Indian Institute of Information Technology Allahabad

Data Structures

Depth First Search (DFS)

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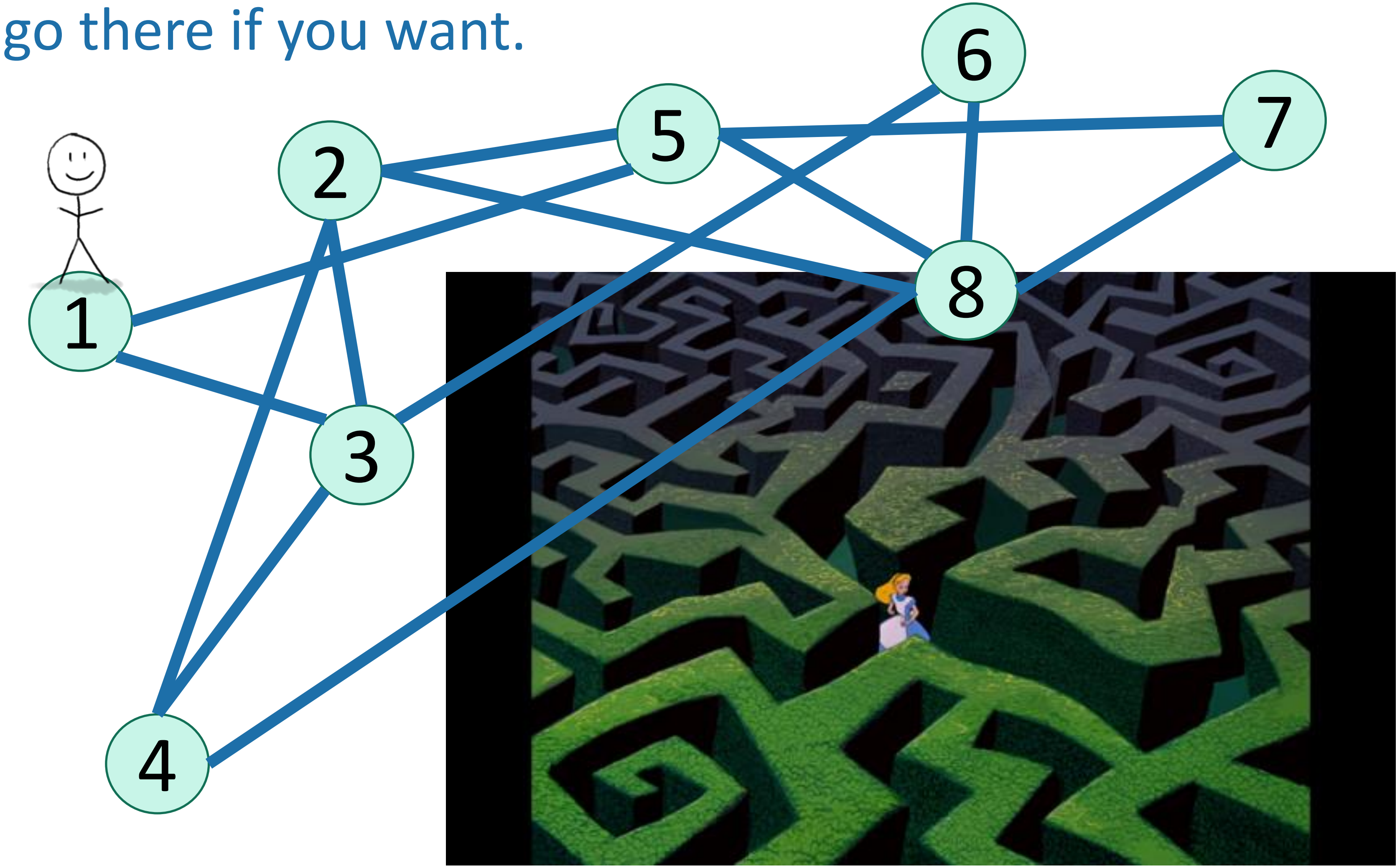
How do we explore a graph?

At each node, you can get a list of neighbors, and choose to go there if you want.



