Indian Institute of Information Technology Allahabad

Data Structures and Algorithms

Graphs

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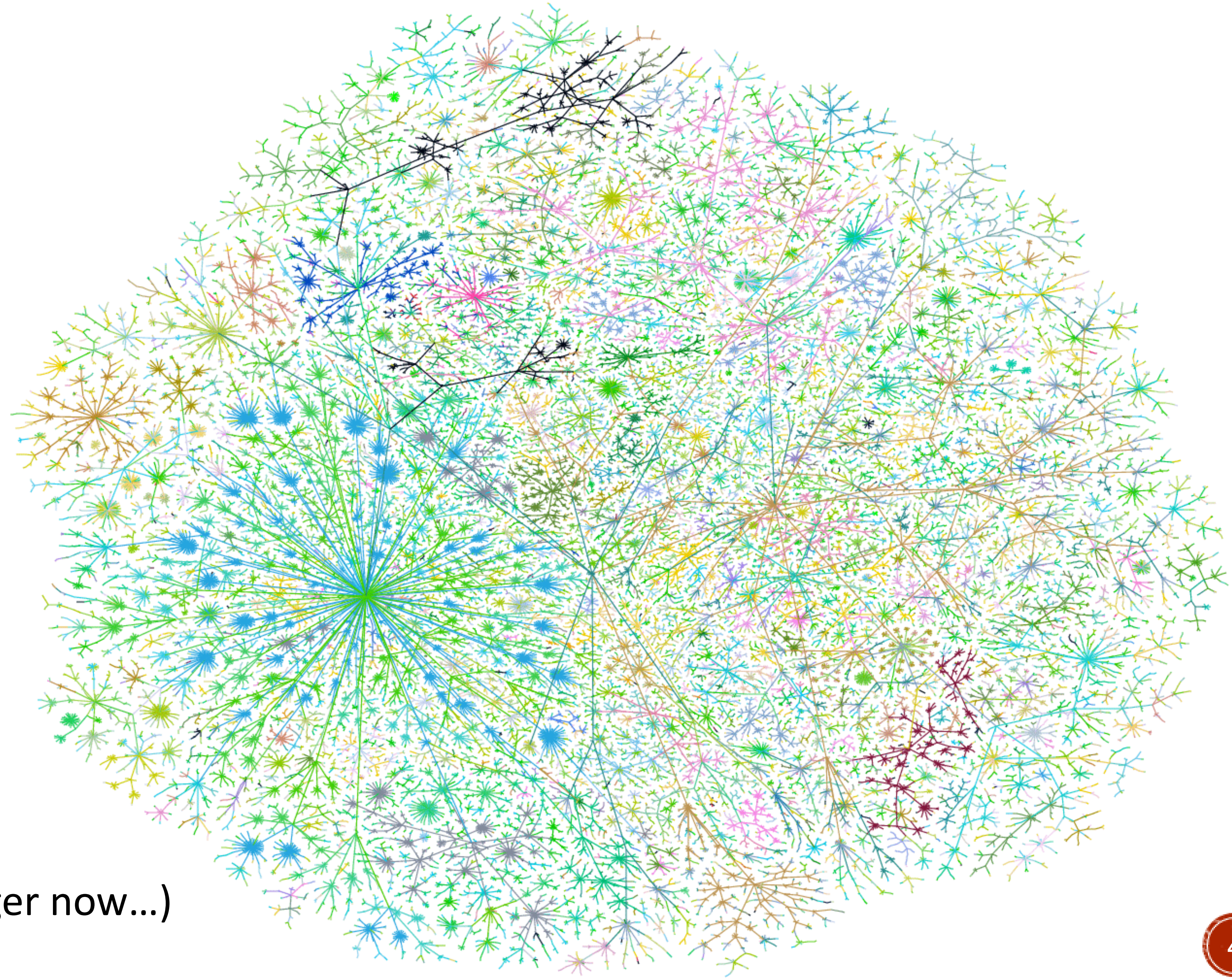
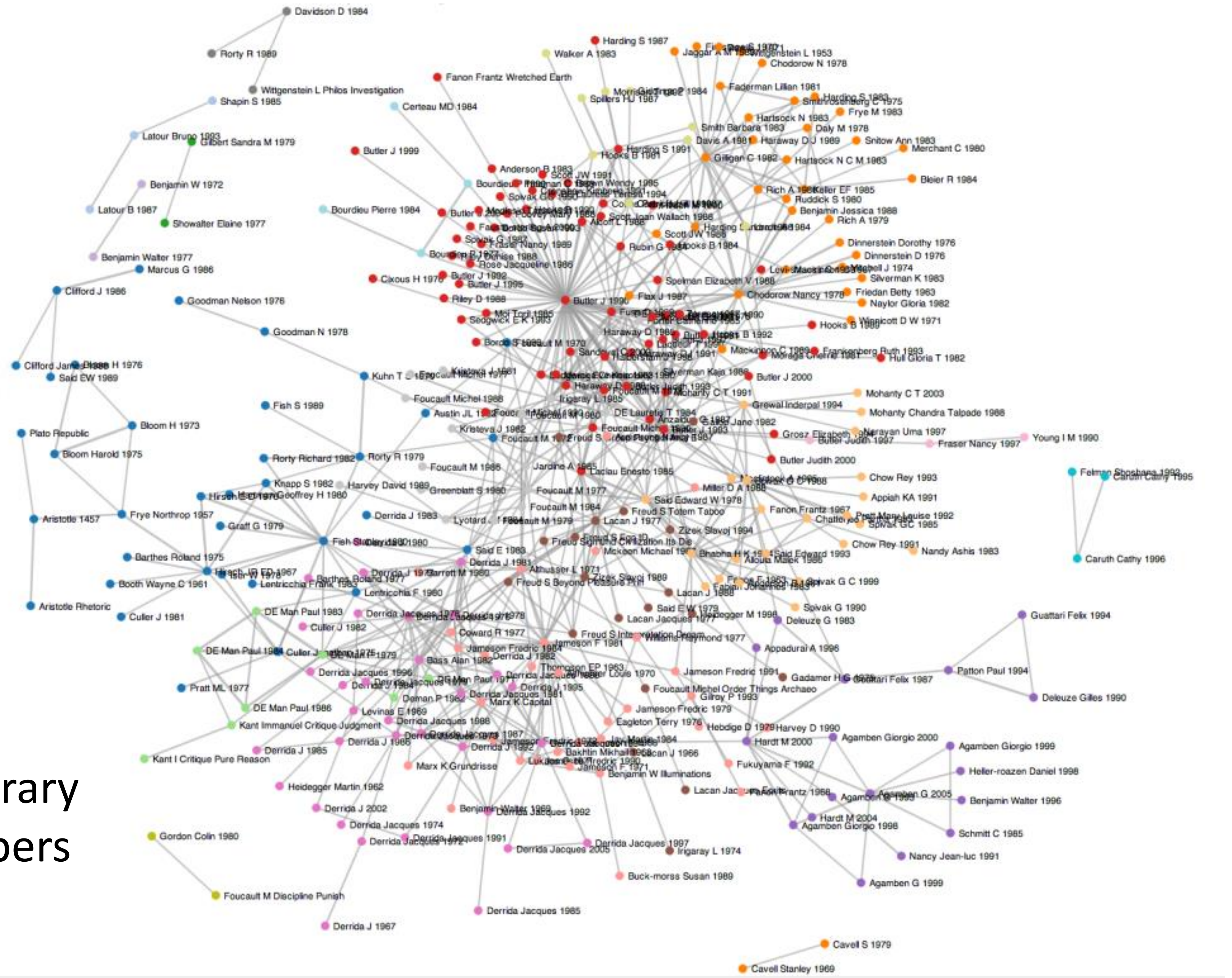
Graphs

Graphs



Graph of the internet
(in 1999…it's a lot bigger now…)

