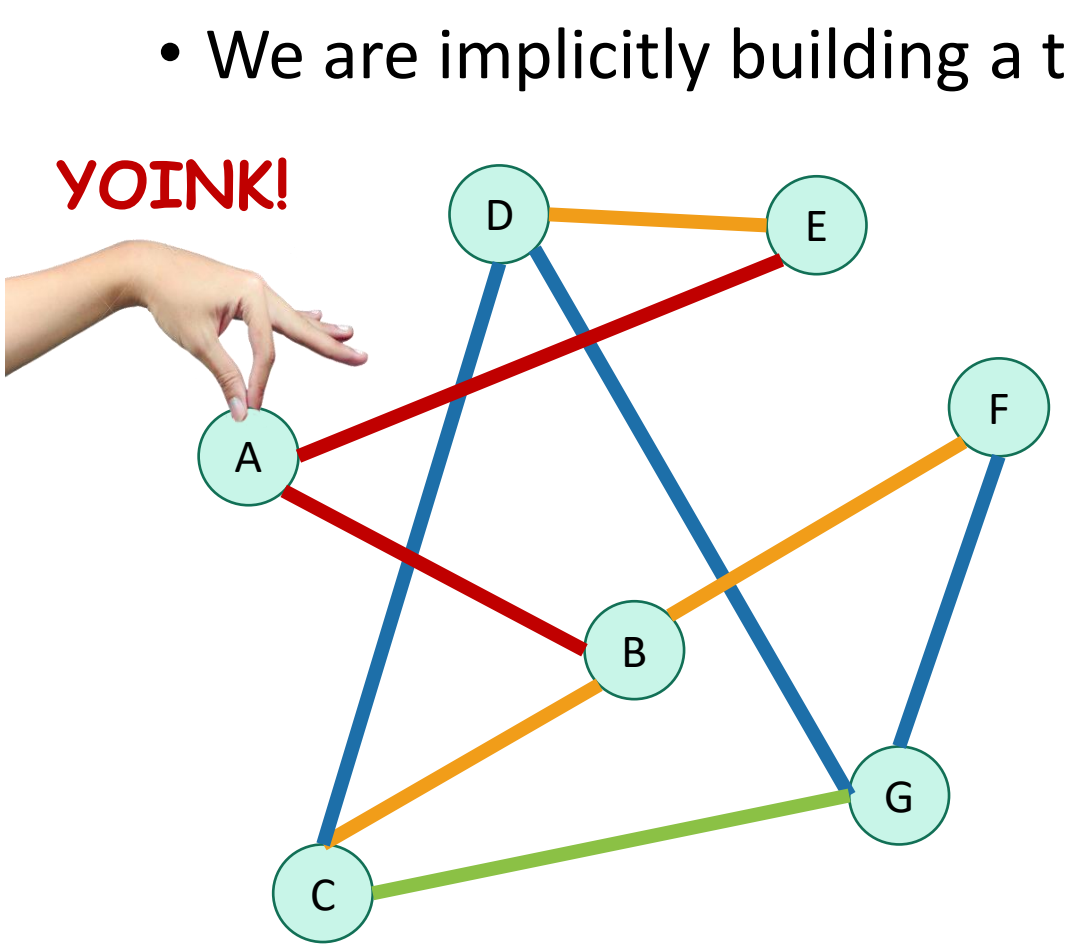

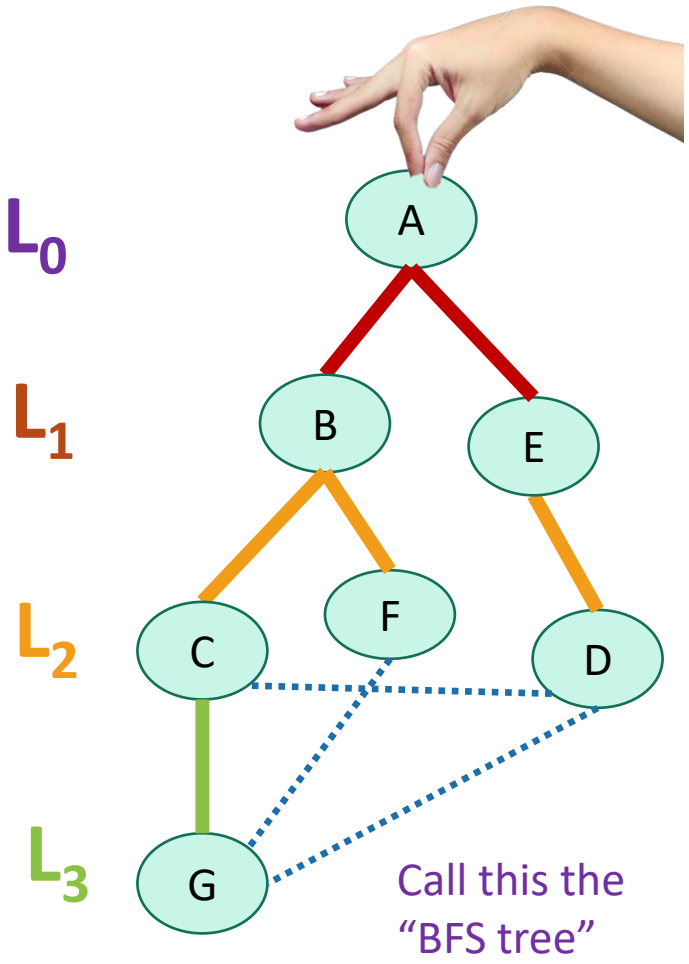

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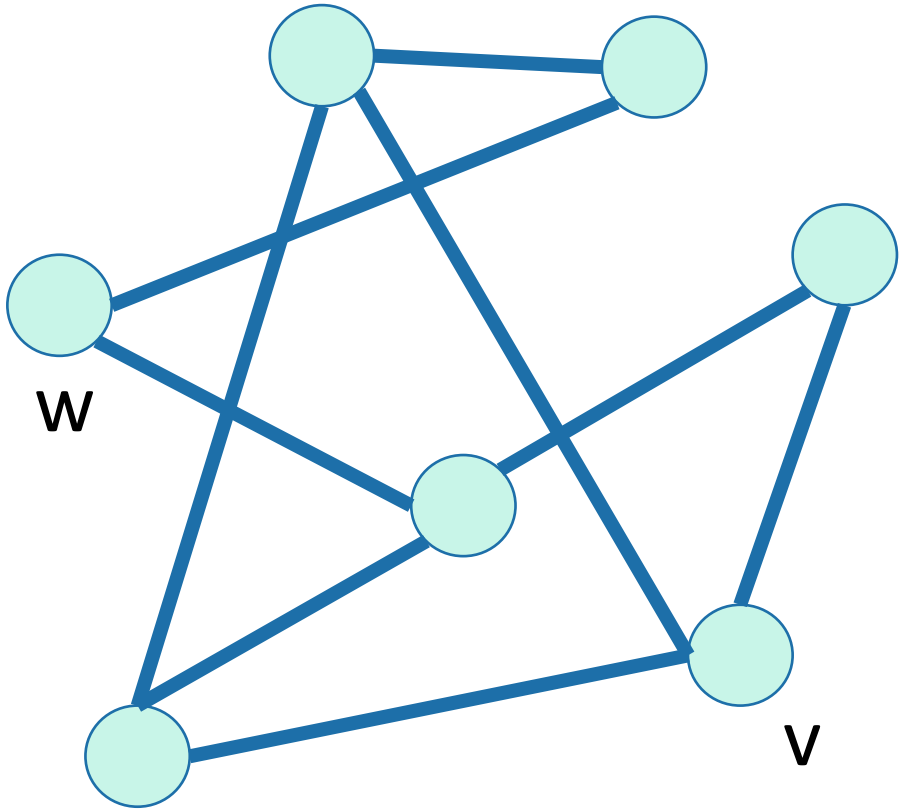
Application of BFS: shortest path

- How long is the shortest path between w and v?



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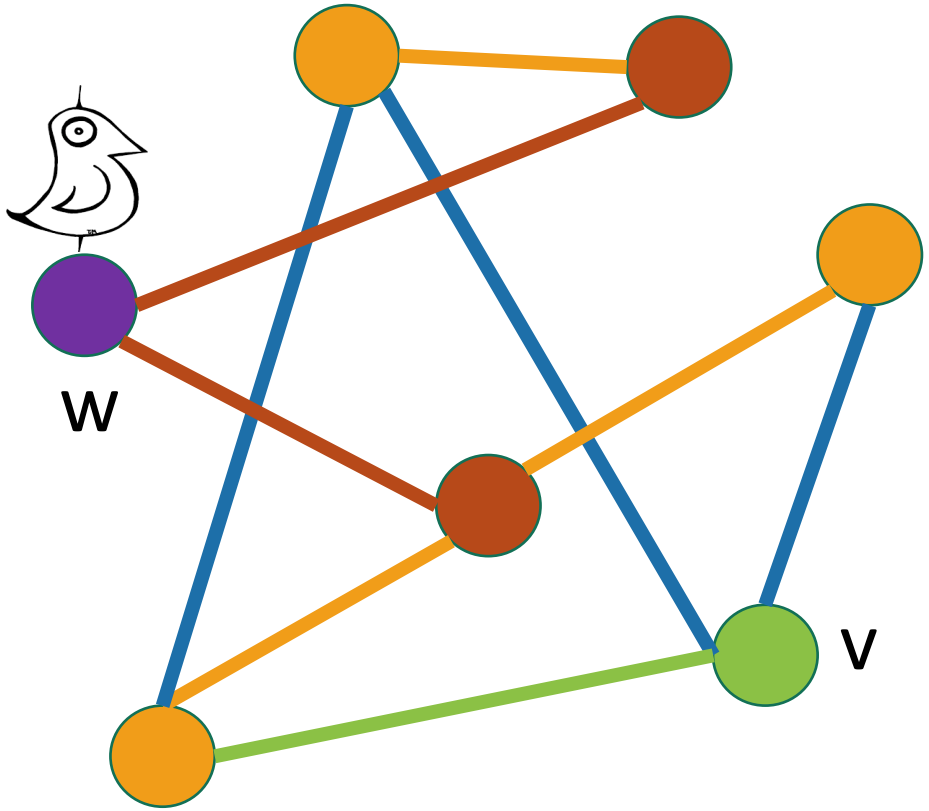