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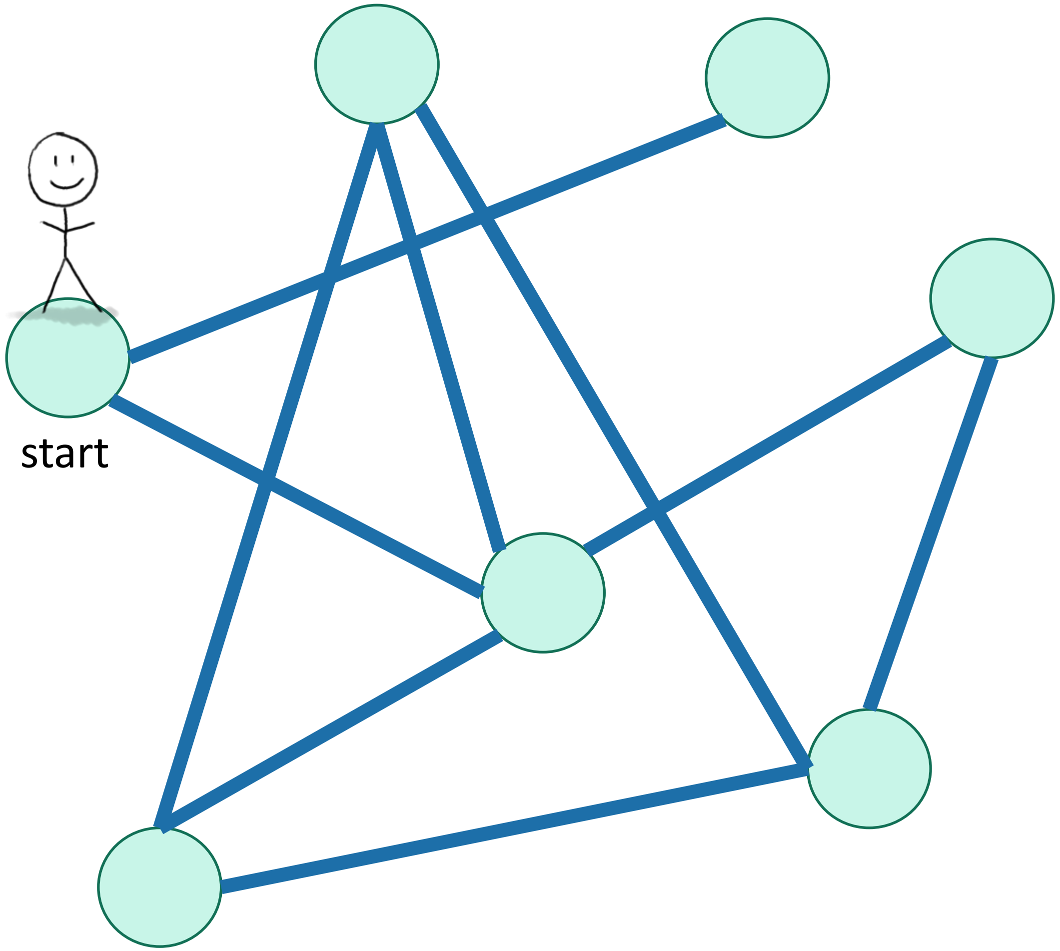

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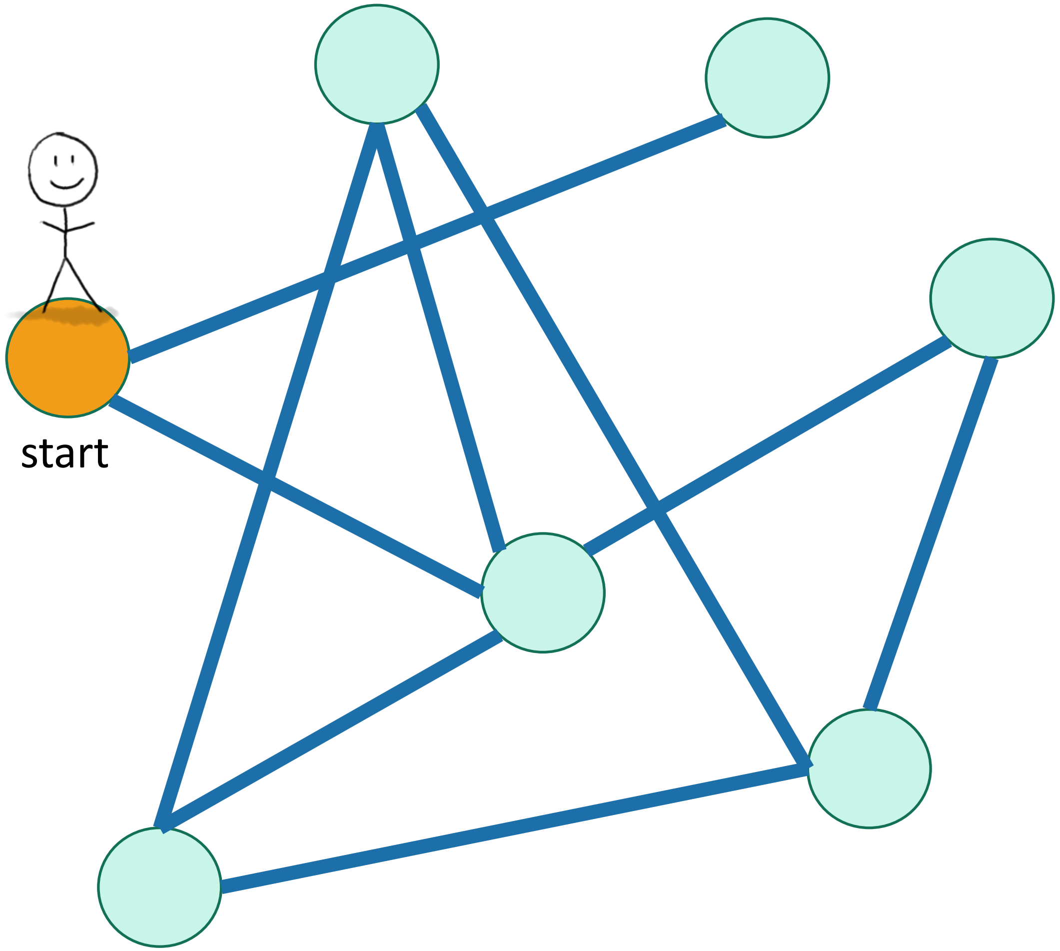