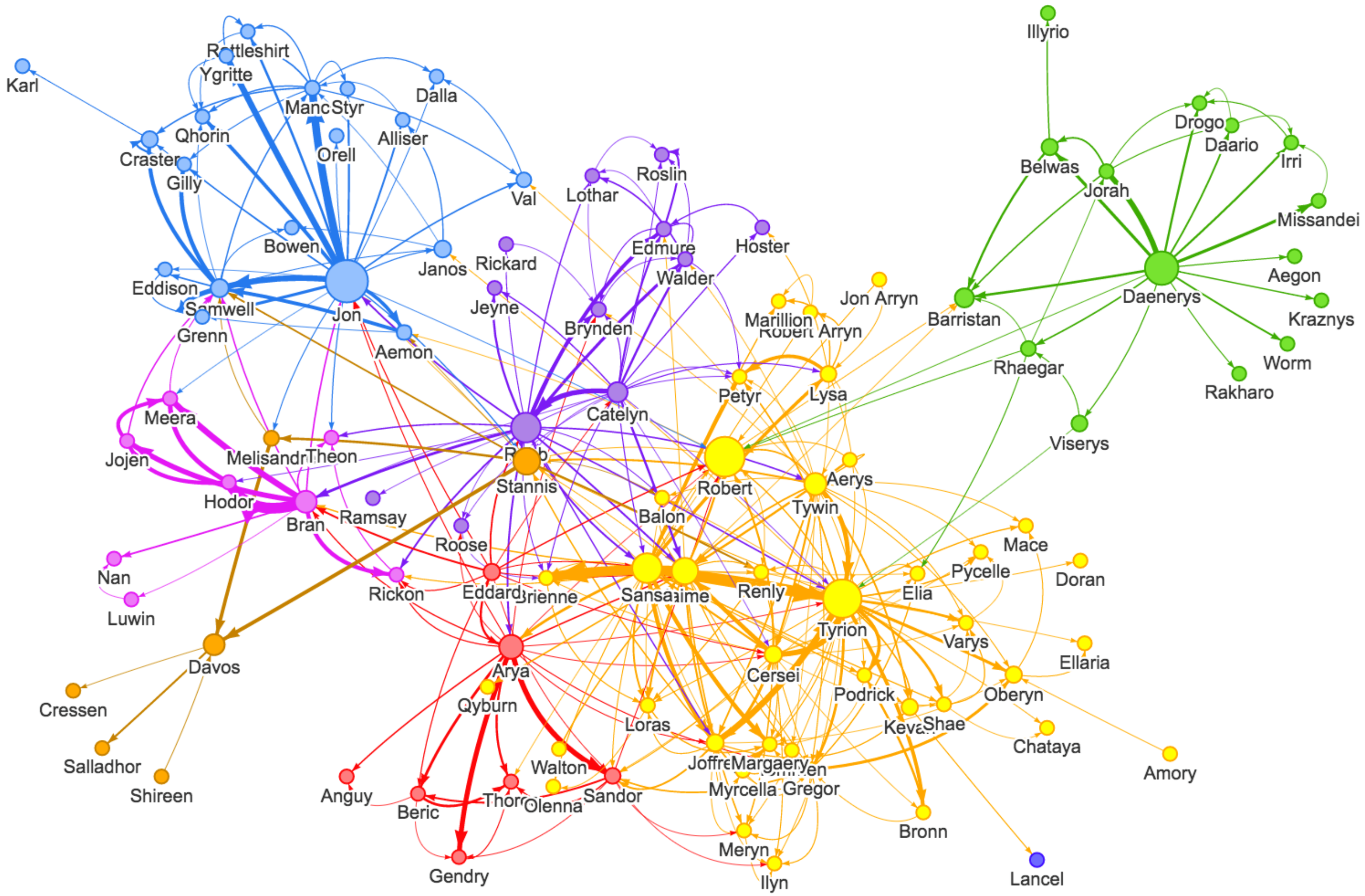
Graphs



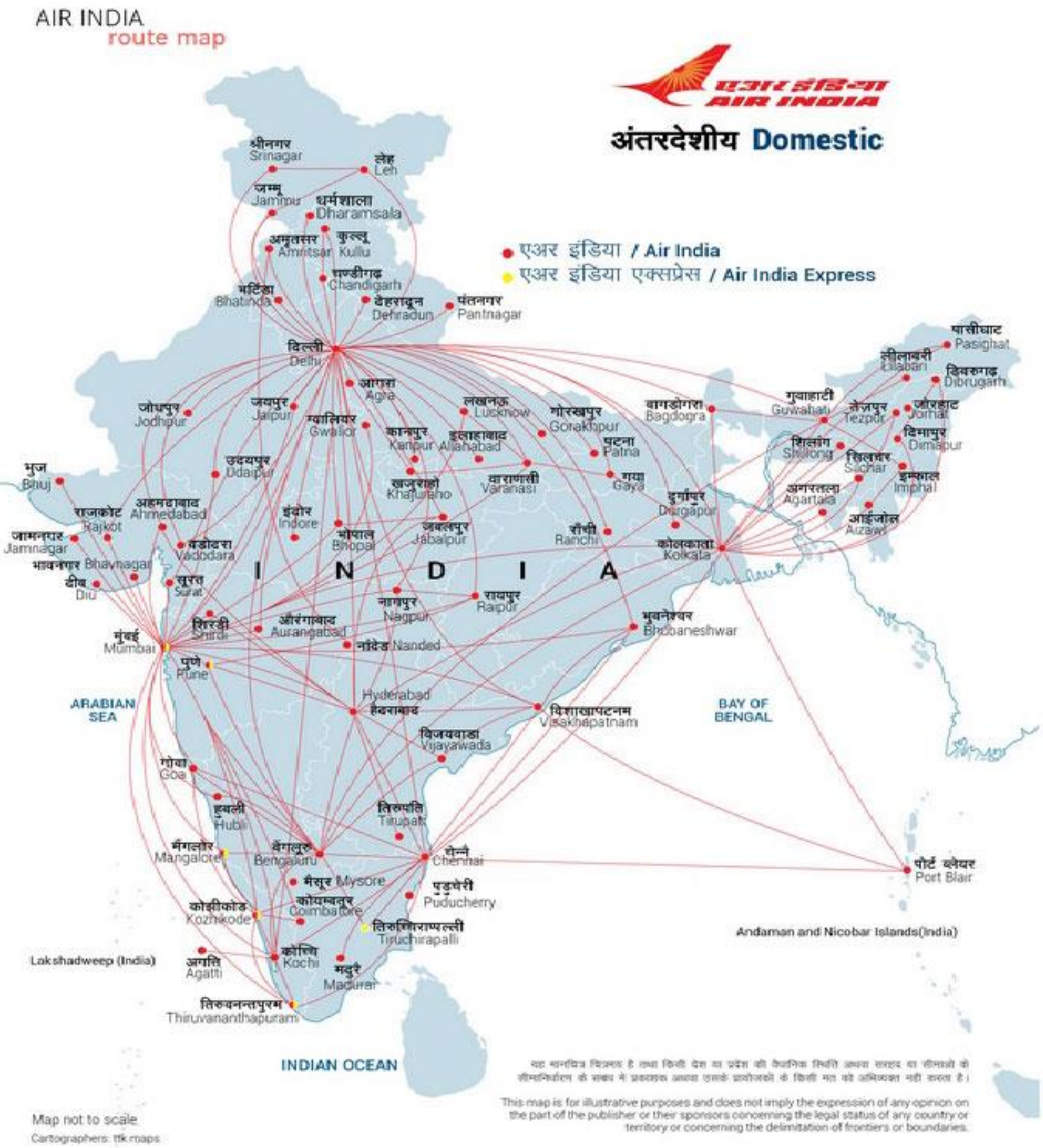
Citation graph of literary theory academic papers

Graphs

Game of Thrones
Character Interaction Network

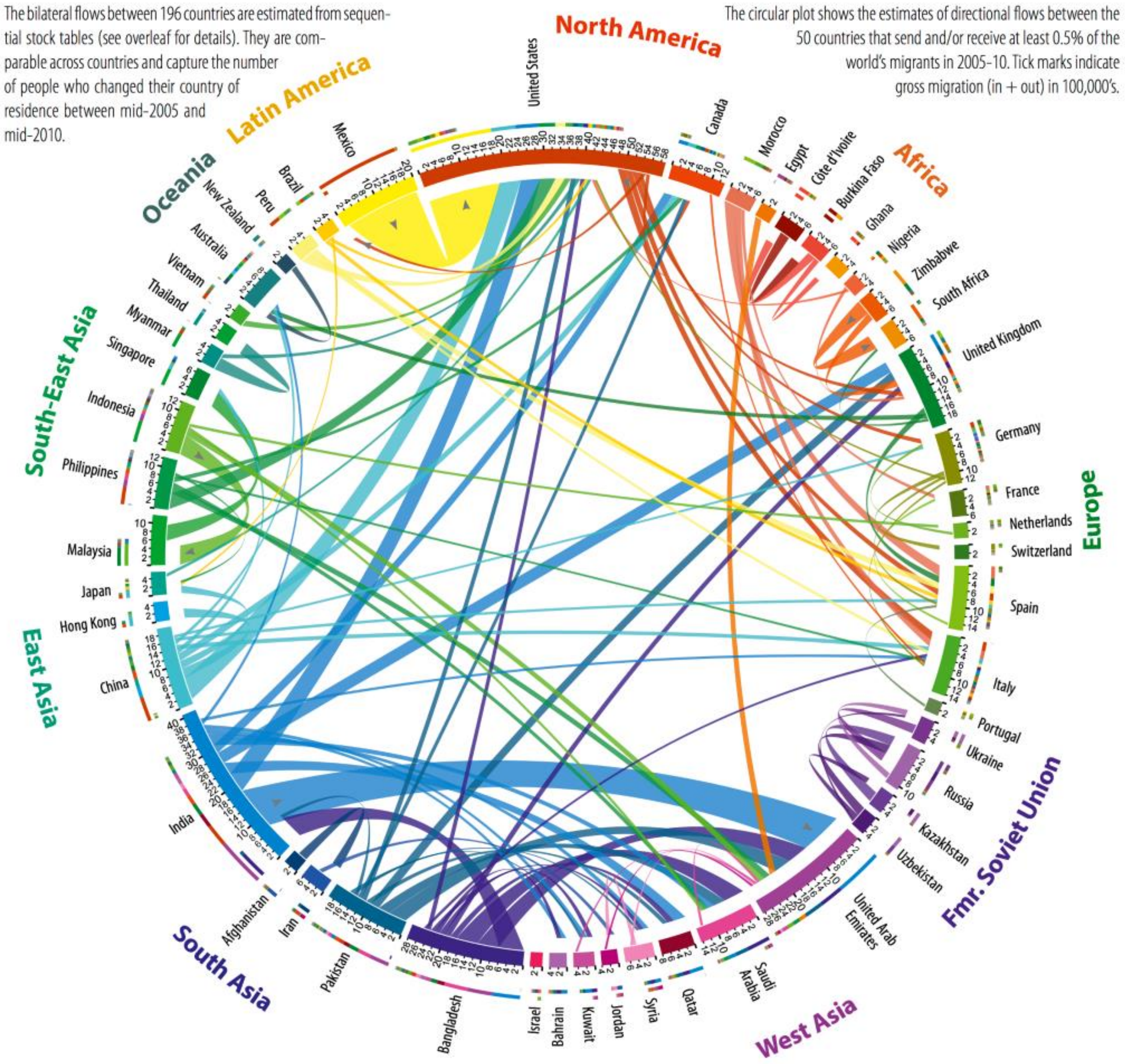


Graphs



AIR INDIA
route map
AIR INDIA
अंतरदेशीय Domestic
एअर इंडिया / Air India
एअर इंडिया एक्सप्रेस / Air India Express
Srinagar
Leh
Jammu
Dharamsala
Amritsar
Kullu
Chandigarh
Bhatinda
Dehradun
Pantnagar
Delhi
Agra
Jodhpur
Jaipur
Gwalior
Lucknow
Gorakhpur
Bagdogra
Guwahati
Pasighat
Dibrugarh
Tezpur
Jorhat
Dimapur
Shillong
Silchar
Imphal
Agartala
Aizawl
Kanpur
Allahabad
Patna
Gaya
Varanasi
Khajuraho
Udaipur
Bhuj
Ahmedabad
Rajkot
Jamnagar
Vadodara
Bhavnagar
Diu
Surat
Indore
Bhopal
Jabalpur
Ranchi
Durgapur
Kolkata
I N D I A
Raipur
Nagpur
Aurangabad
Shirdi
Mumbai
Nanded
Pune
Bhubaneshwar
ARABIAN SEA
Hyderabad
Visakhapatnam
BAY OF BENGAL
Vijayawada
Goa
Hubli
Tirupati
Mangalore
Bengaluru
Chennai
Port Blair
Mysore
Puducherry
Kozhikode
Coimbatore
Tiruchirapalli
Andaman and Nicobar Islands(India)
Lakshadweep (India)
Agatti
Kochi
Madurai
Thiruvananthapuram
INDIAN OCEAN
Map not to scale
This map is for illustrative purposes and does not imply the expression of any opinion on the part of the publisher or their sponsors concerning the legal status of any country or territory or concerning the delimitation of frontiers or boundaries.