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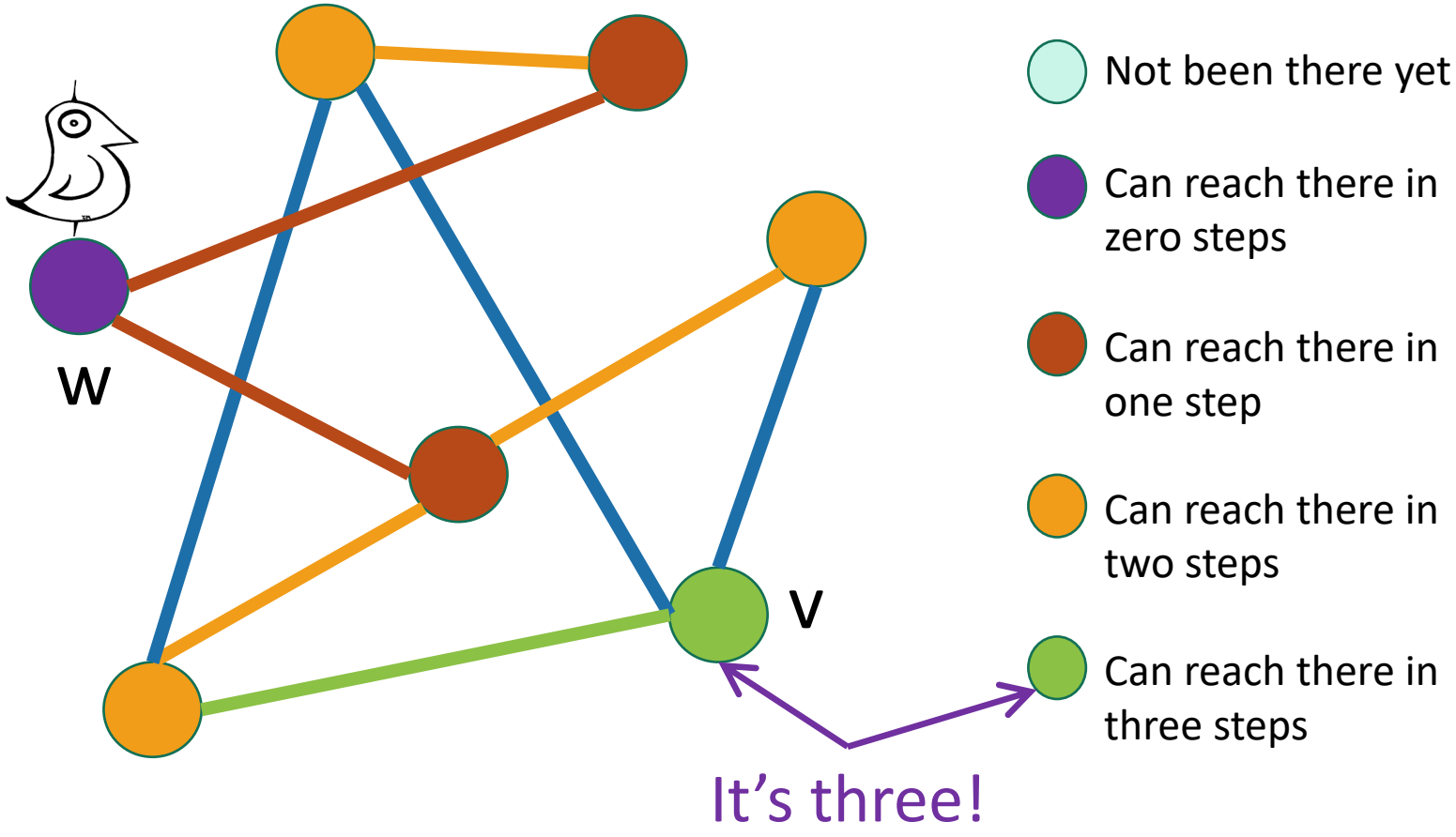

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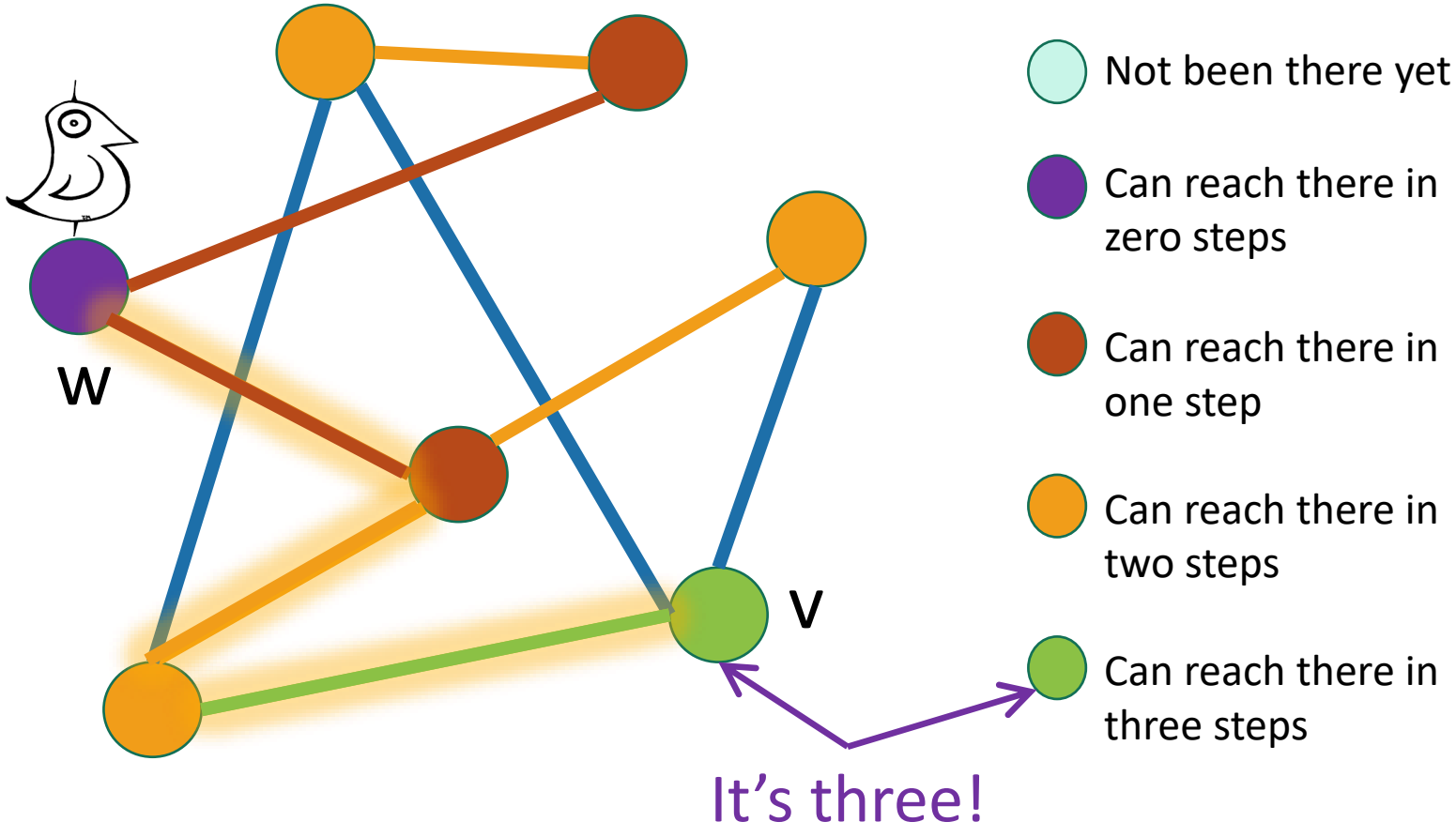
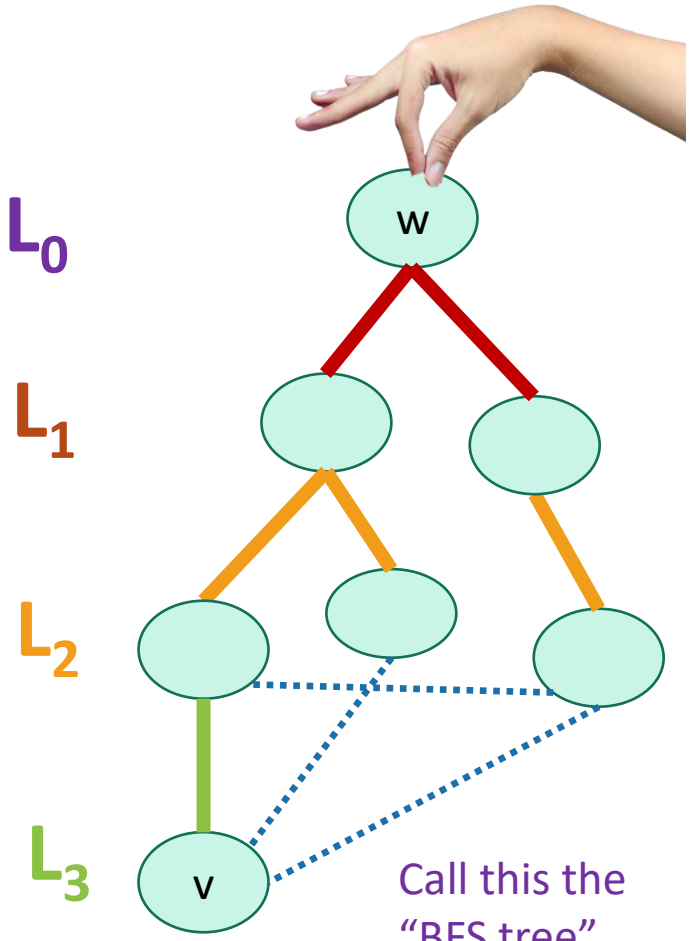


To find the distance between w and all other vertices v

The **distance** between two vertices is the number of edges in the shortest path between them.



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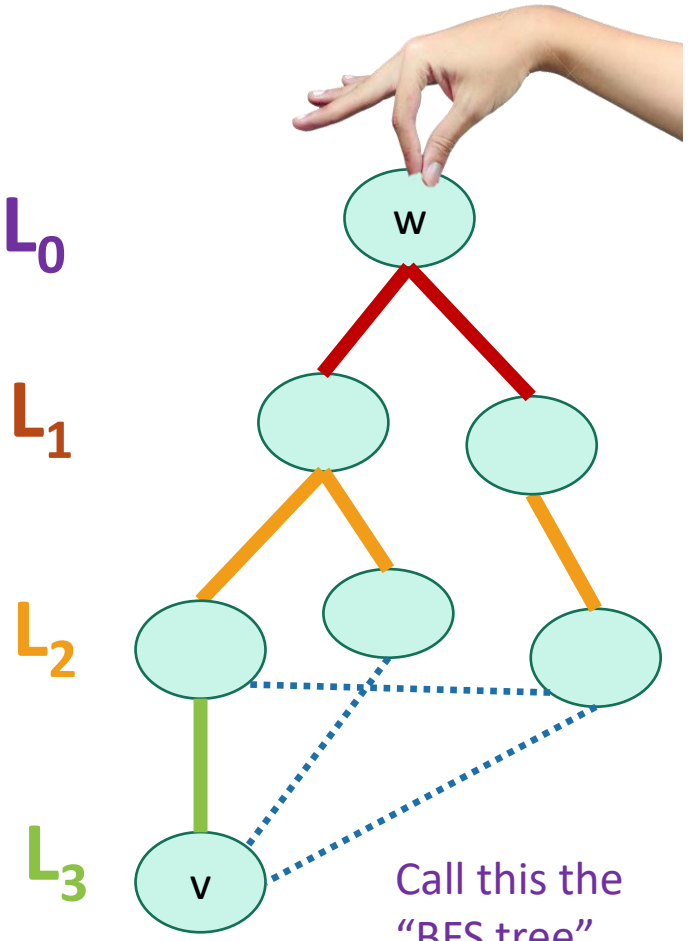

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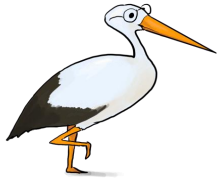
Modify the BFS pseudocode to return shortest paths!