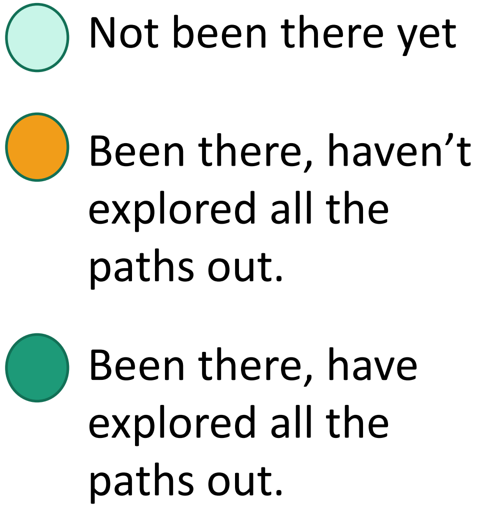

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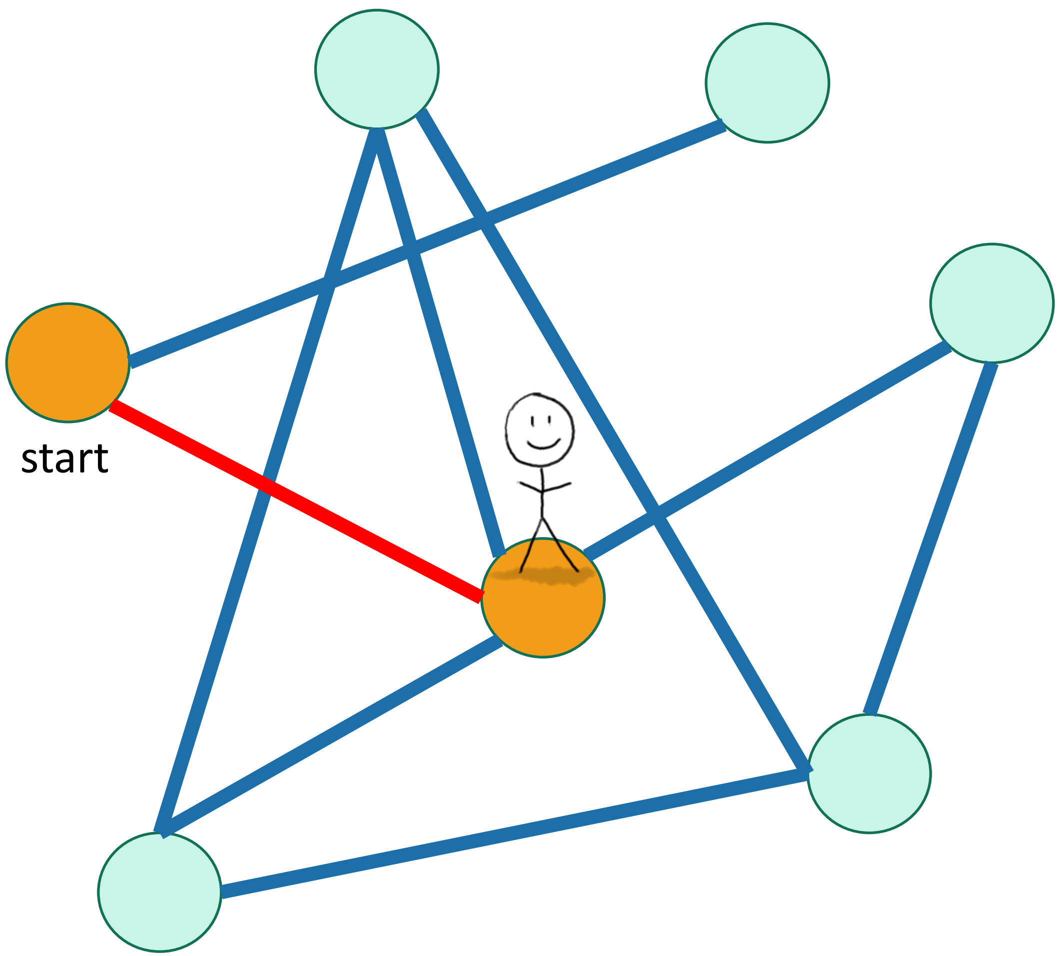