Graphs

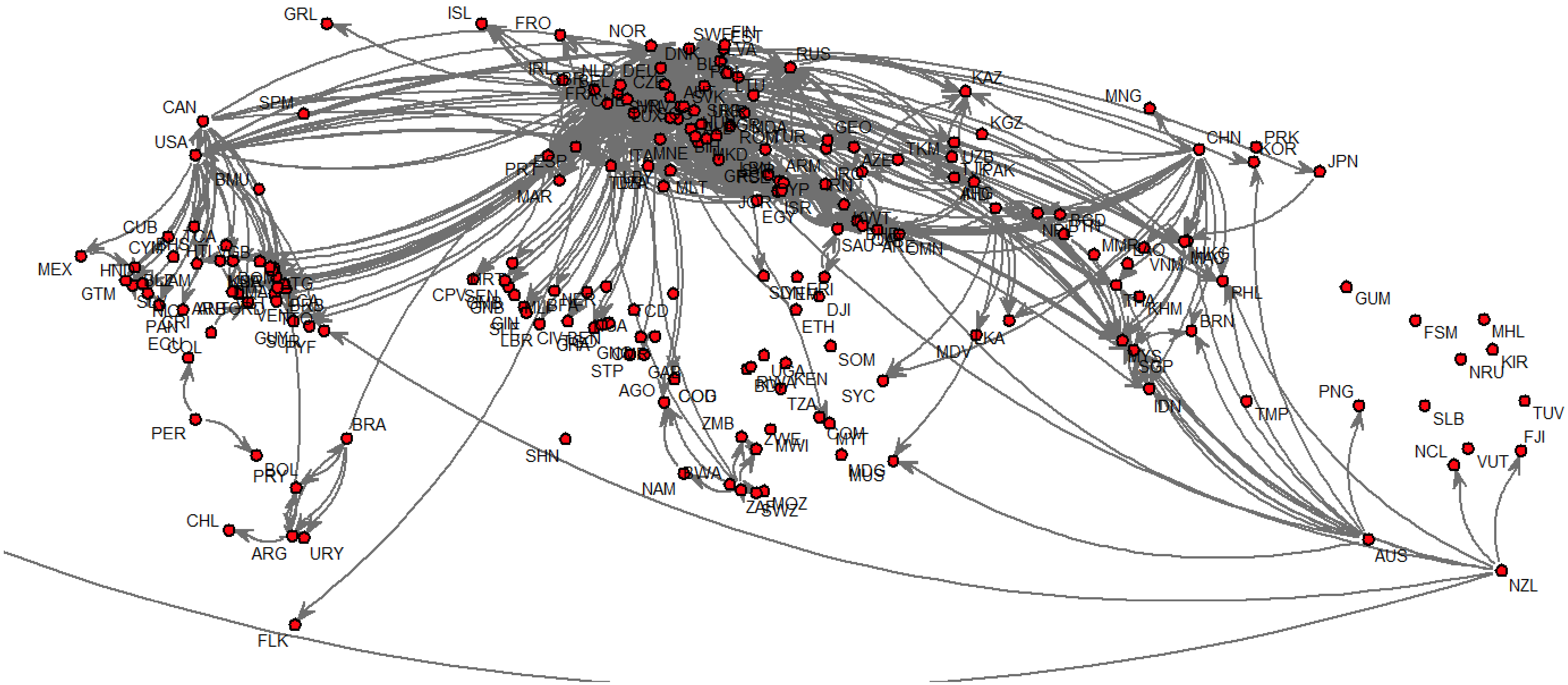


Immigration flows

Graphs

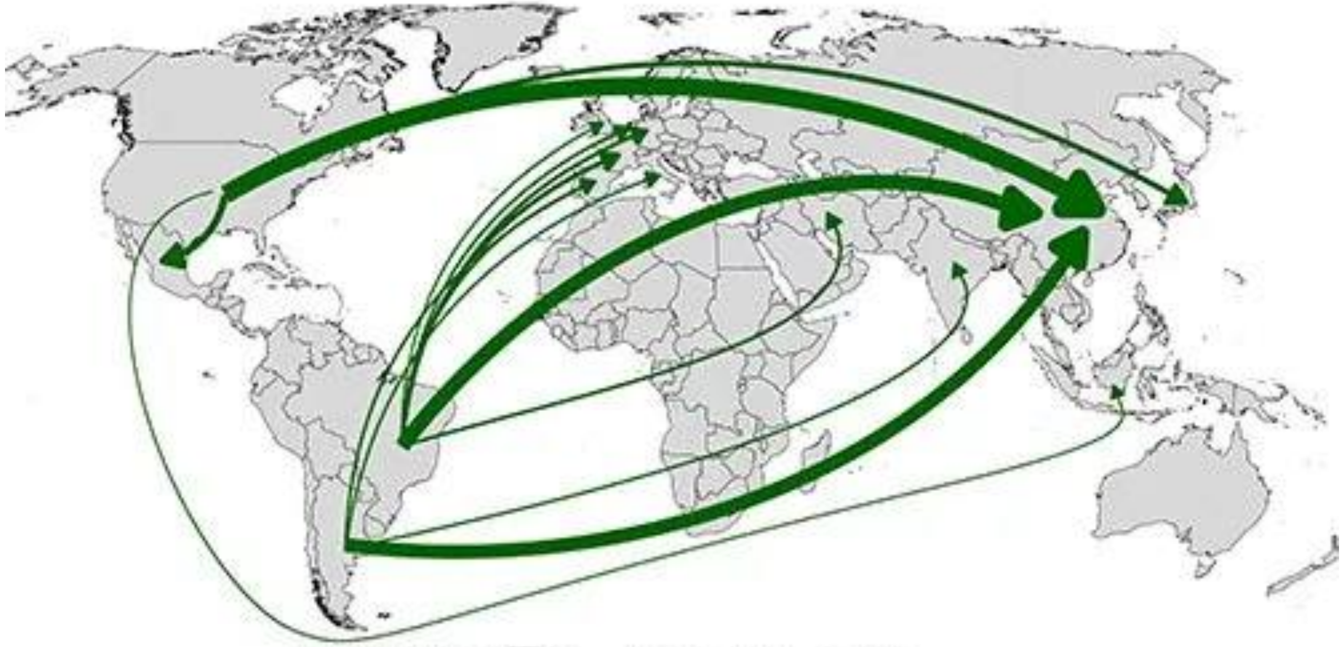
Potato trade

World trade in fresh potatoes, flows over 0.1 m US$ average 2005-2009

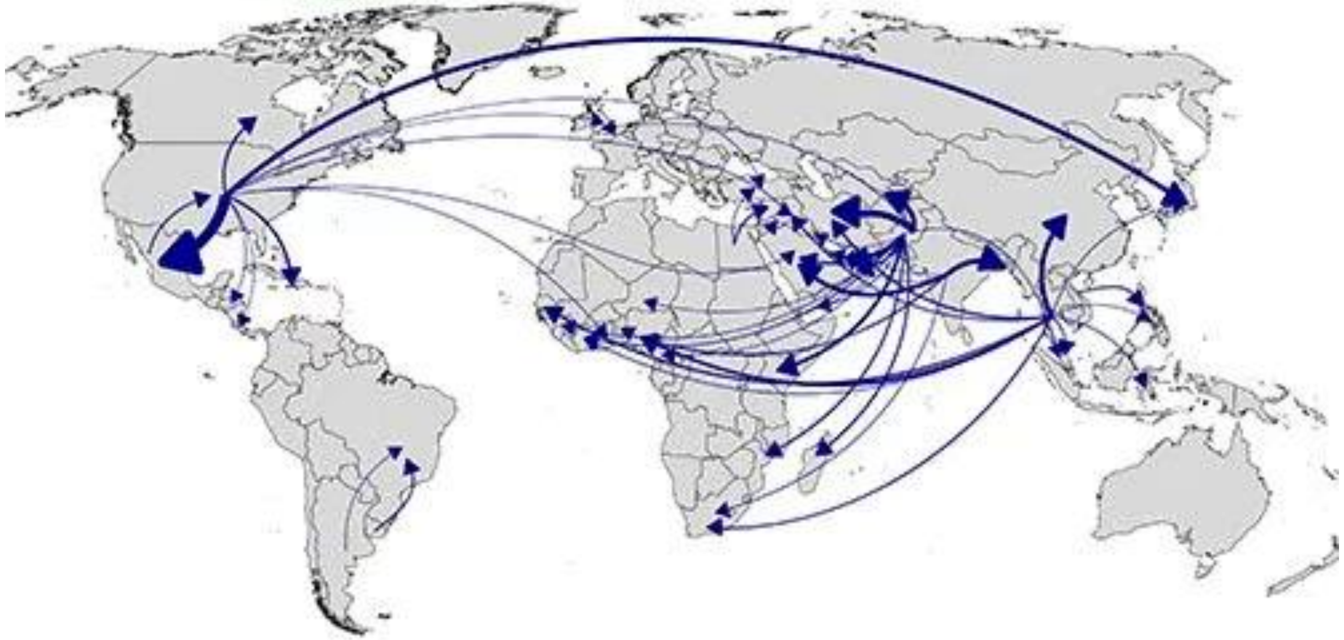