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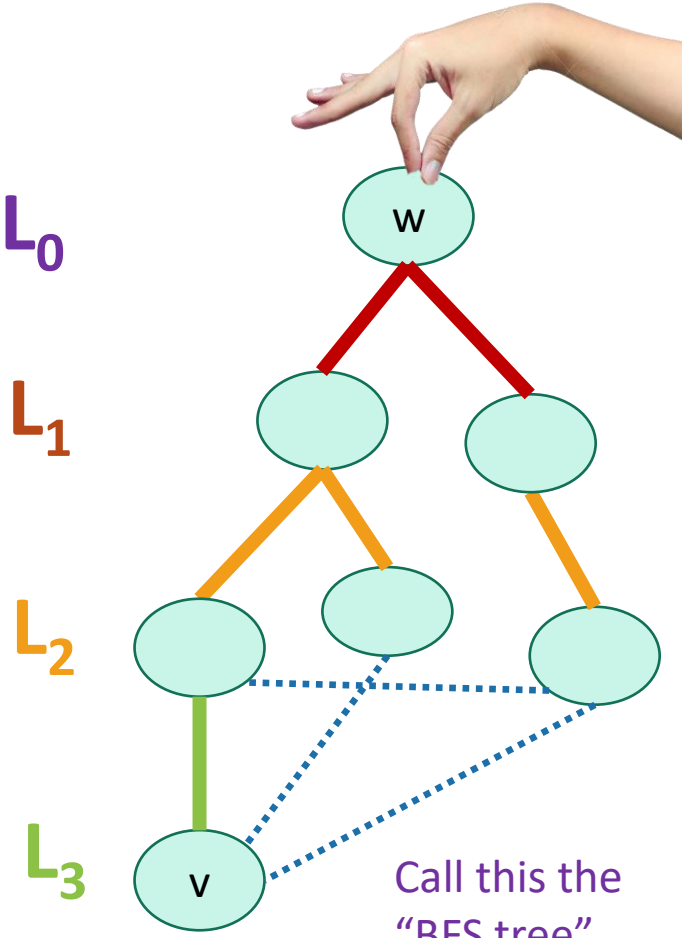

What have we learned?

- The BFS tree is useful for computing distances between pairs of vertices.
- We can find the shortest path between u and v in time O(m).

Another application of BFS

- Testing bipartite-ness

Exercise: fish

- You have a bunch of fish and two fish tanks.
- Some pairs of fish will fight if put in the same tank.
  - Model this as a graph: connected fish will fight.

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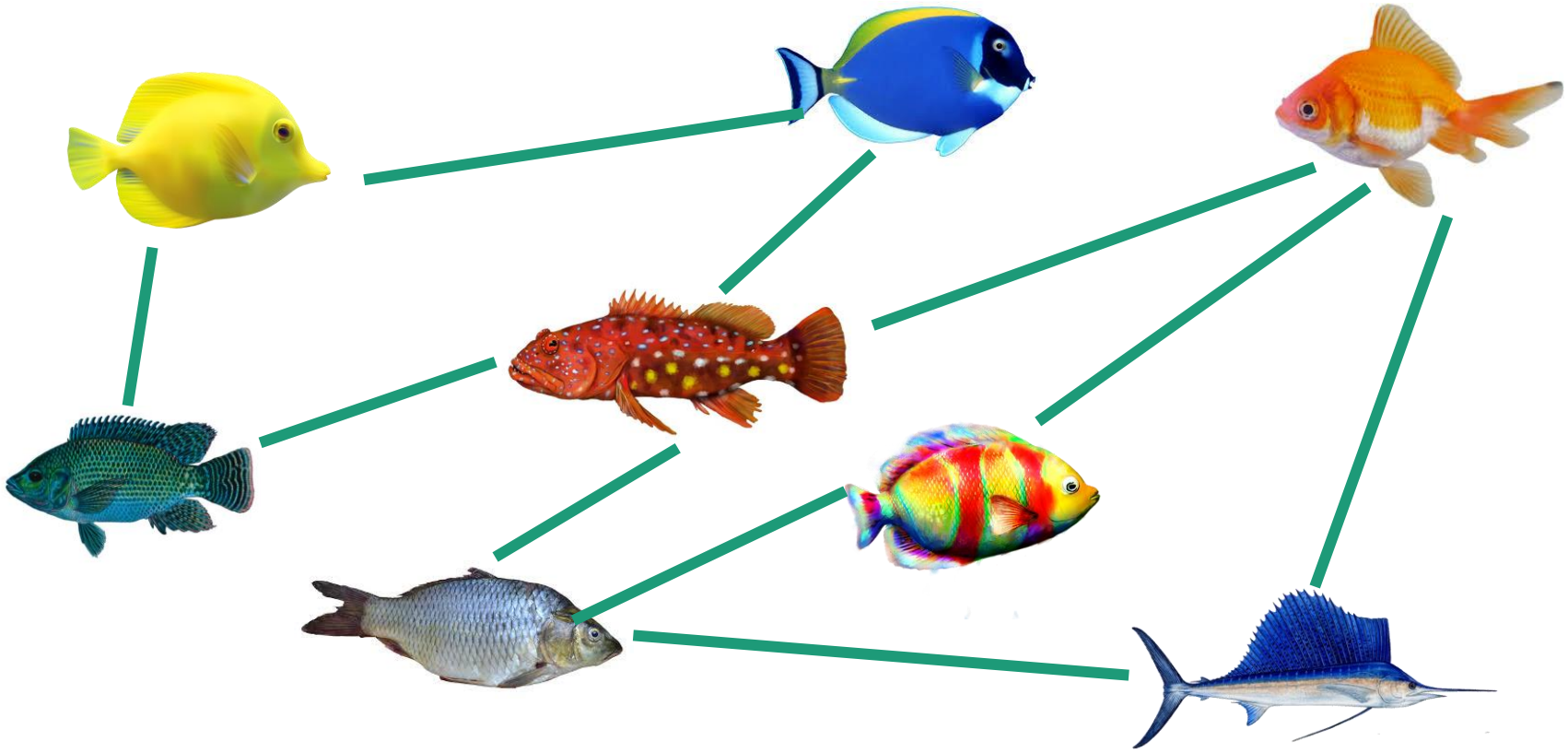

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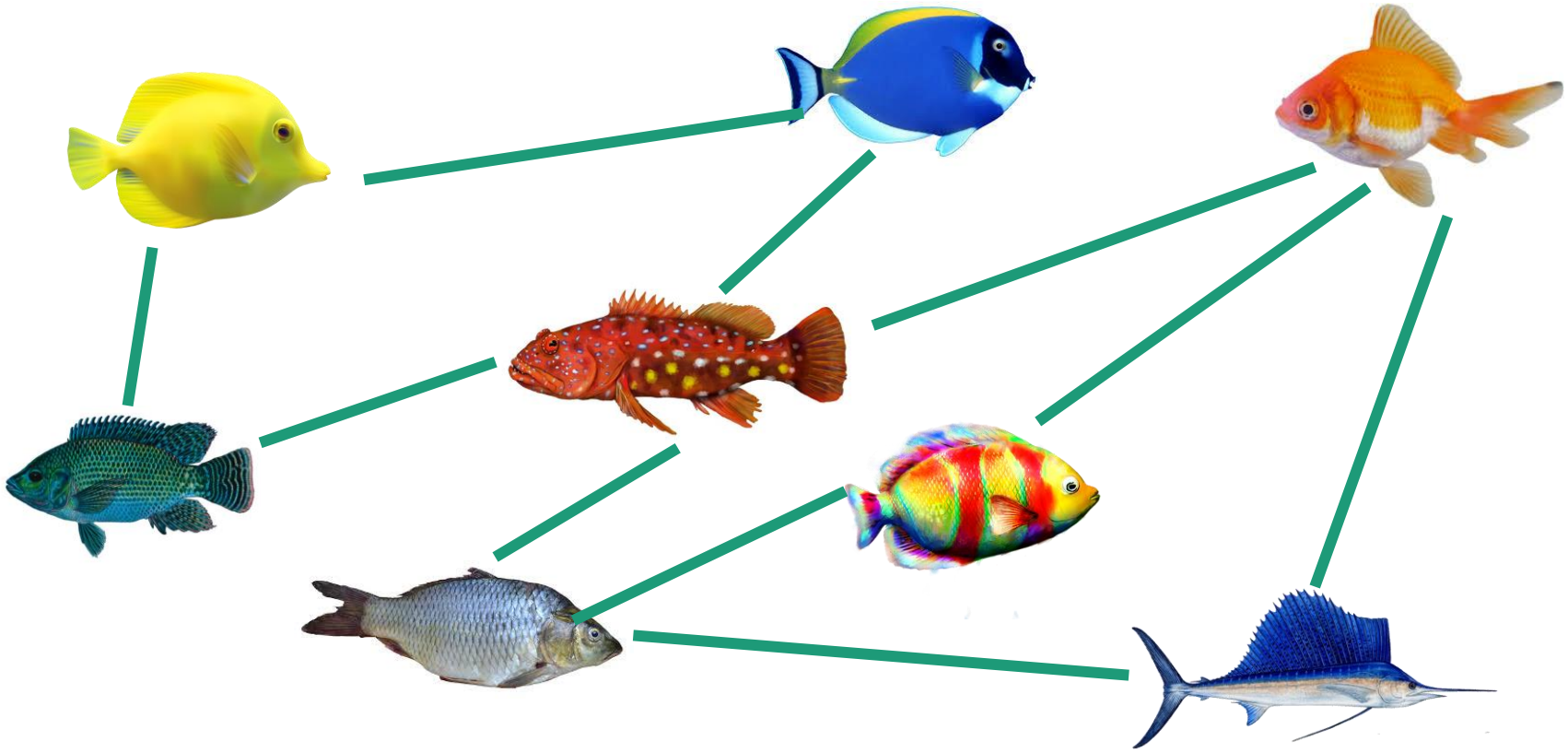
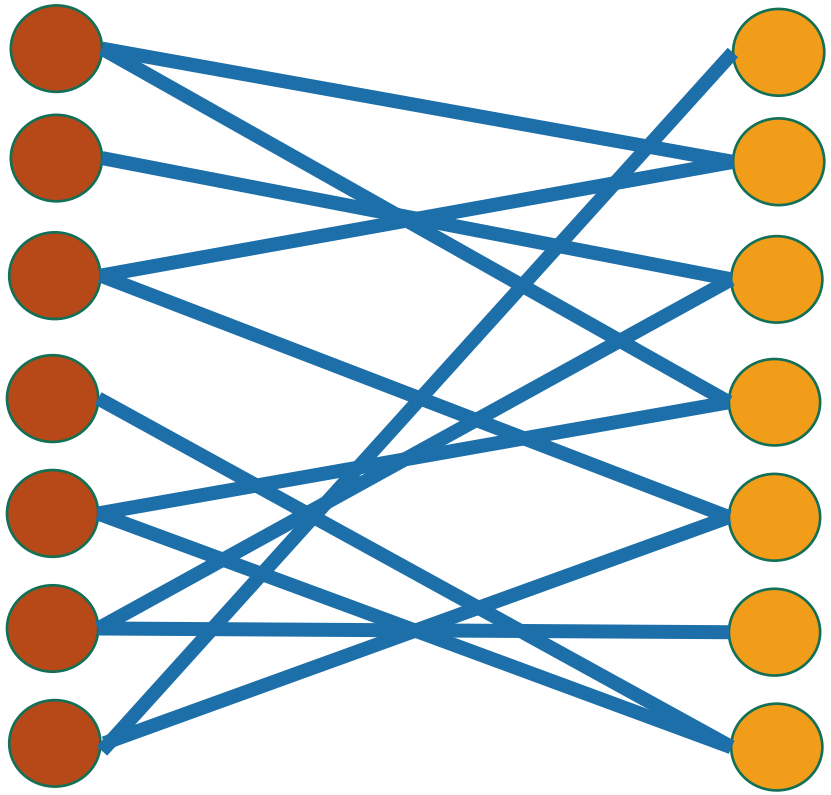
- Can you put the fish in the two tanks so that there is no fighting?