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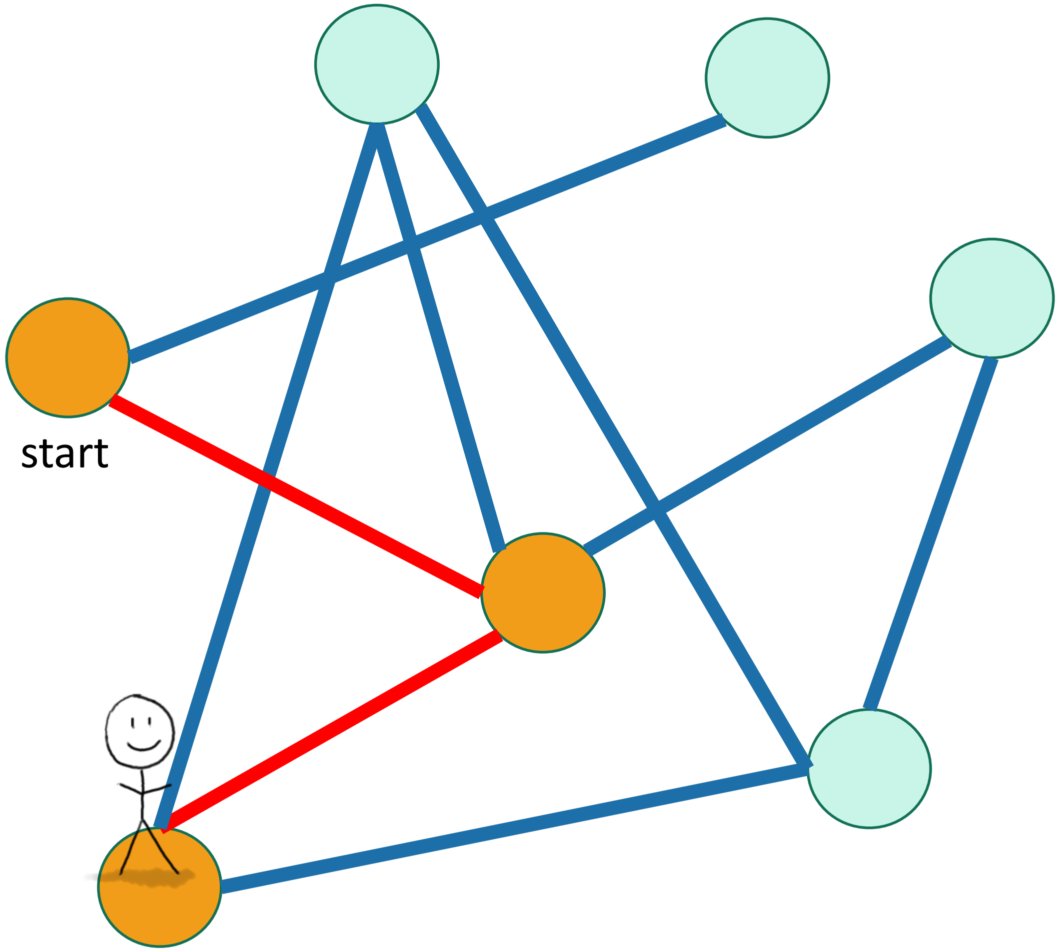

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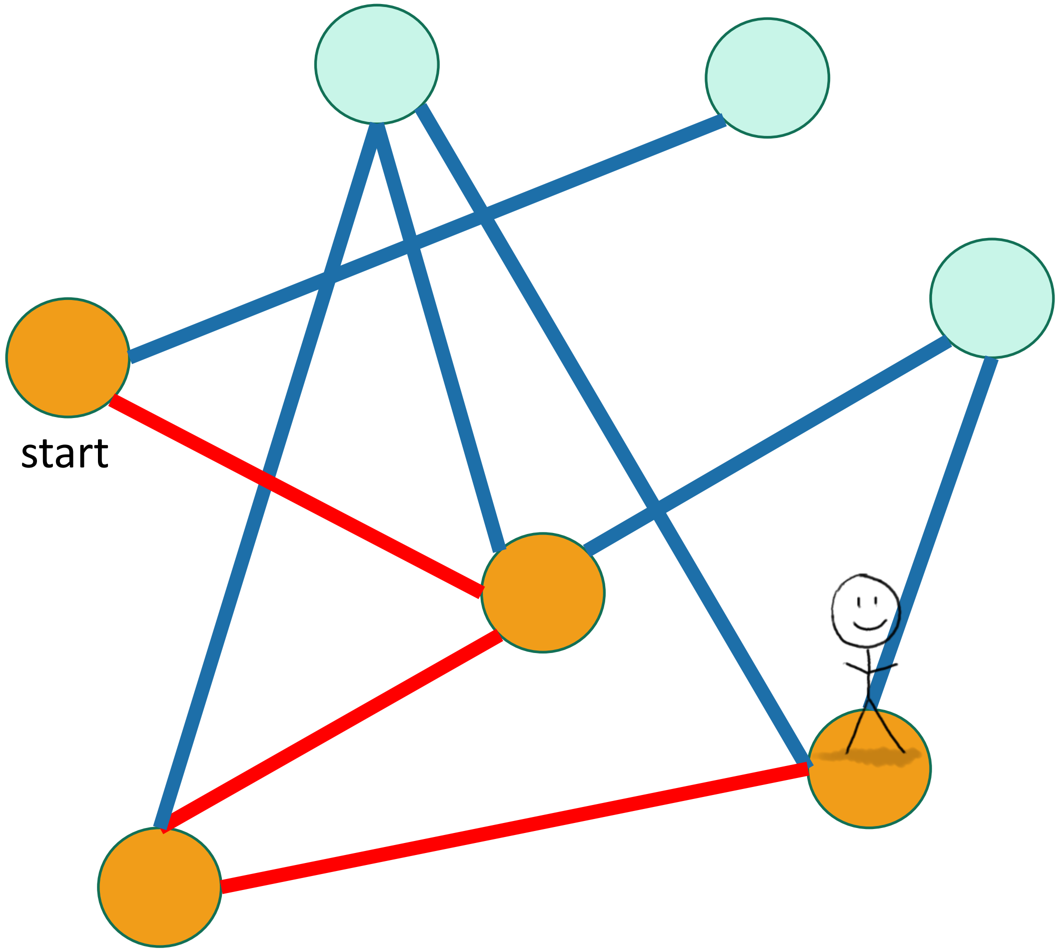