Graphs

Soybeans

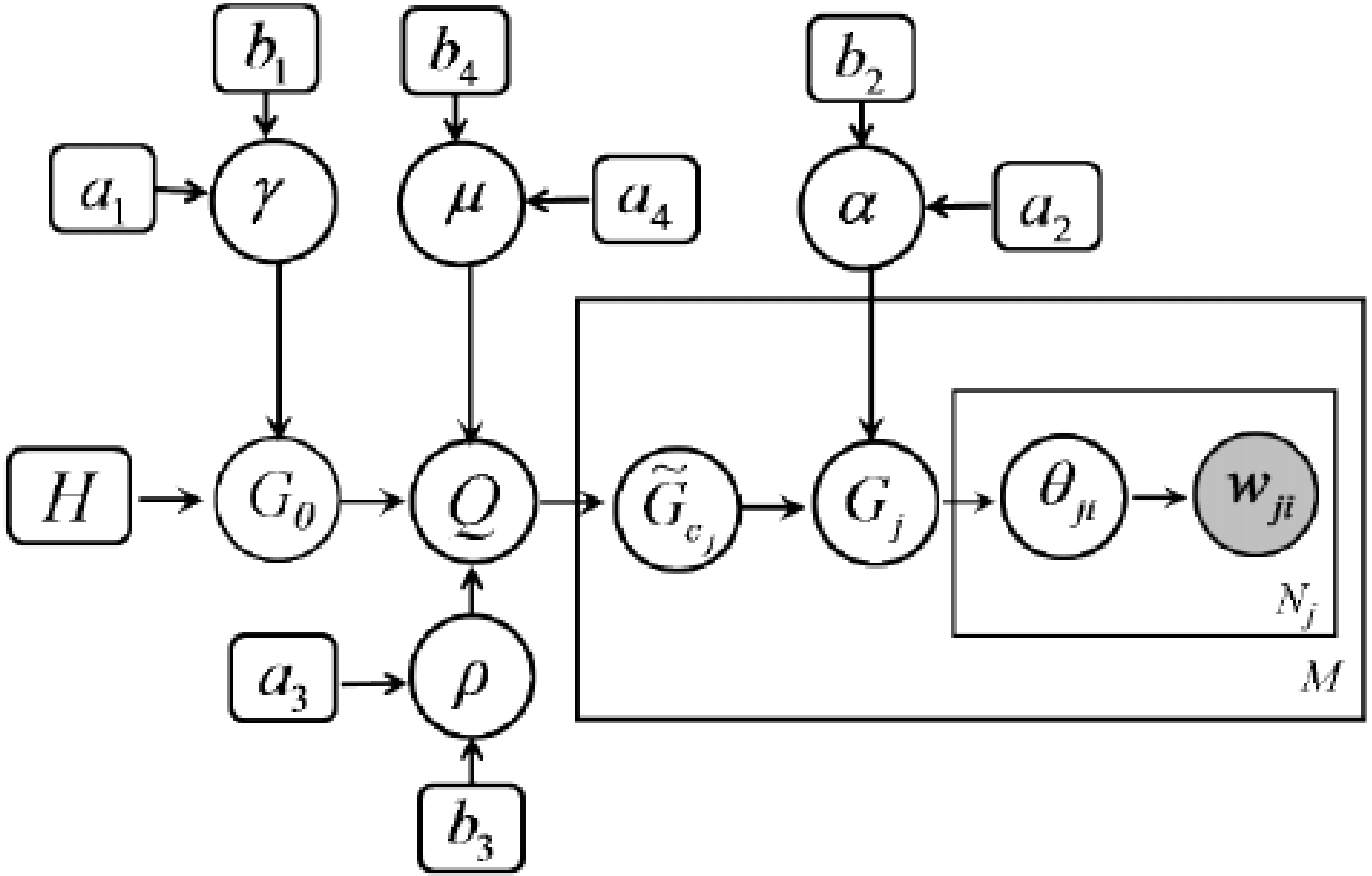


Water



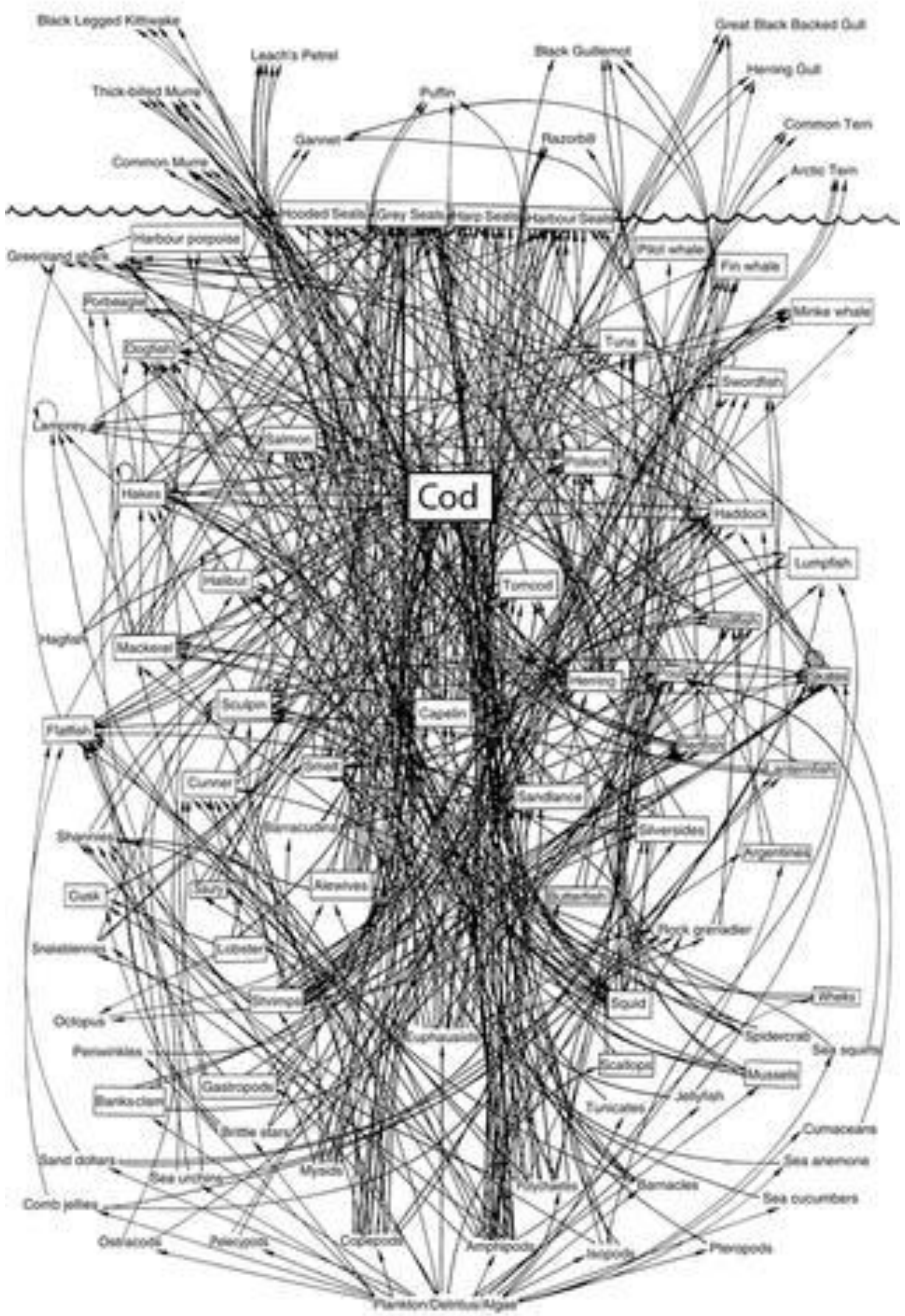
Graphs

Graphical models



Graphs

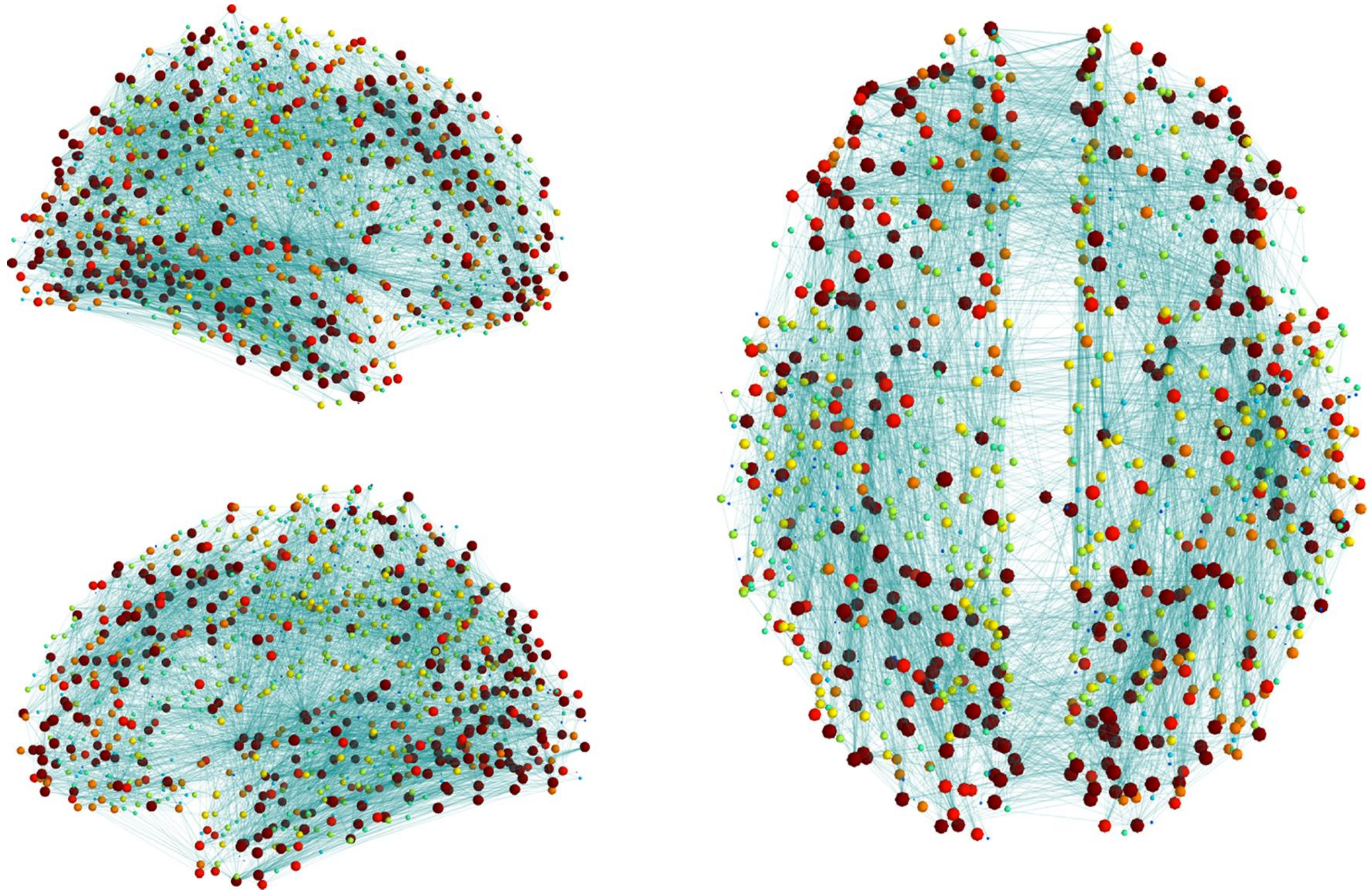
What eats what in the Atlantic ocean?