Bipartite graphs

- A bipartite graph looks like this:



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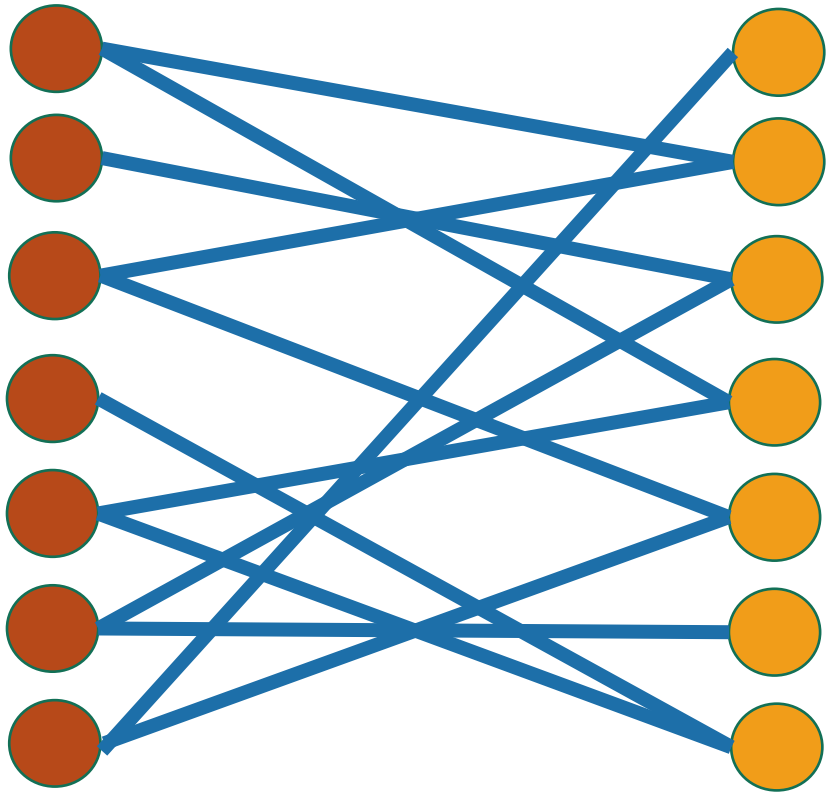


Can color the vertices red and orange so that there are no edges between any same-colored vertices

Example:

are in tank A

are in tank B

if the fish fight

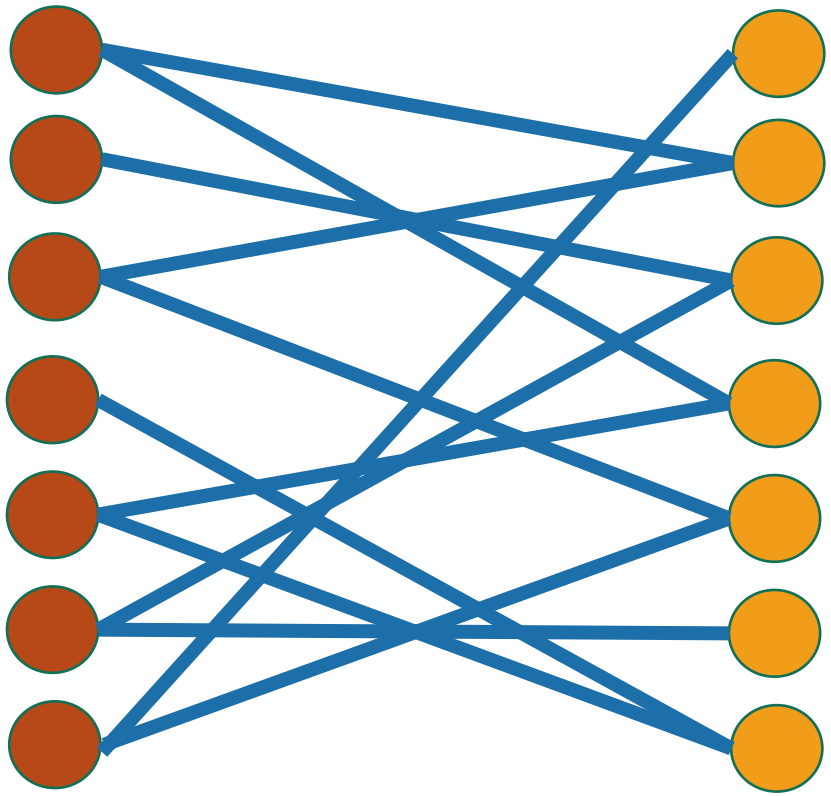
Example:

are students

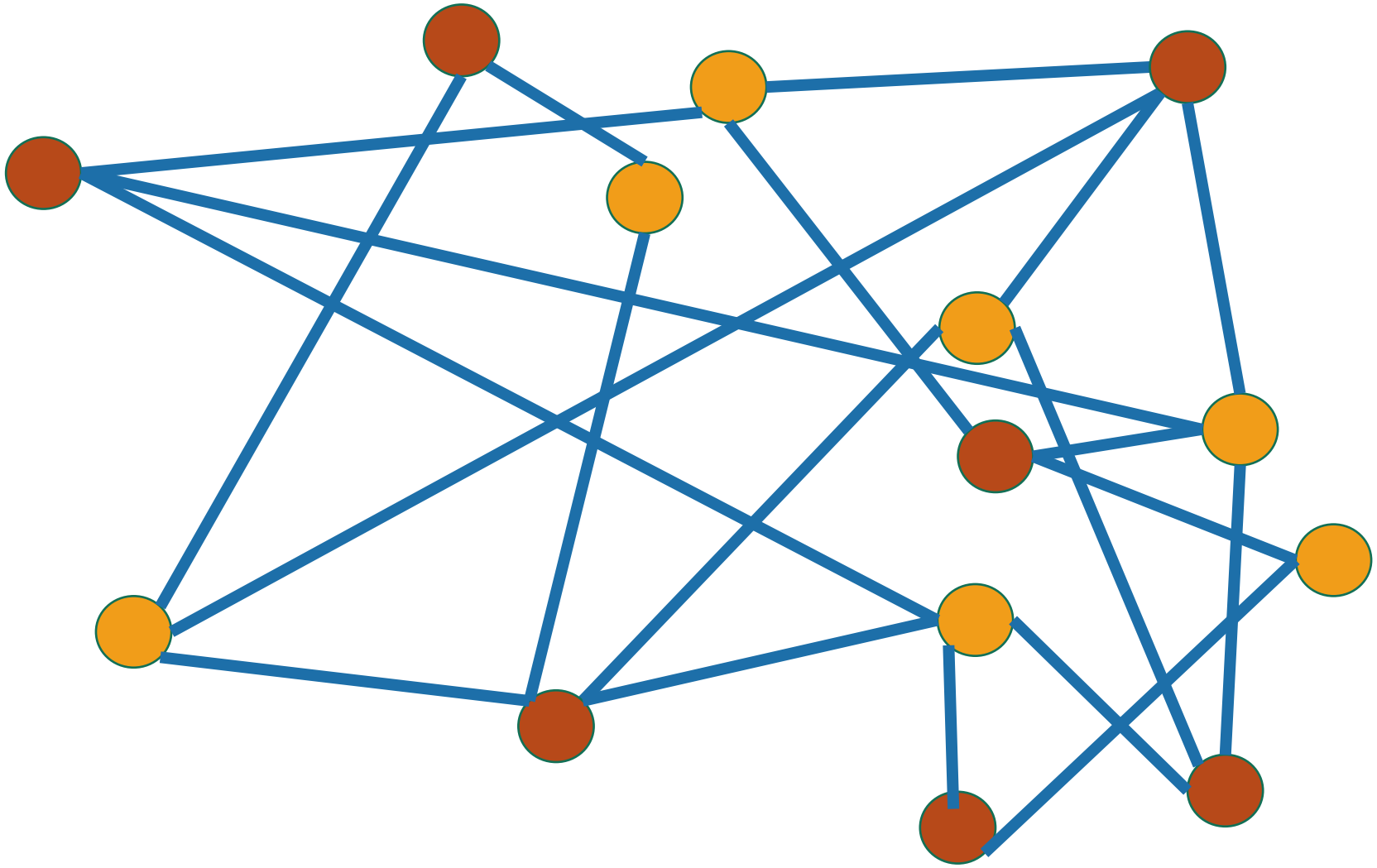
are classes

if the student is enrolled in the class

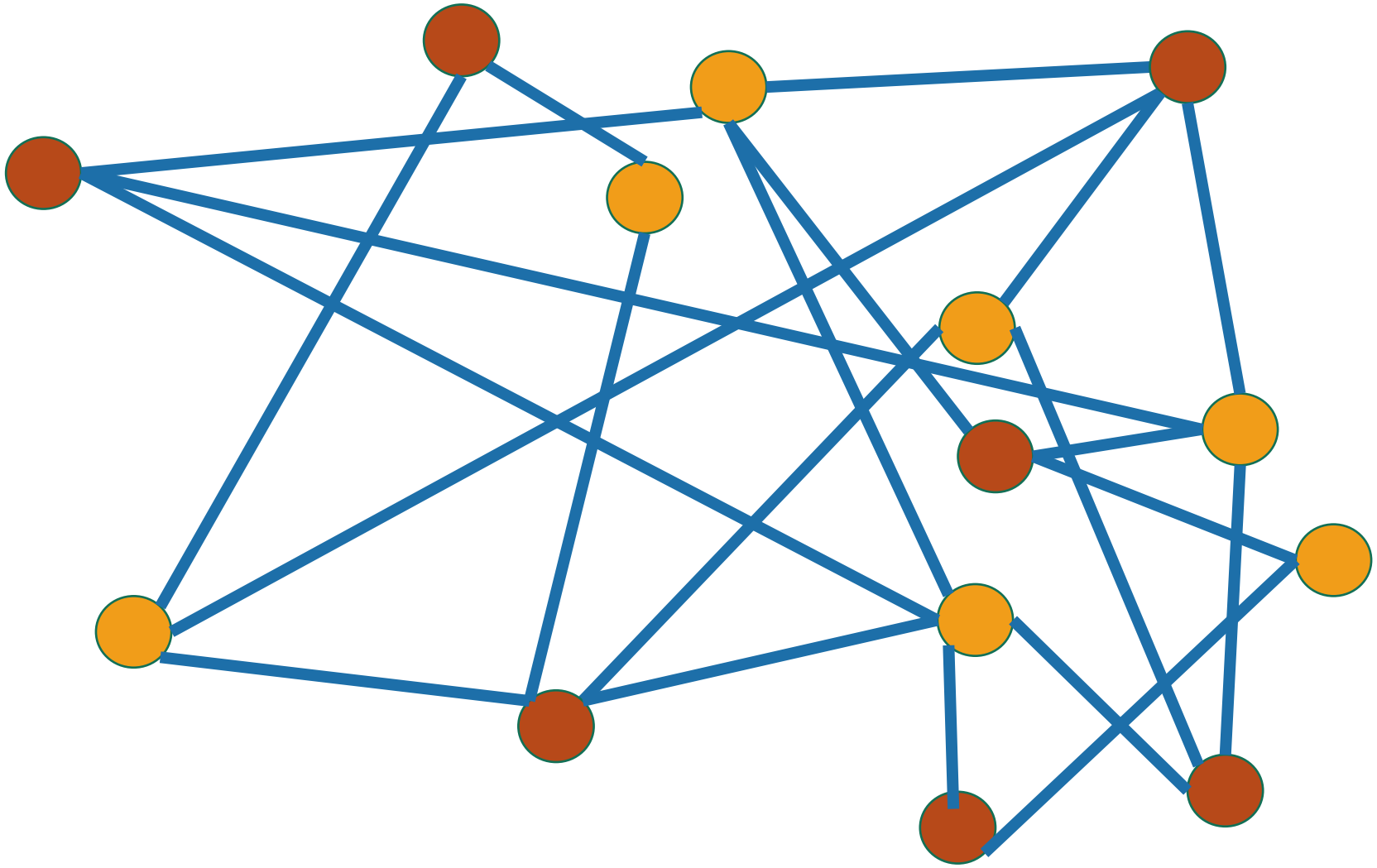
Is this graph bipartite?