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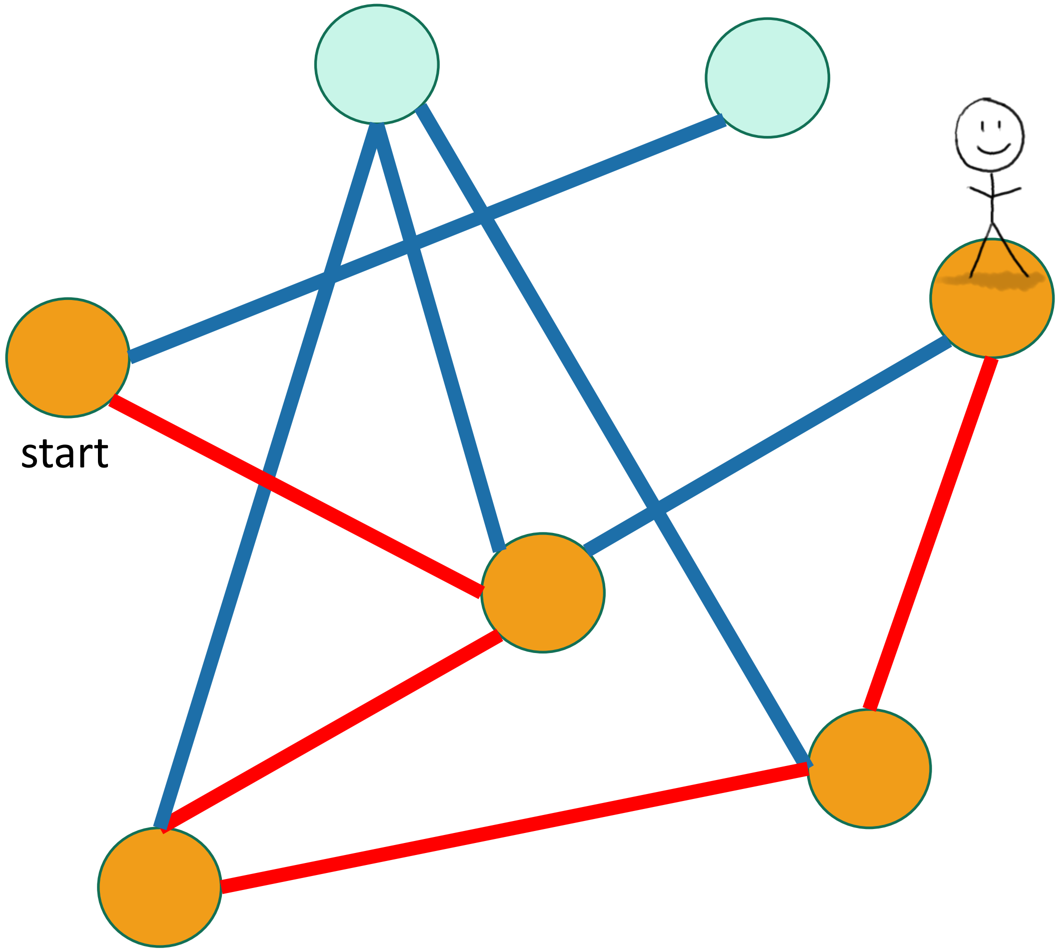