A simplified food web for the Northwest Atlantic. © IMMA

Graphs

Neural connections in the brain

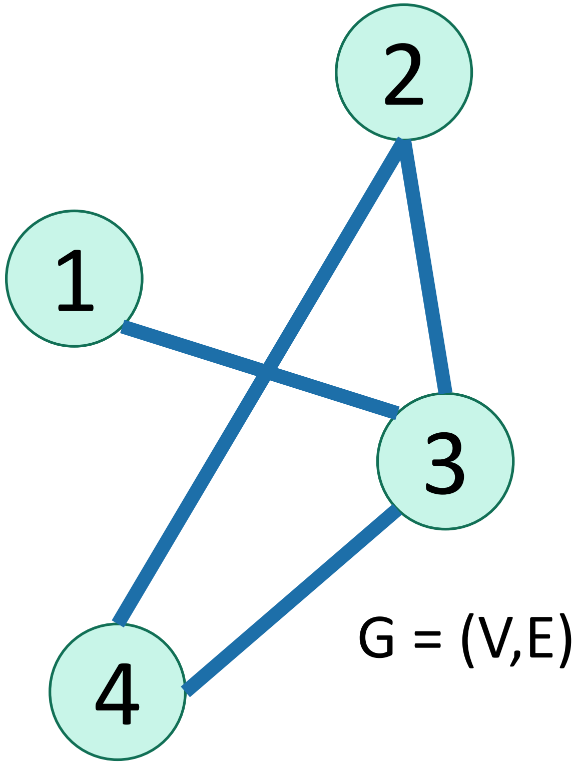


Graphs

- **There are a lot of graphs.**

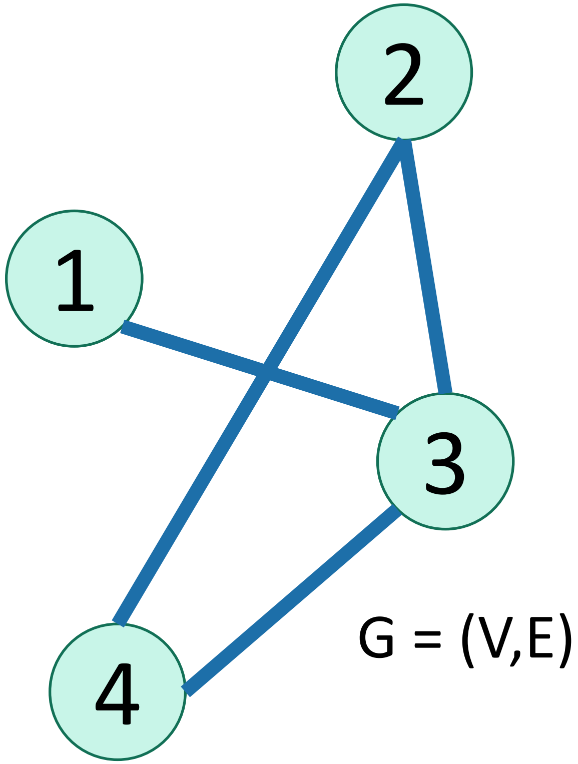
- We want to answer questions about them.
  - Efficient routing?
  - Community detection/clustering?
  - Signing up for classes without violating pre-req constraints
  - How to distribute fish in tanks so that none of them will fight.

Undirected Graphs



Undirected Graphs

- Has vertices and edges
  - V is the set of vertices
  - E is the set of edges
  - Formally, a graph is G = (V,E)



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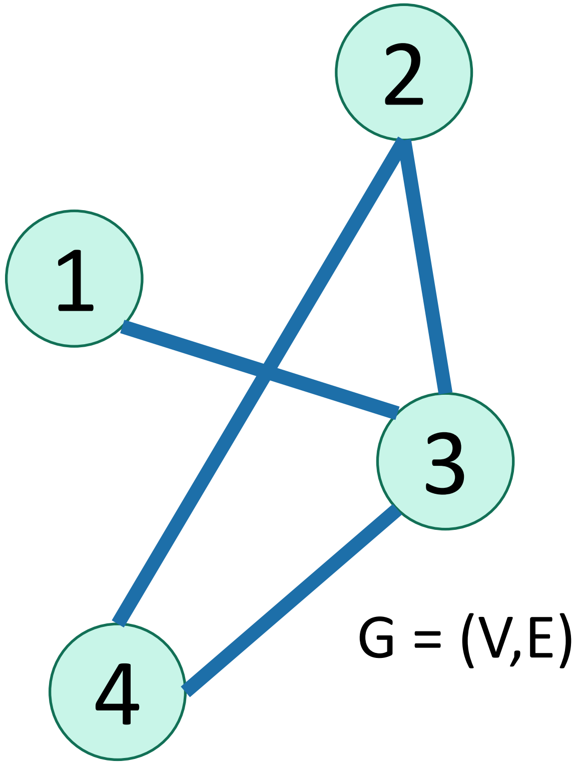
- Example
  - V = {1,2,3,4}
  - E = { {1,3}, {2,4}, {3,4}, {2,3} }



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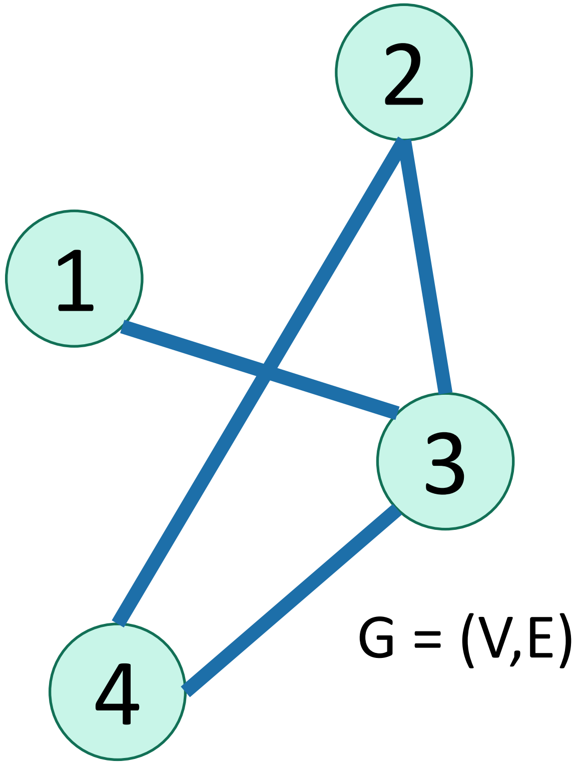
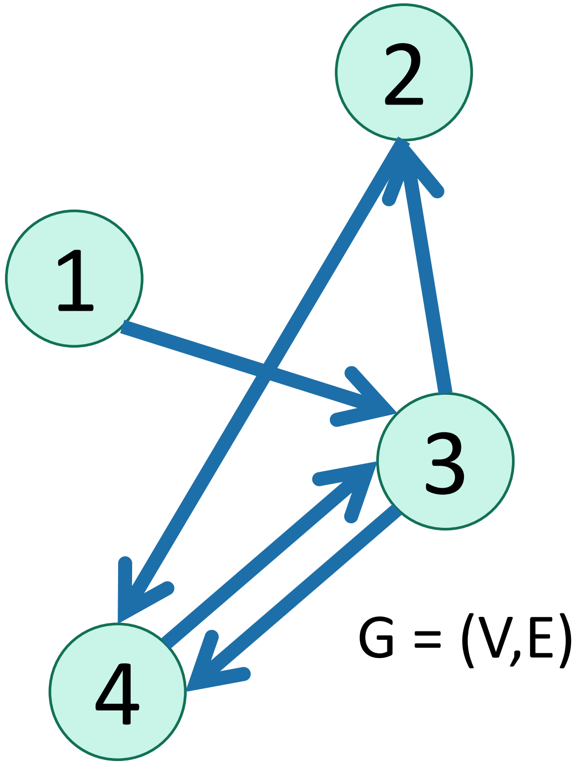


- Example
  - V = {1,2,3,4}
  - E = { {1,3}, {2,4}, {3,4}, {2,3} }

- The **degree** of vertex 4 is 2.
  - There are 2 edges coming out.
- Vertex 4's **neighbors** are 2 and 3

Directed Graphs



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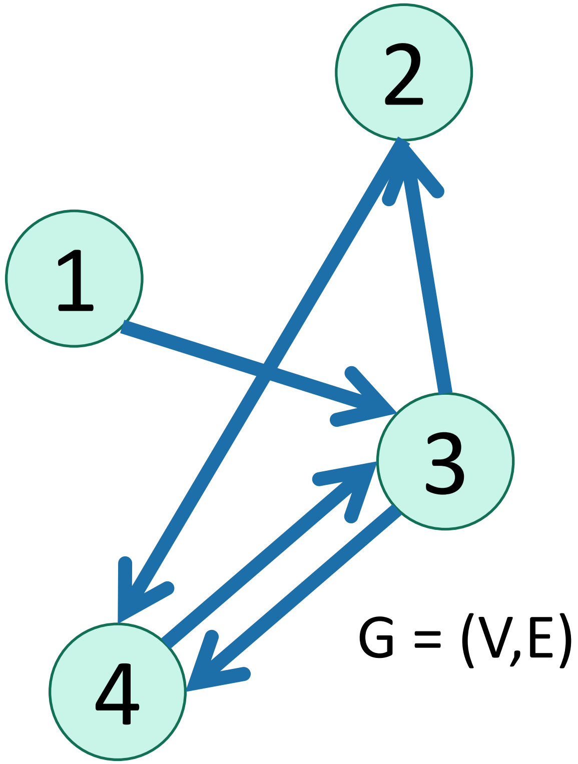