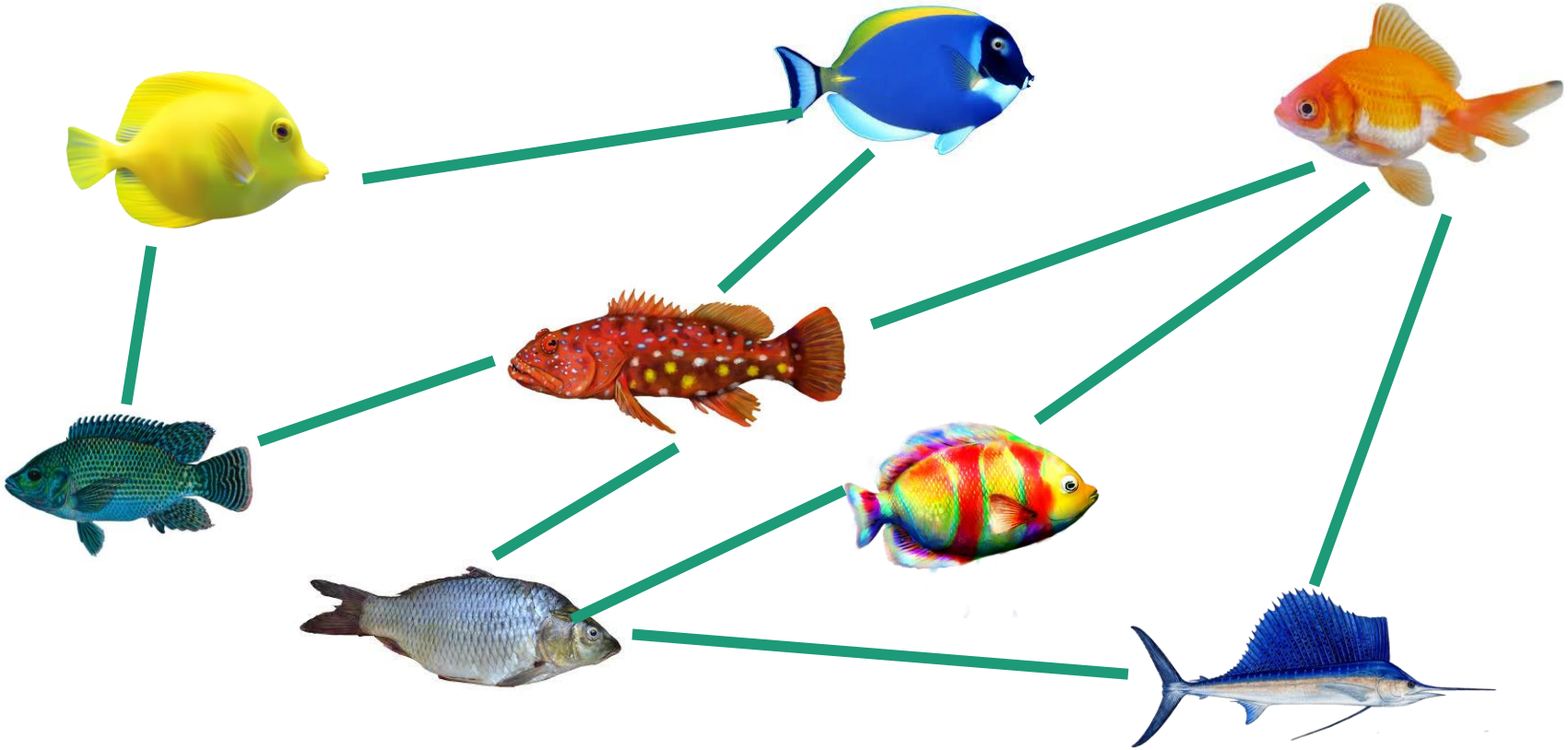
How about this one?



How about this one?



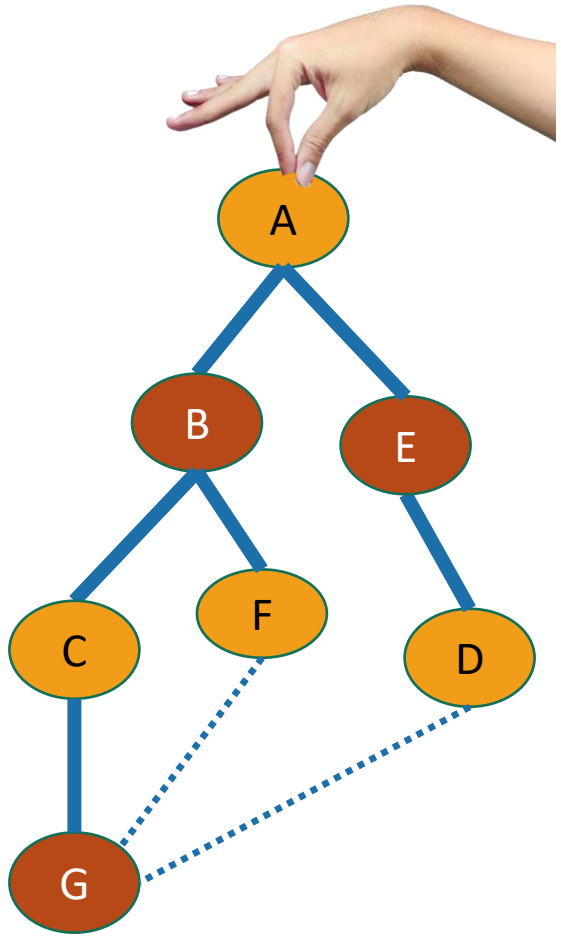
This one?



Application of BFS:

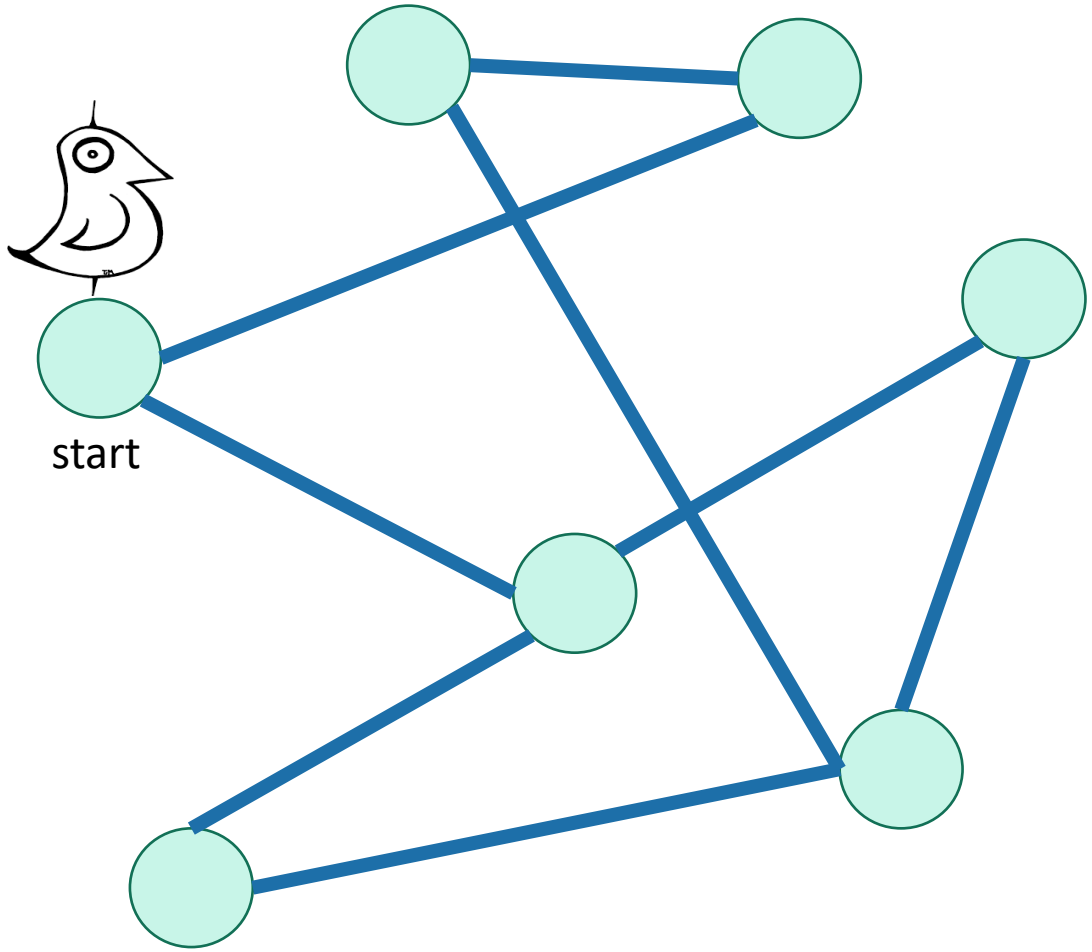
Testing Bipartiteness

- Color the levels of the BFS tree in alternating colors.
- If you never color two connected nodes the same color, then it is bipartite.
- Otherwise, it's not.



Breadth-First Search

For testing bipartite-ness



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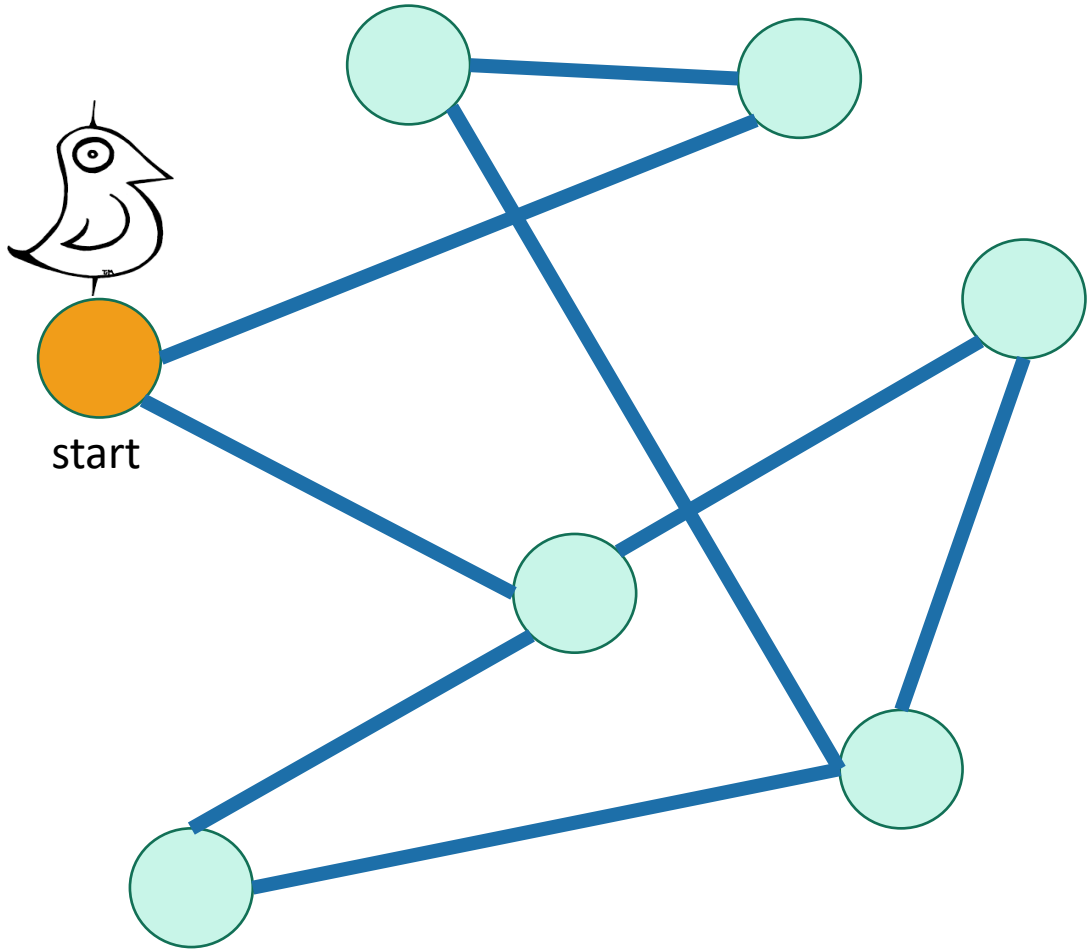
- Can reach there in three steps