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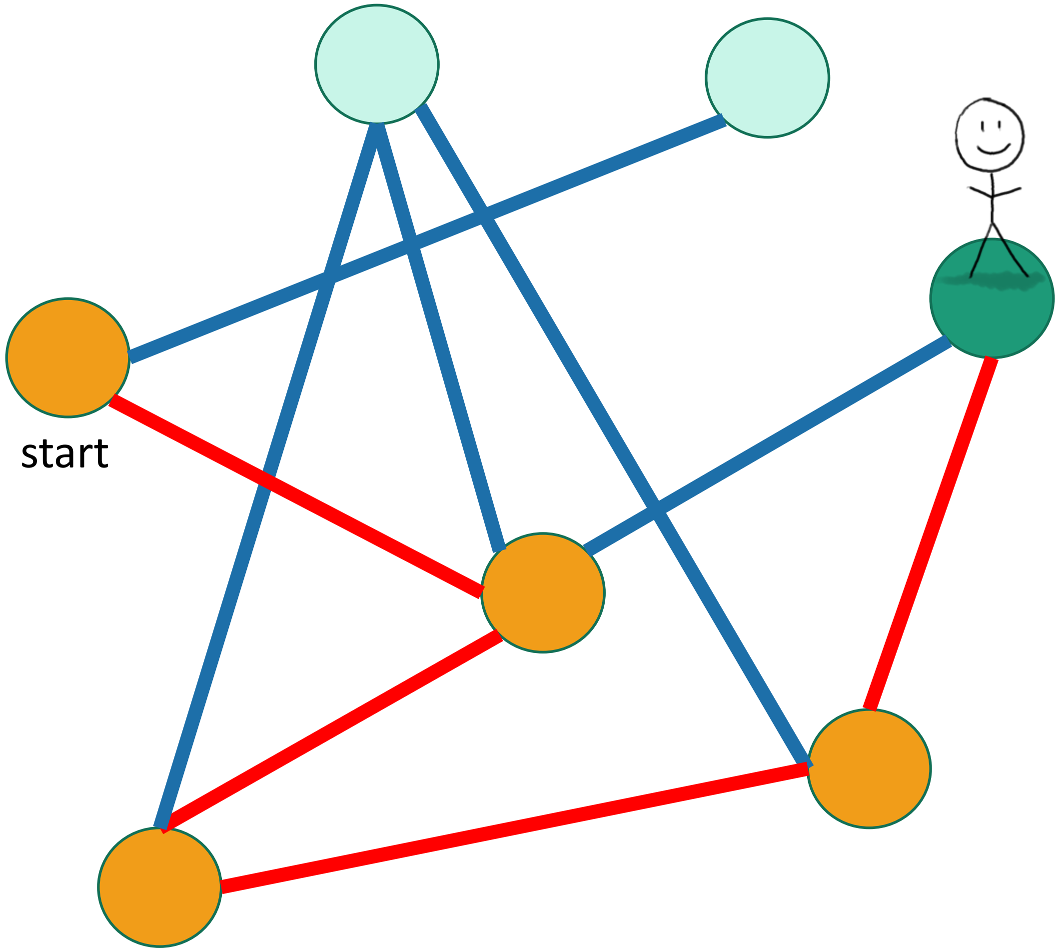

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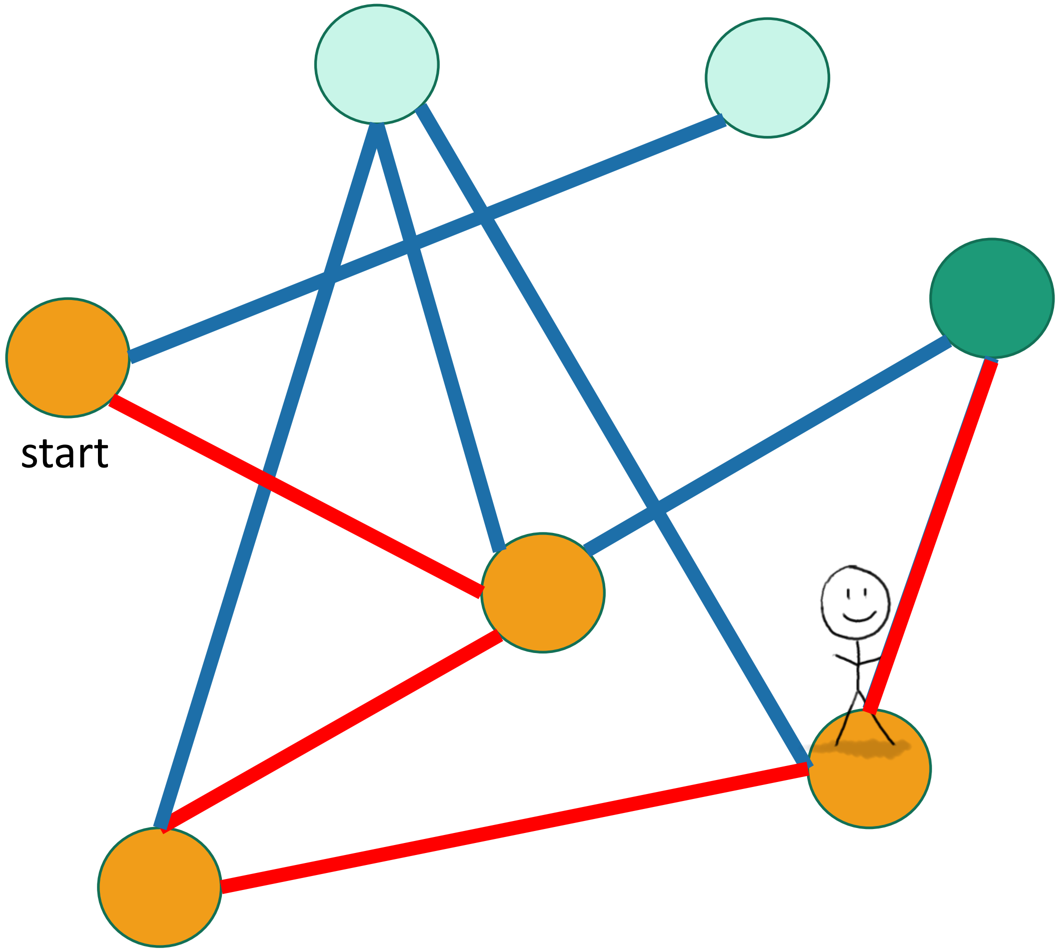