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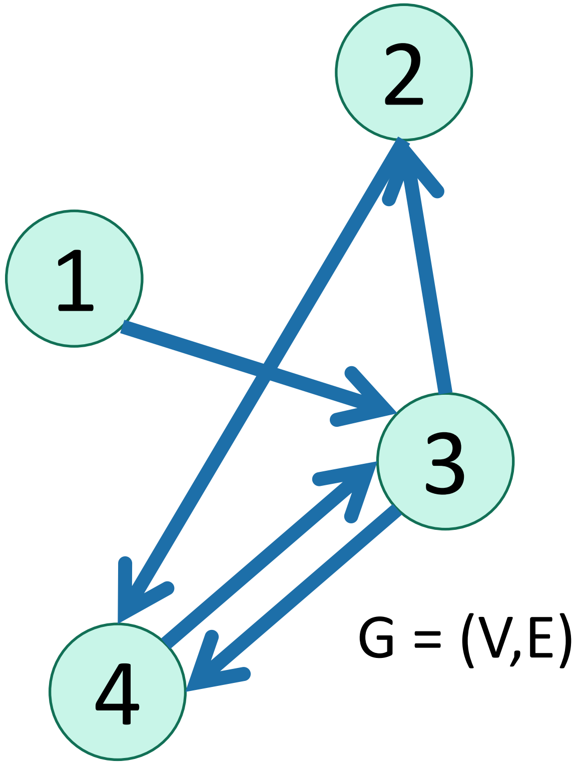

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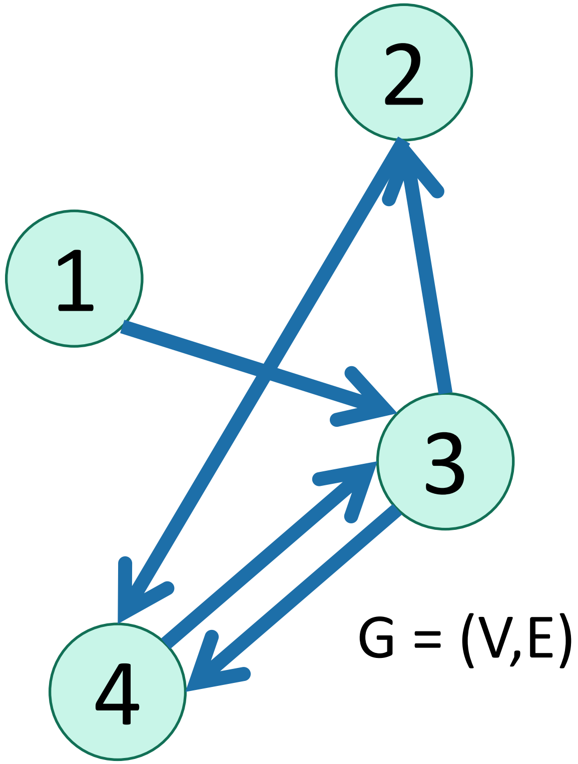
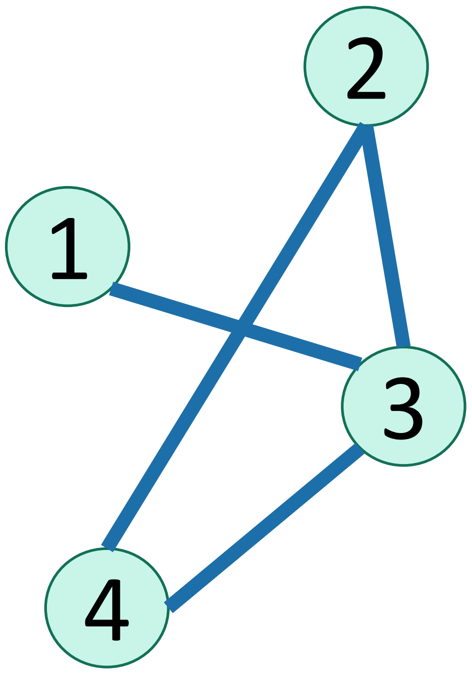


- Example
  - V = {1,2,3,4}
  - E = { (1,3), (2,4), (3,4), (4,3), (3,2) }

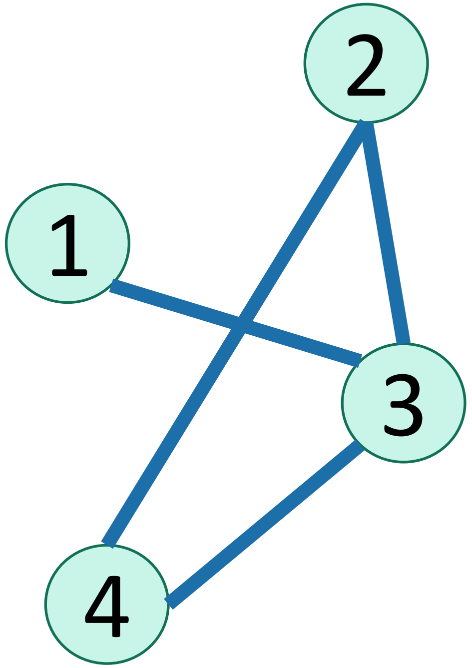
- The **in-degree** of vertex 4 is 2.
- The **out-degree** of vertex 4 is 1.
- Vertex 4's **incoming neighbors** are 2,3
- Vertex 4's **outgoing neighbor** is 3.

How do we represent graphs?



How do we represent graphs?

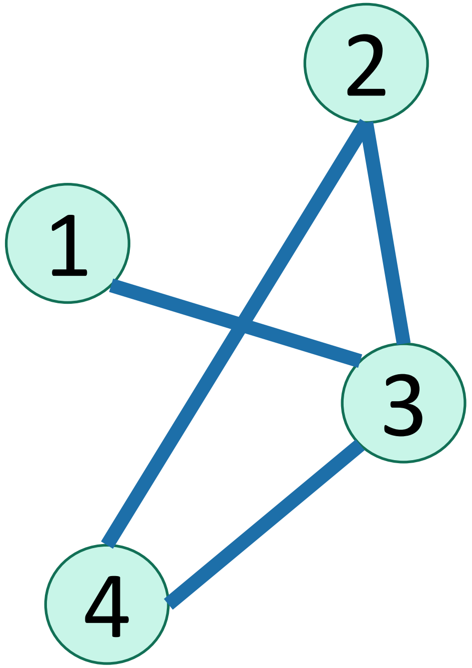
- Option 1: adjacency matrix.



How do we represent graphs?

- Option 1: adjacency matrix.

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}
\end{array}
$$



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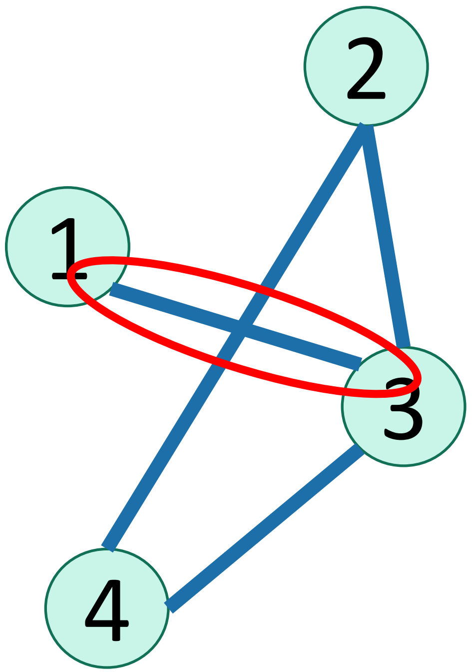
	1	2	3	4
1	0	0	1	0
2	0	0	1	1
3	1	1	0	1
4	0	1	1	0



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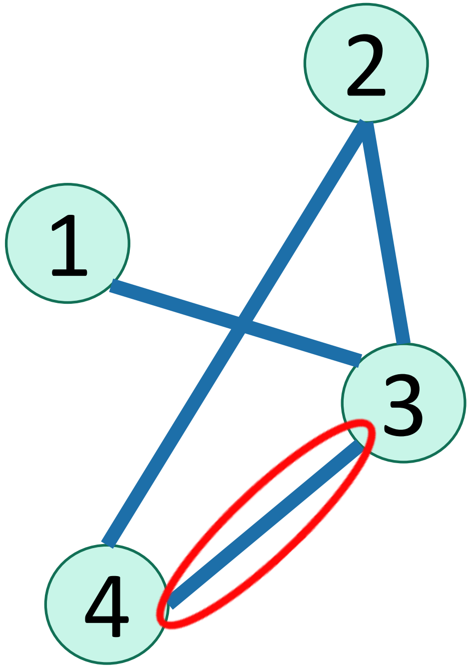


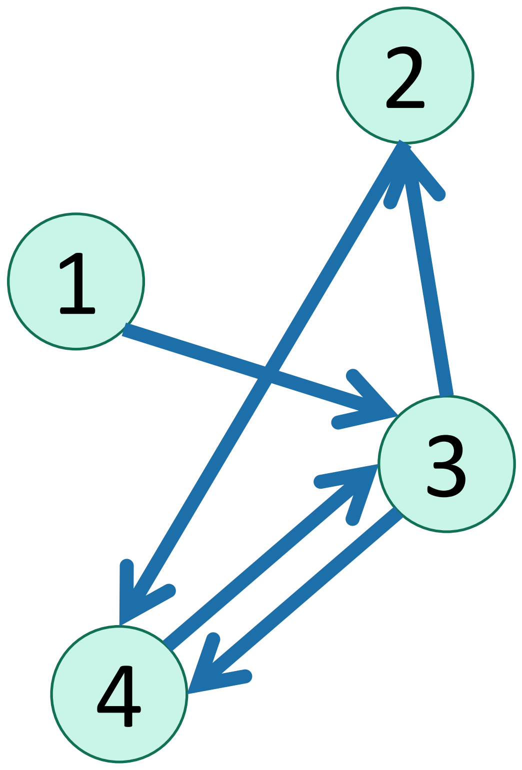
How do we represent graphs?

- Option 1: adjacency matrix.