Breadth-First Search

For testing bipartite-ness



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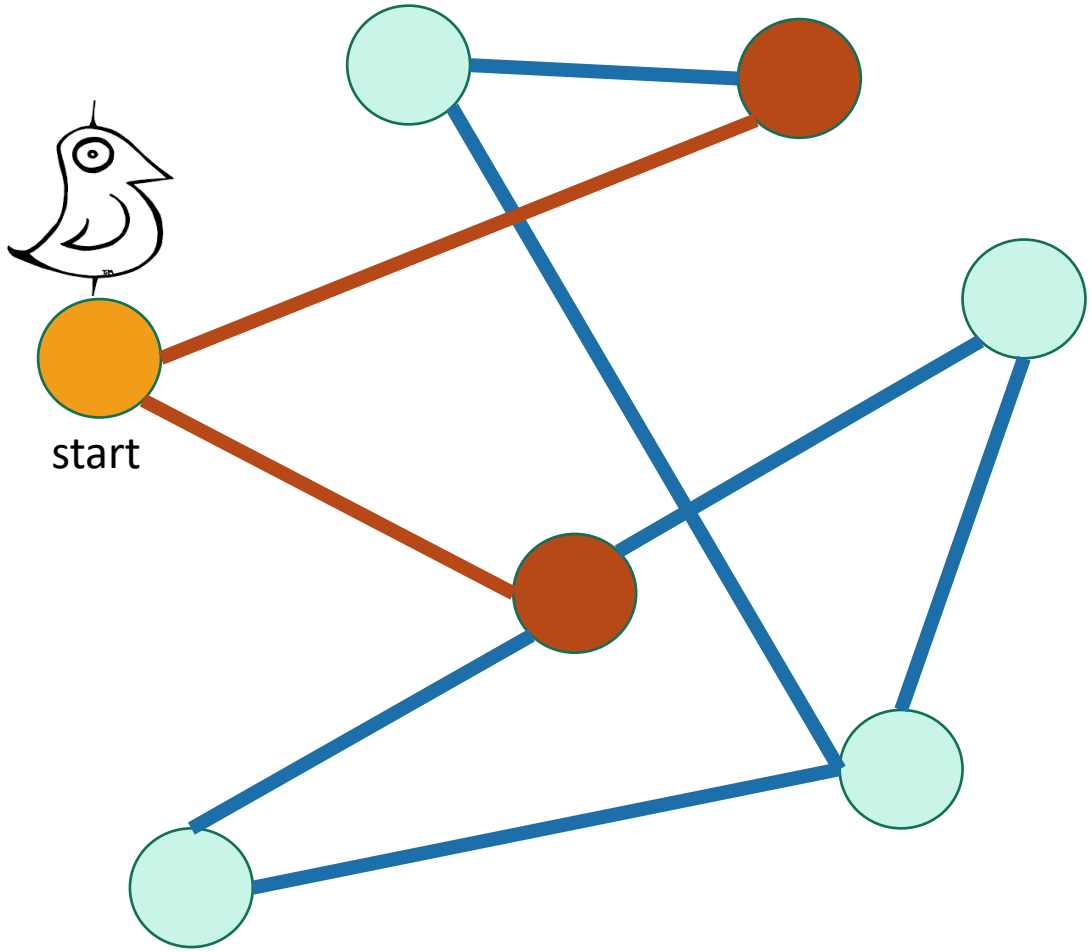

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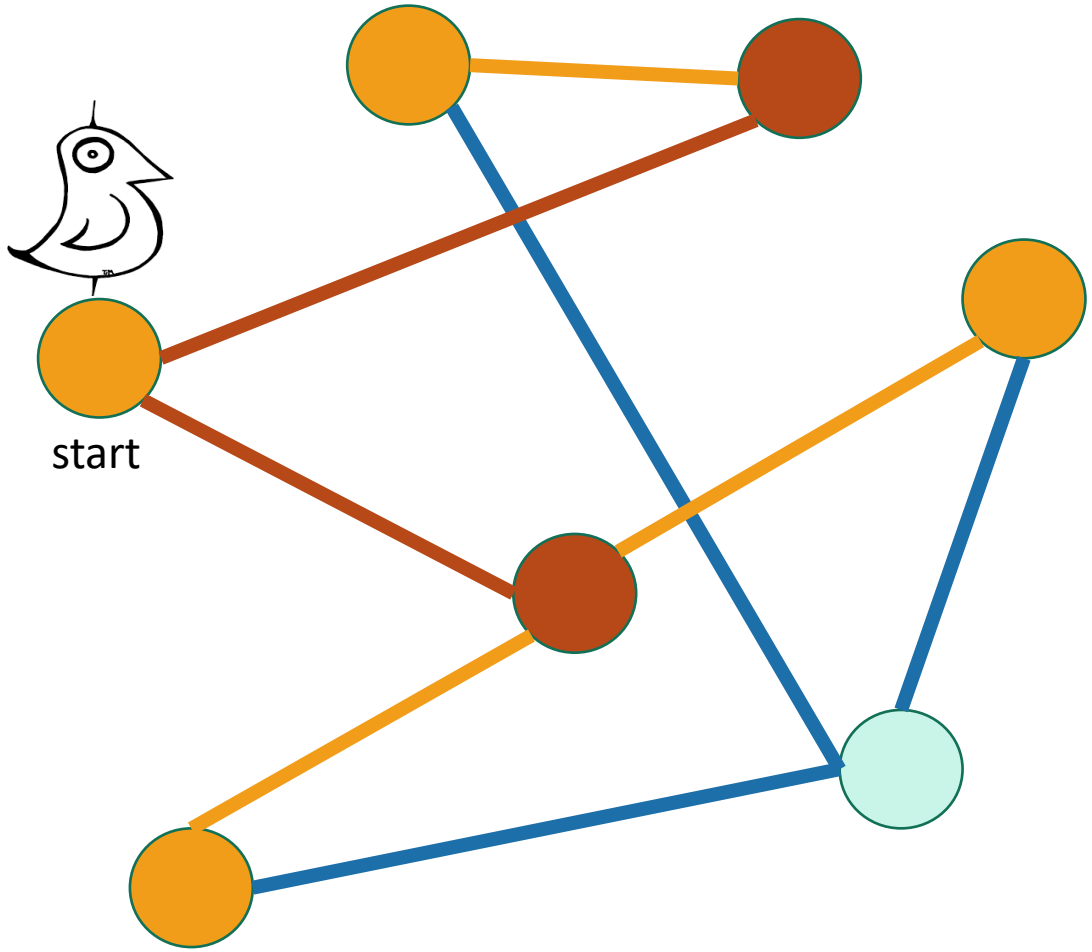

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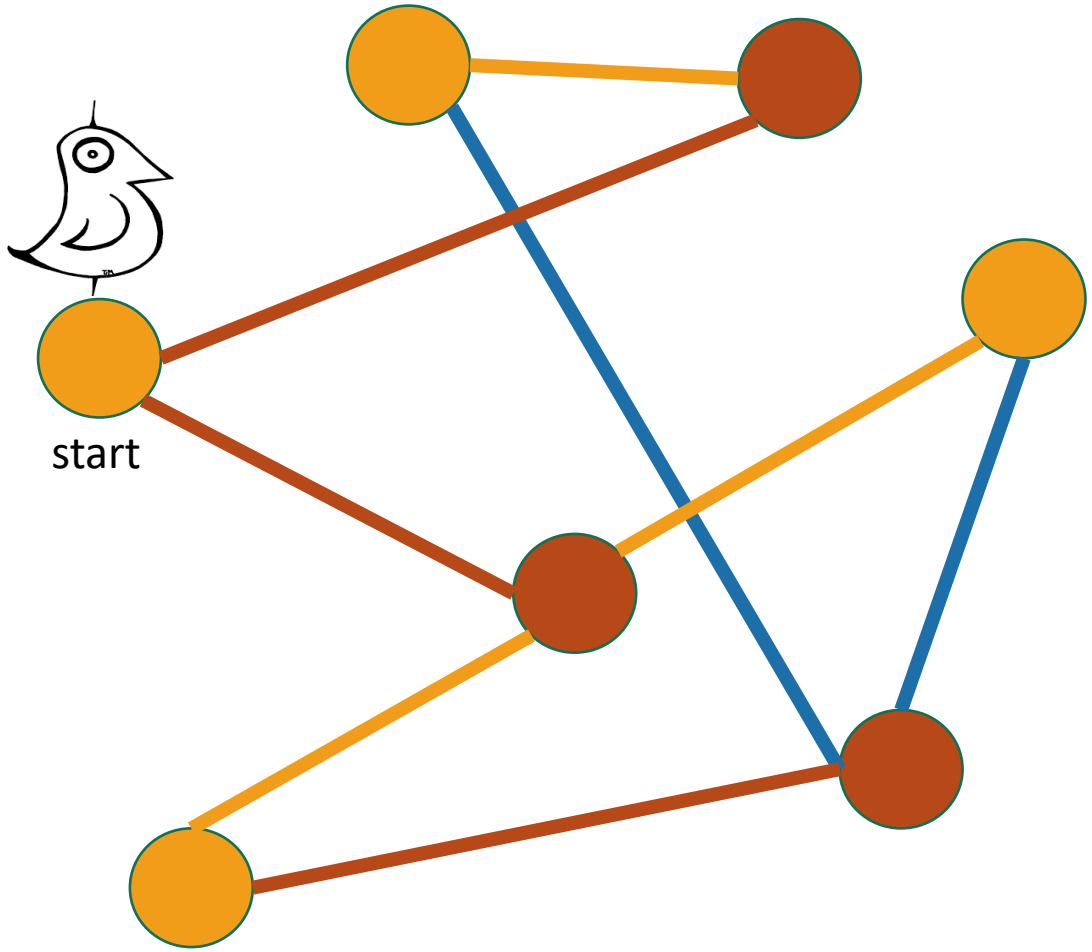