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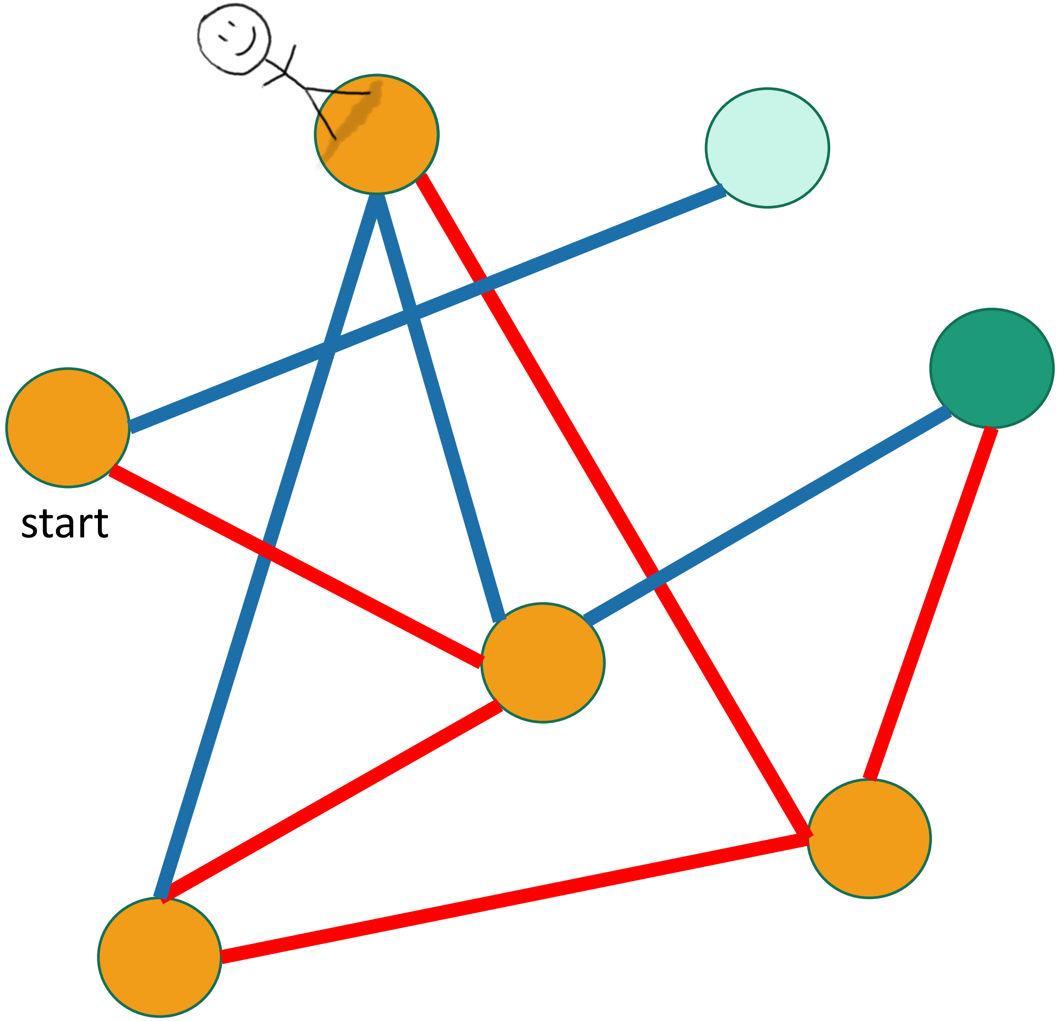