Destination

Source

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



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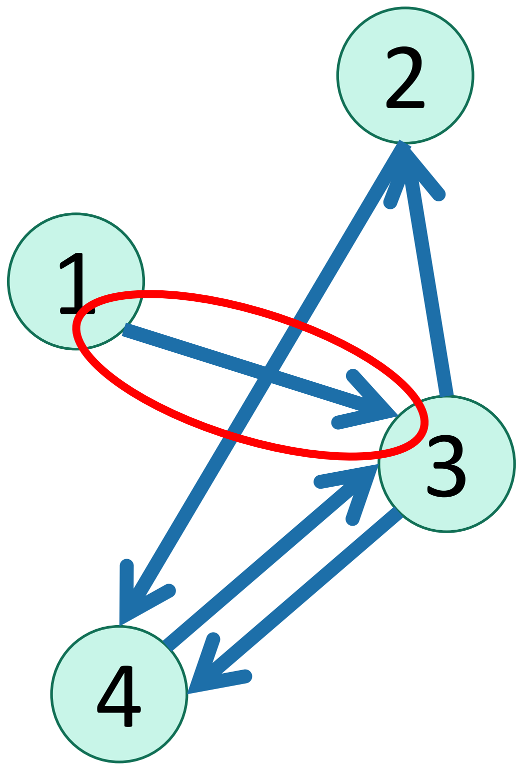
Destination

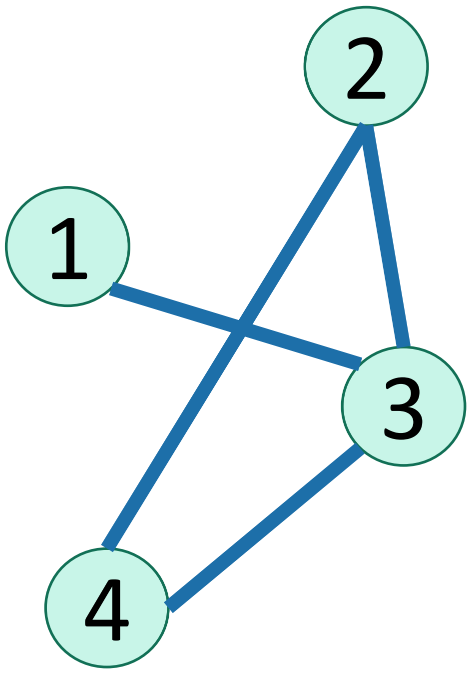
Source

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$



How do we represent graphs?

- Option 2: adjacency lists.



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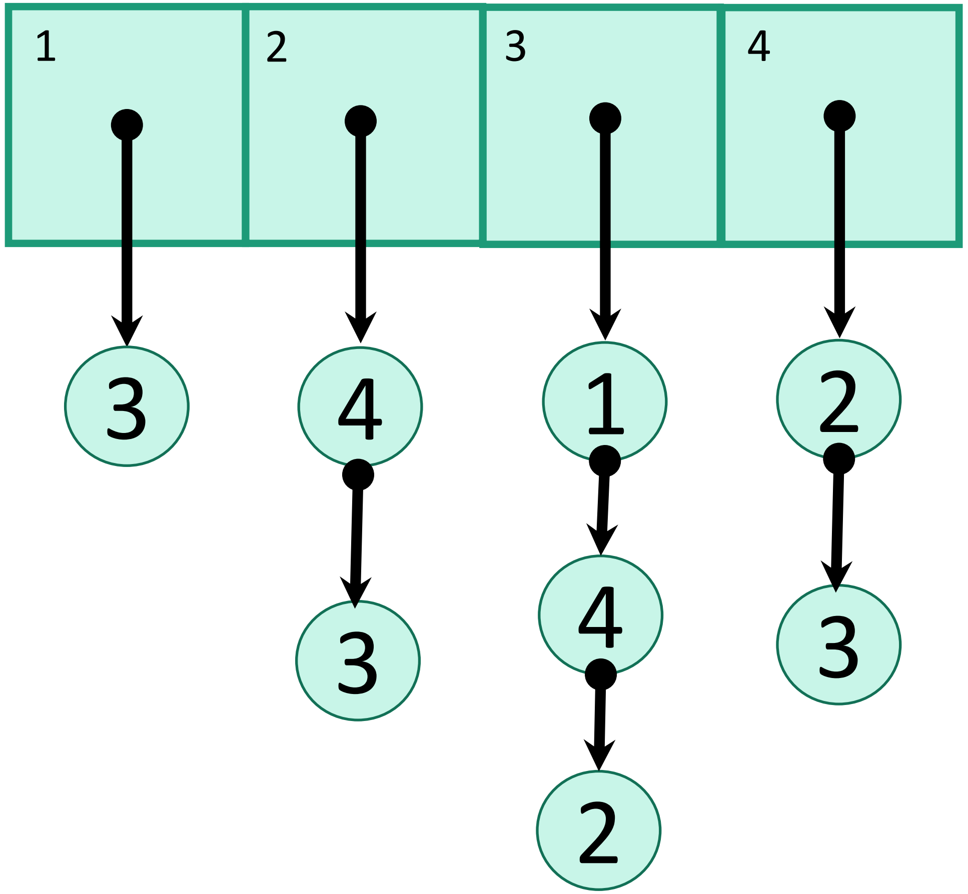

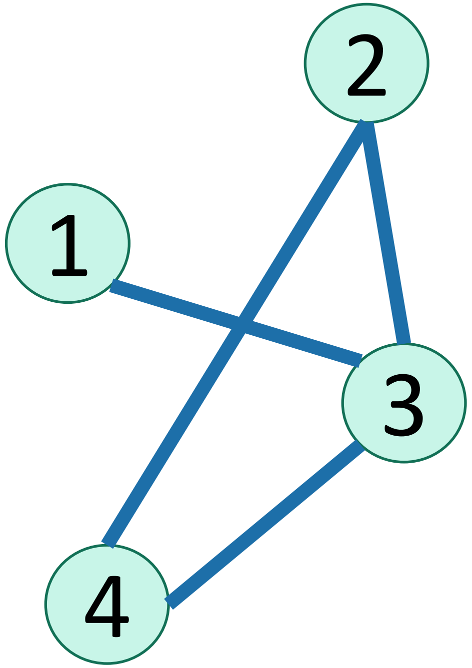

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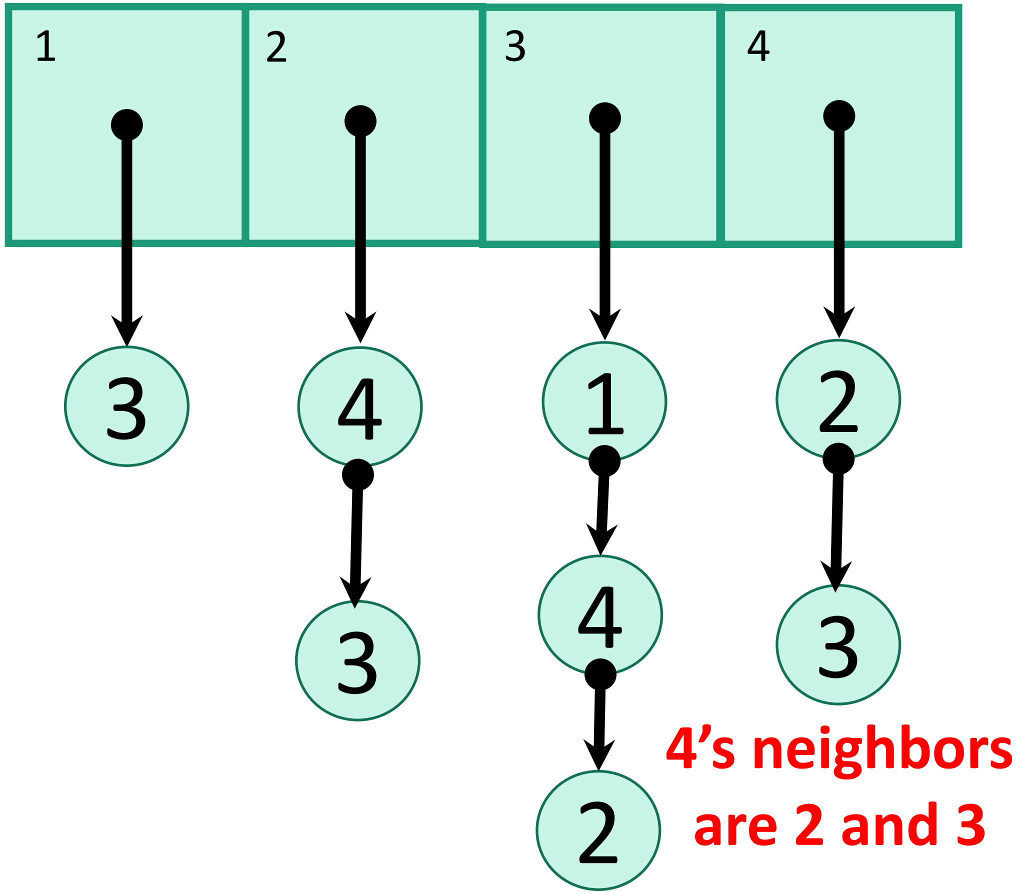