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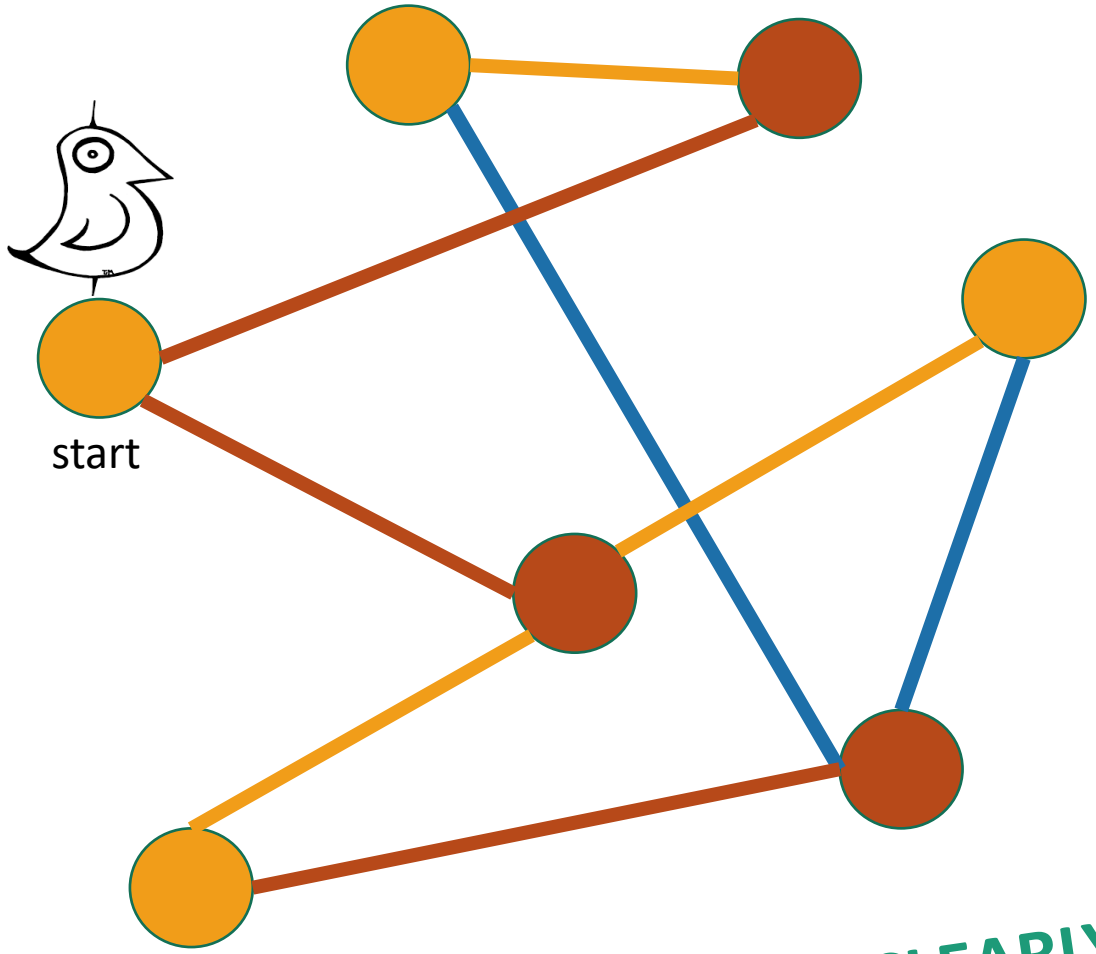

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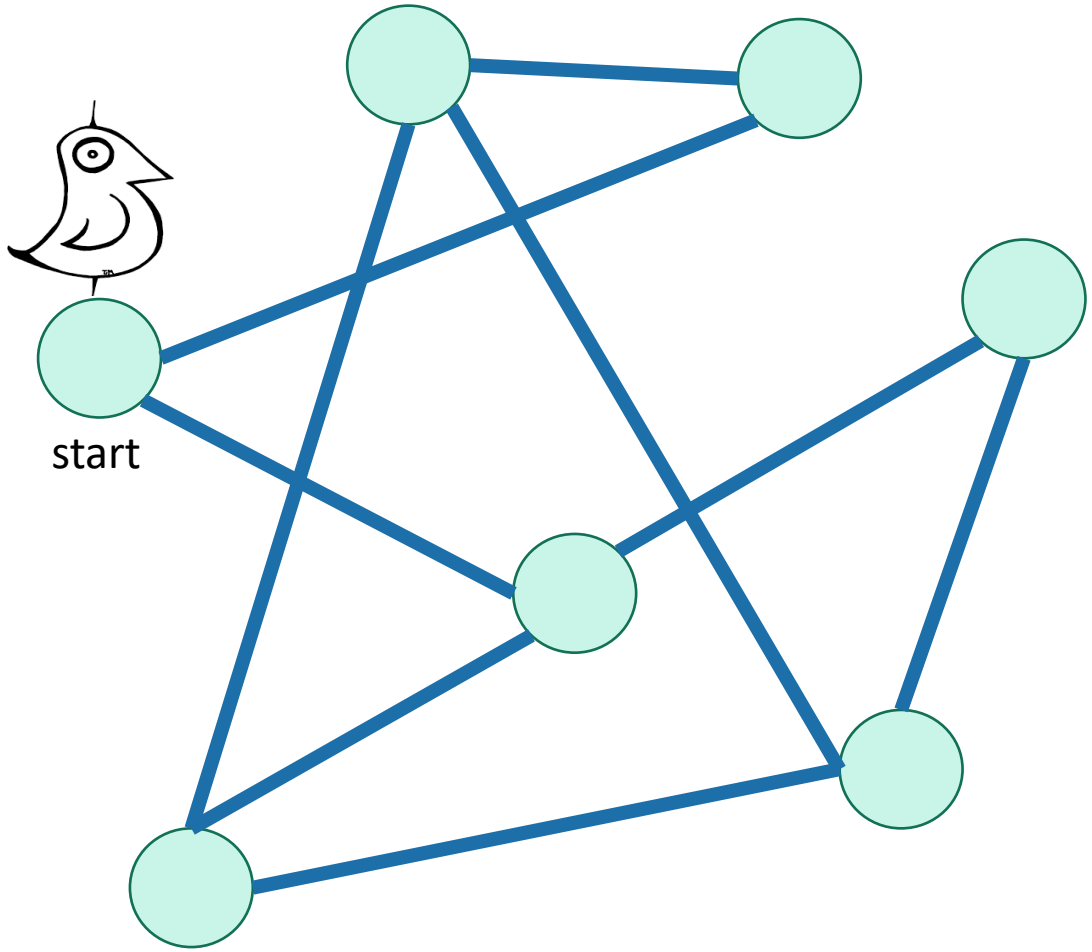
CLEARLY BIPARTITE!

Breadth-First Search

For testing bipartite-ness



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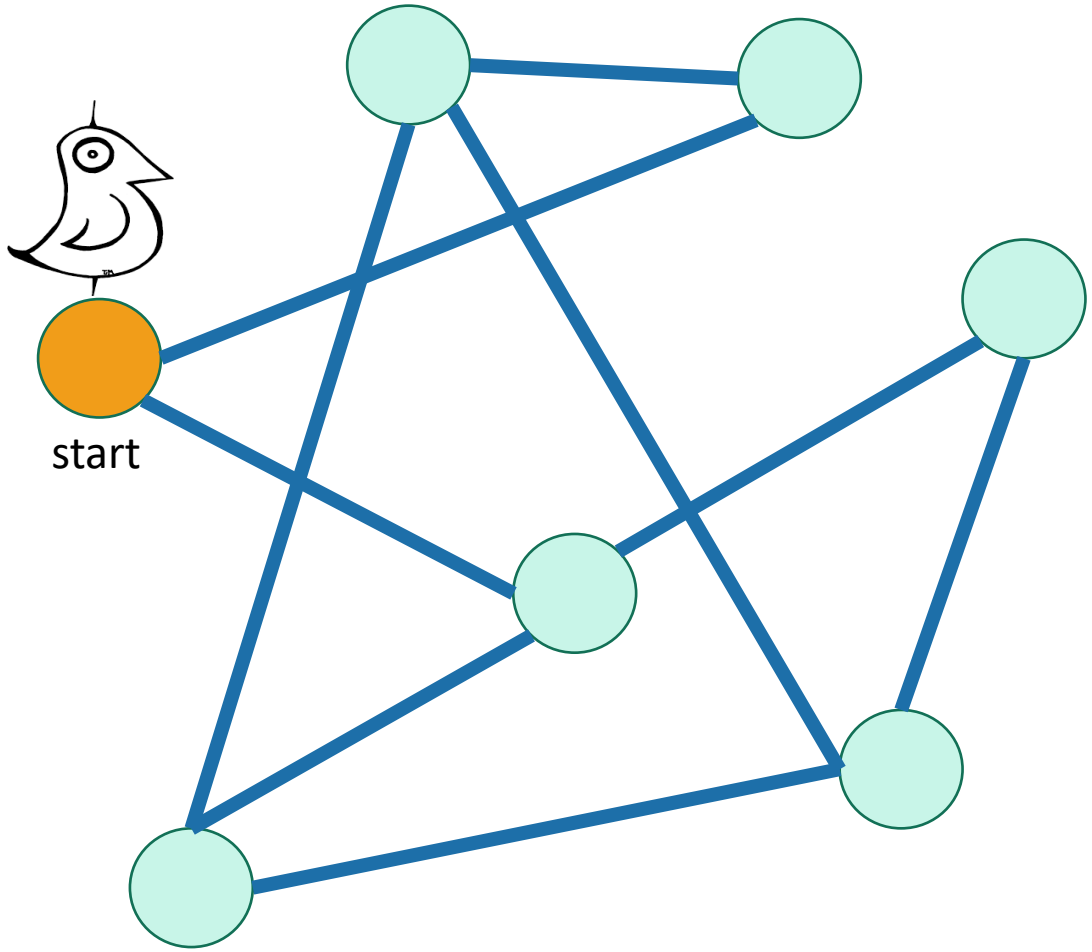
For testing bipartite-ness



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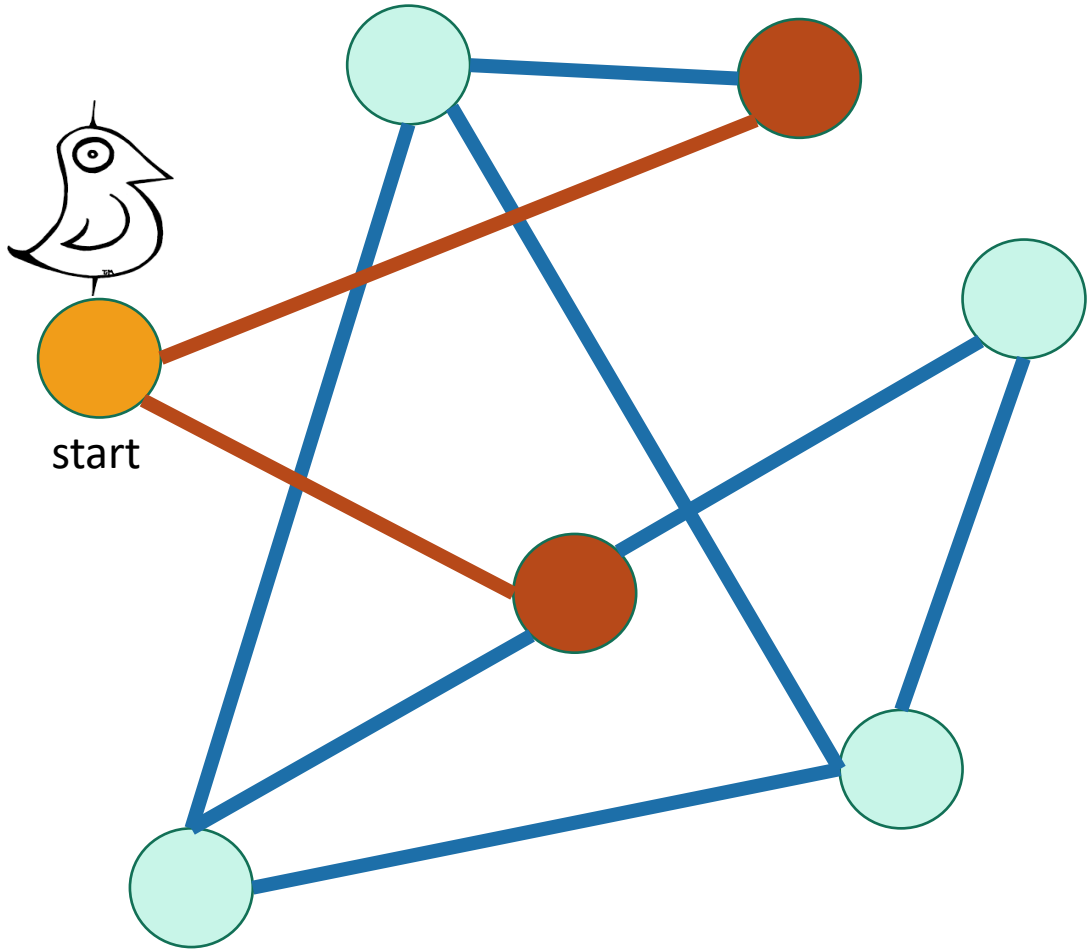