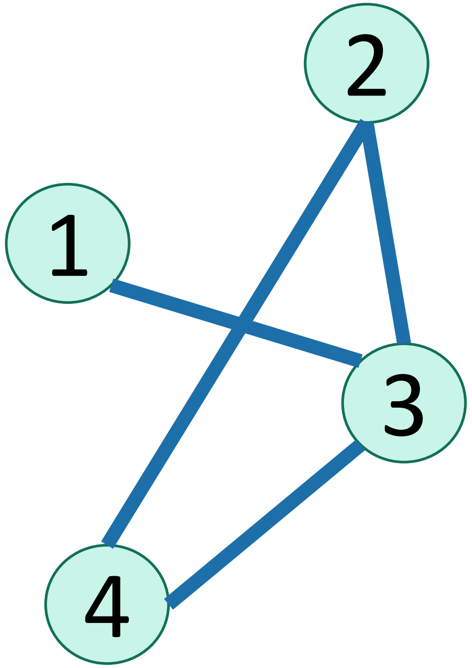

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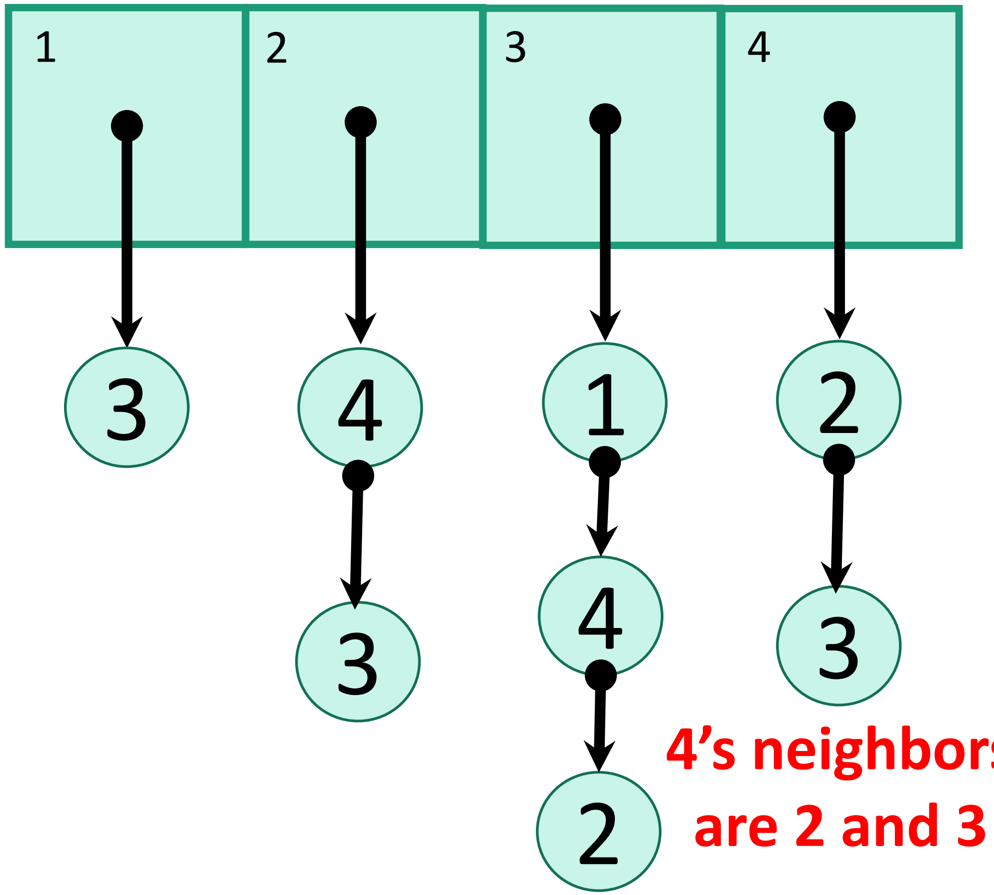
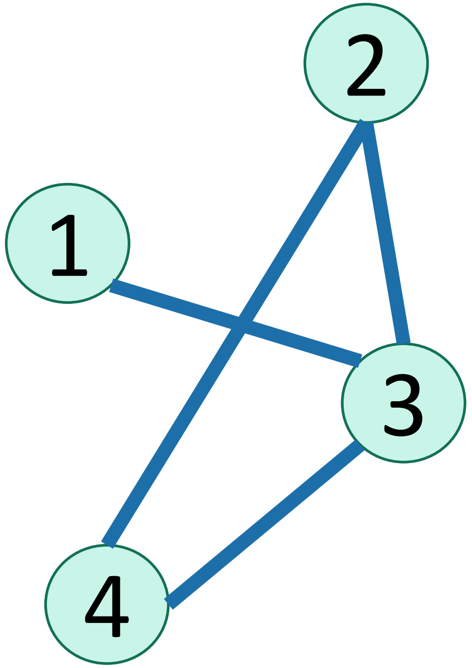


4's neighbors are 2 and 3



How would you modify this for directed graphs?

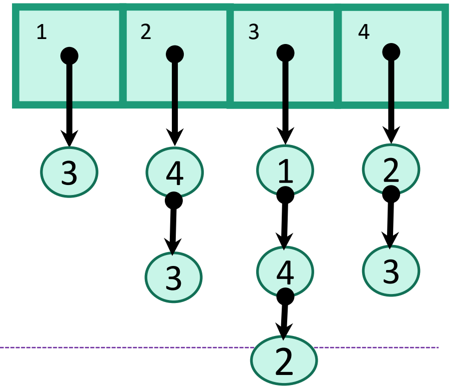
In either case

- Vertices can store other information
  - Attributes (name, IP address, …)
  - helper info for algorithms that we will perform on the graph

In either case

- Vertices can store other information
  - Attributes (name, IP address, …)
  - helper info for algorithms that we will perform on the graph
- Want to be able to do the following operations:
  - **Edge Membership**: Is edge e in E?
  - **Neighbor Query**: What are the neighbors of vertex v?

Trade-offs

Say there are n vertices and m edges.	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$	
Edge membership Is e = {v,w} in E?		
Neighbor query Give me v's neighbors.		
Space requirements		

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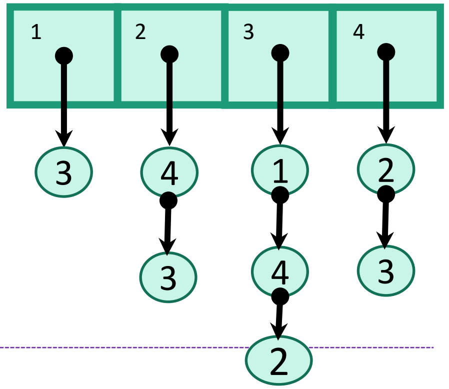
Say there are n vertices and m edges.	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$	
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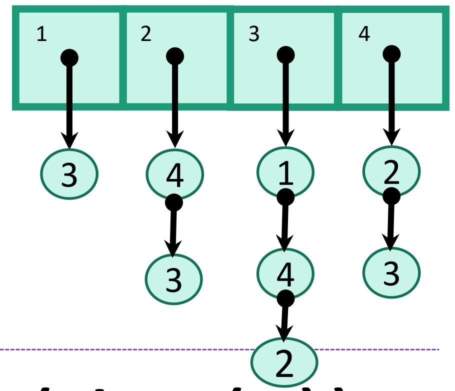
Say there are n vertices and m edges.	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$	
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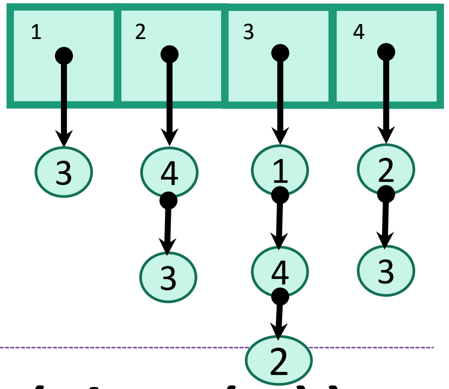
Say there are n vertices and m edges.	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$	
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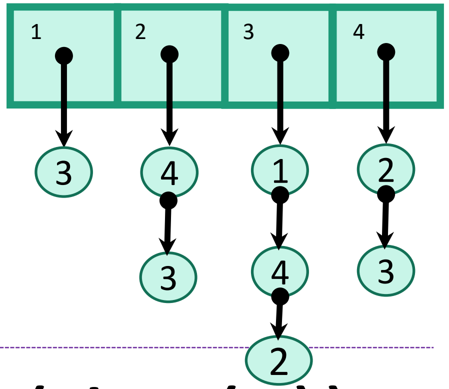
Say there are n vertices and m edges.	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$	
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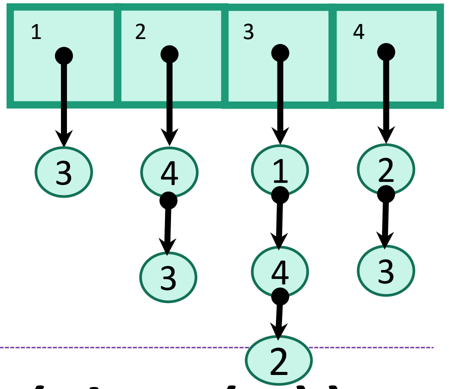
Say there are n vertices and m edges.	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$	
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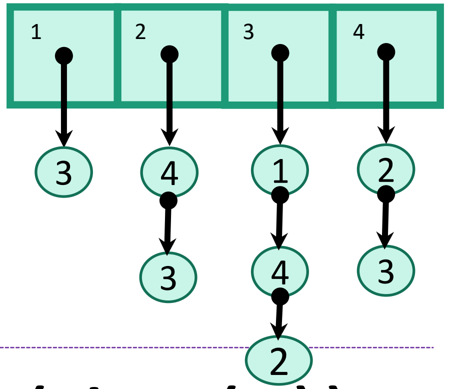
Say there are n vertices and m edges.	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$	
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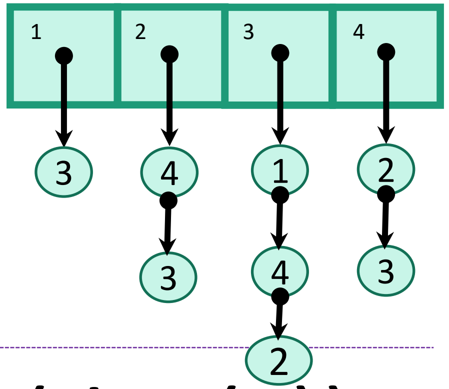
Space requirements	$O(n^2)$	O(n + m)

Trade-offs

Say there are n vertices and m edges.	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$	Generally better for **sparse** graphs 
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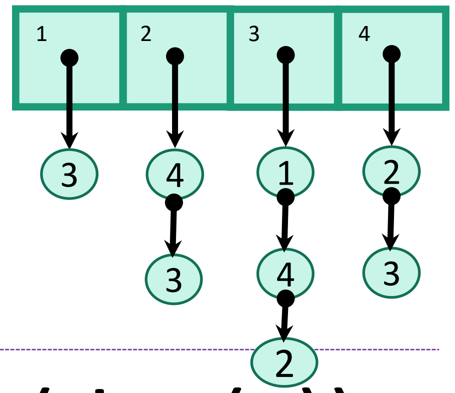
Say there are n vertices and m edges.	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$	
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Neighbor query Give me v's neighbors.	O(n)	O(deg(v))
Space requirements	$O(n^2)$	O(n + m) **We'll assume this representation for the rest of the class**

Acknowledgement

- Stanford University

Thank You