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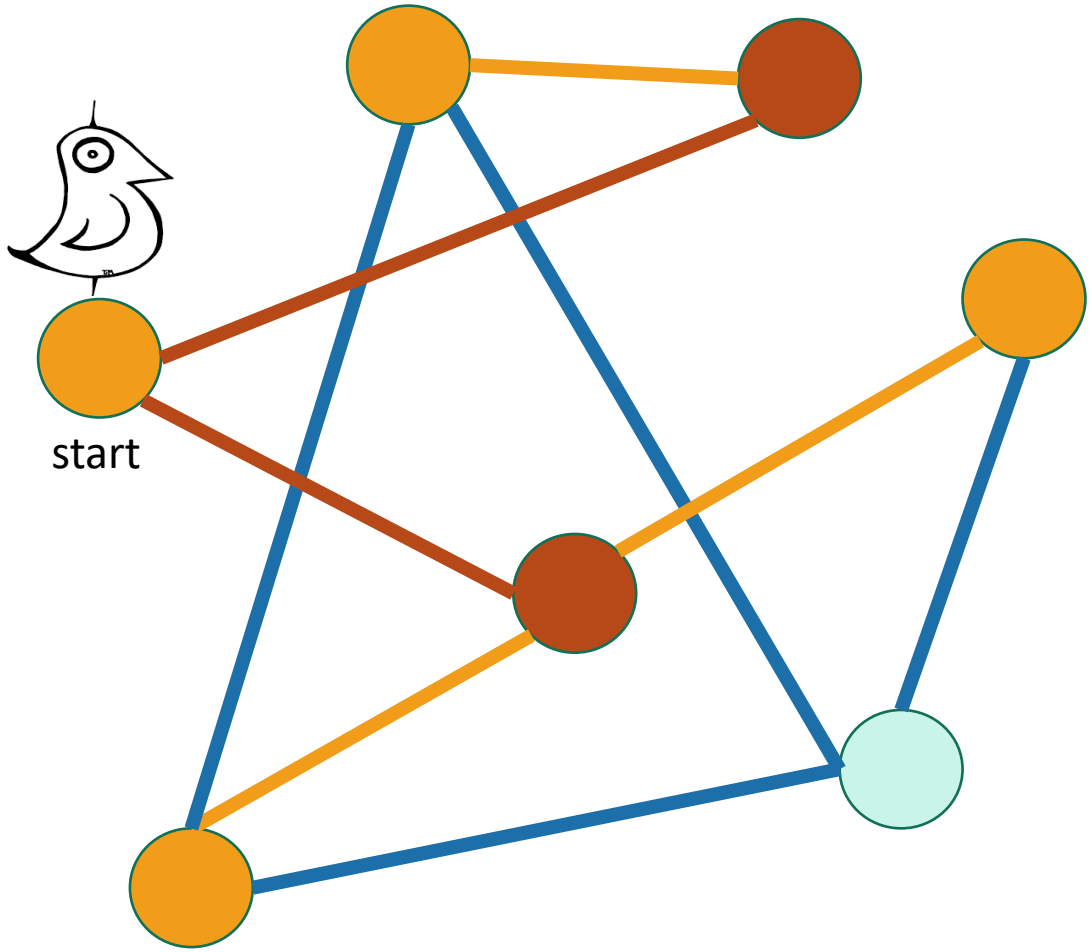

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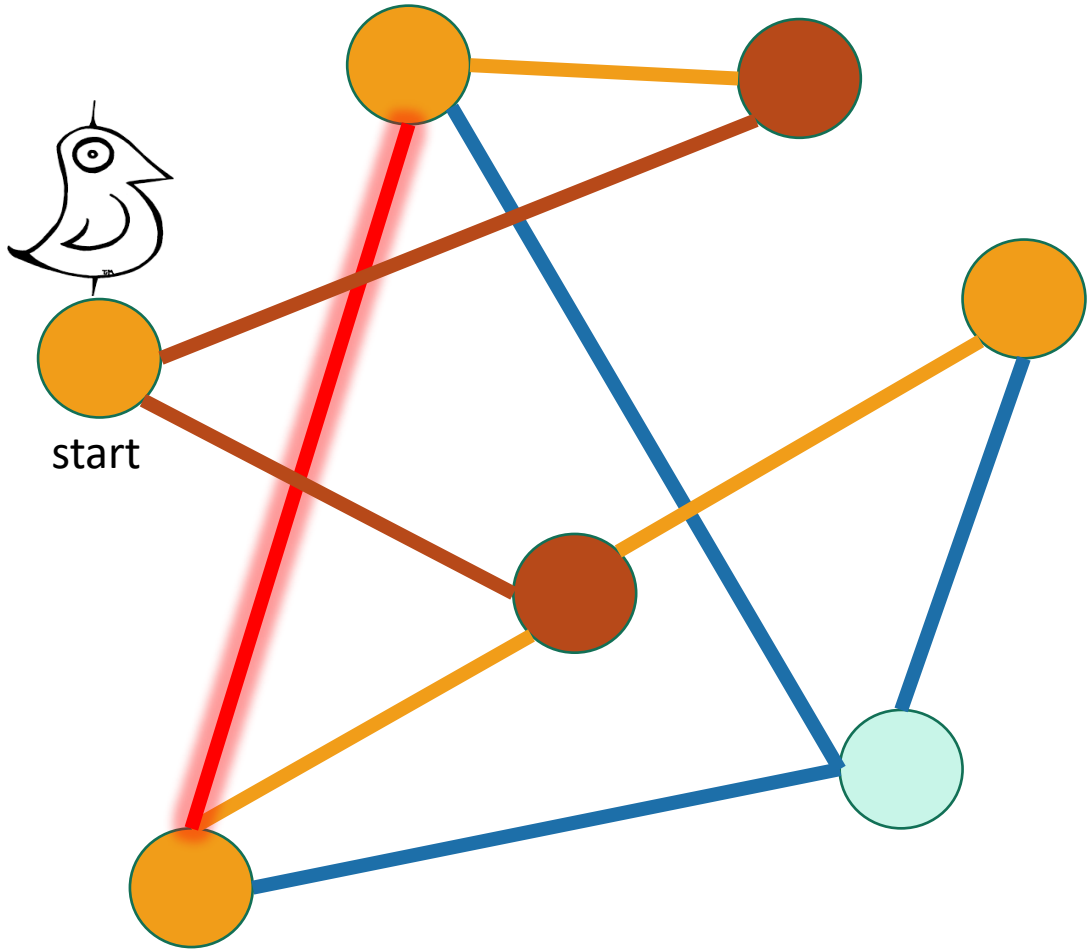

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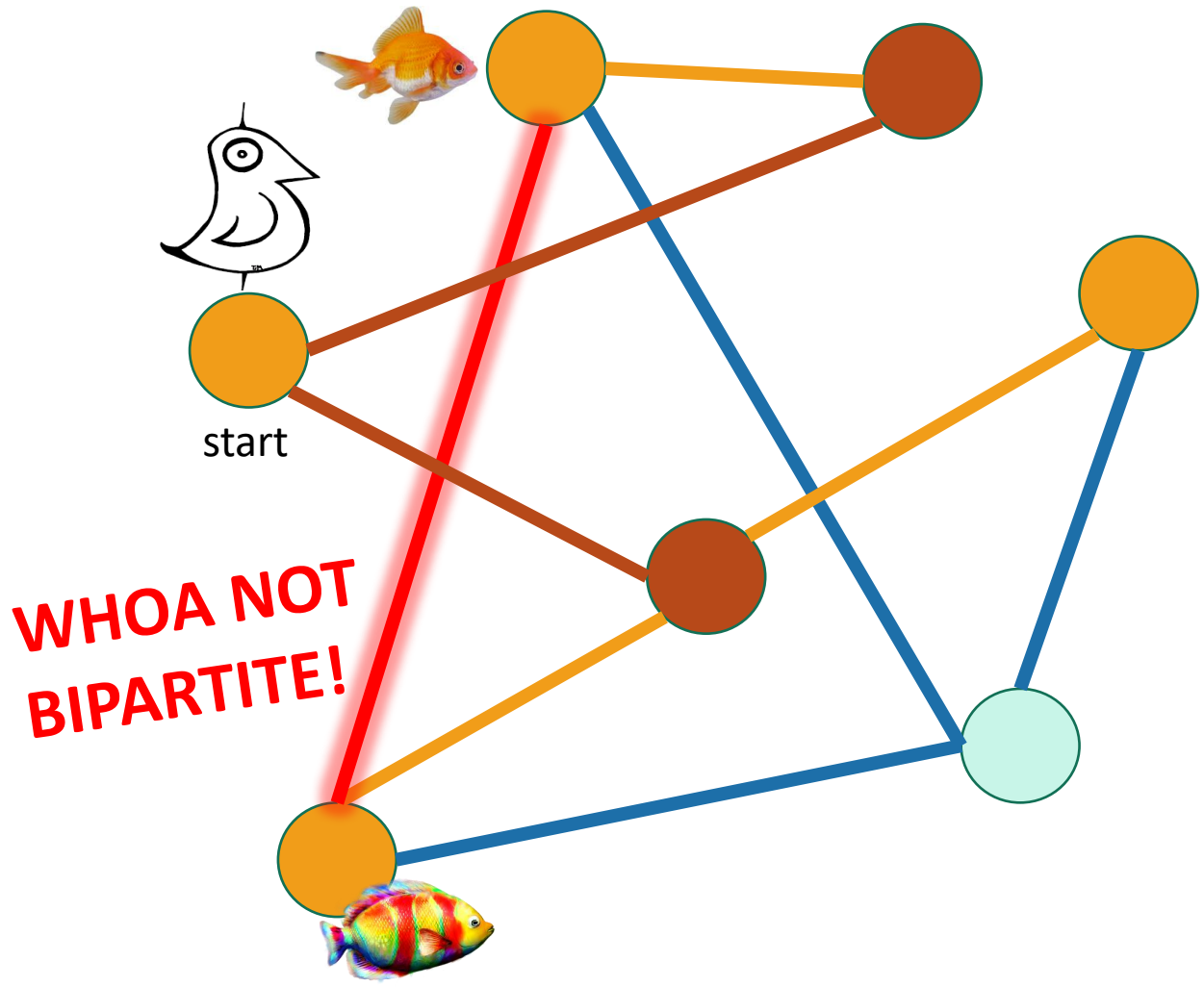

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What have we learned?

BFS can be used to detect
bipartite-ness in time O(n + m).

Acknowledgement

- Stanford University

Thank You