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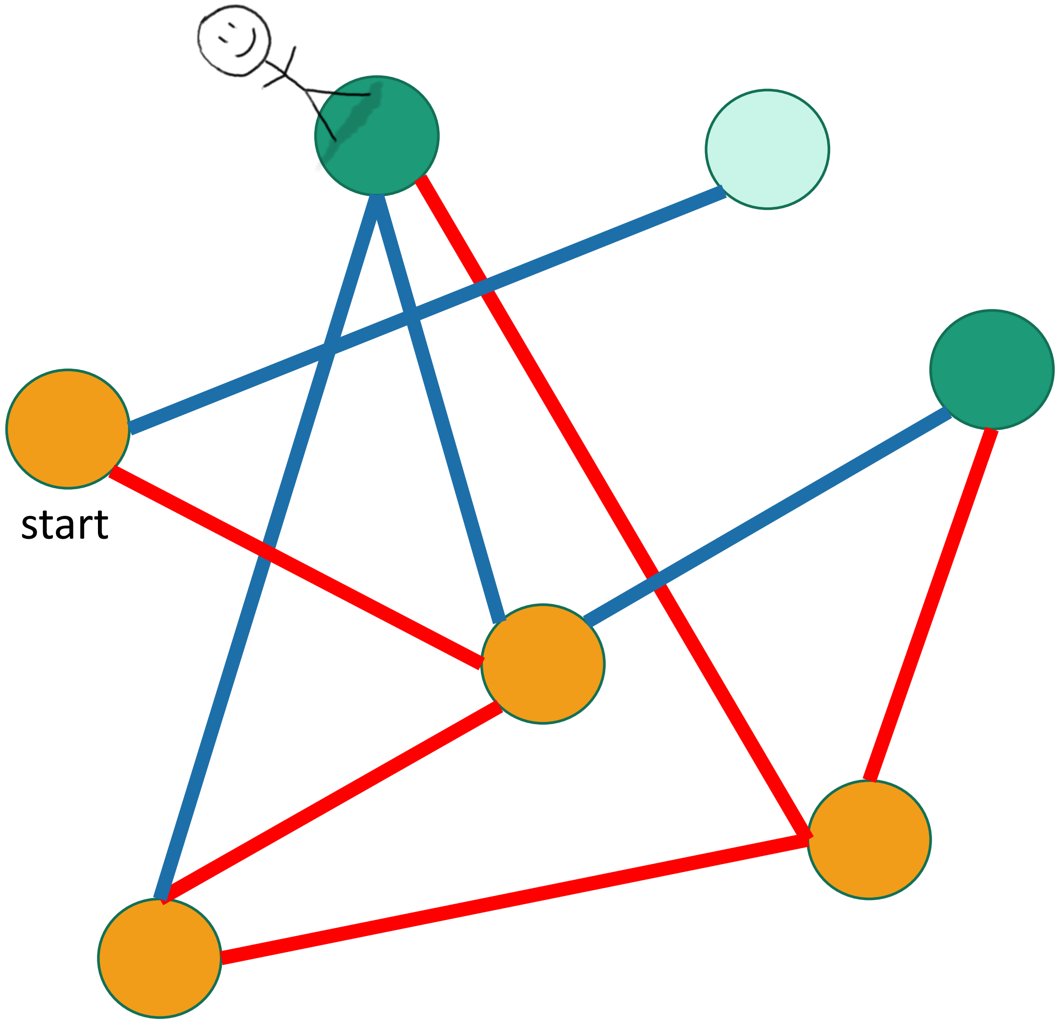

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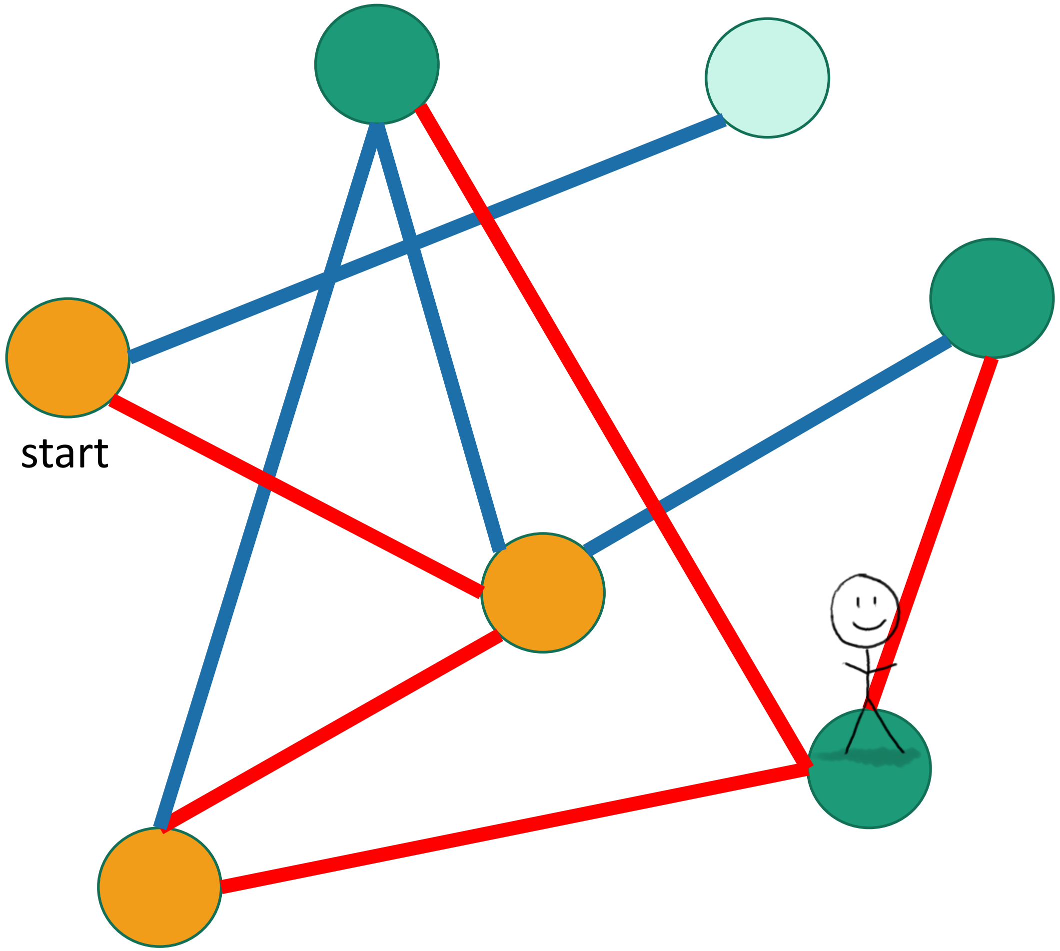

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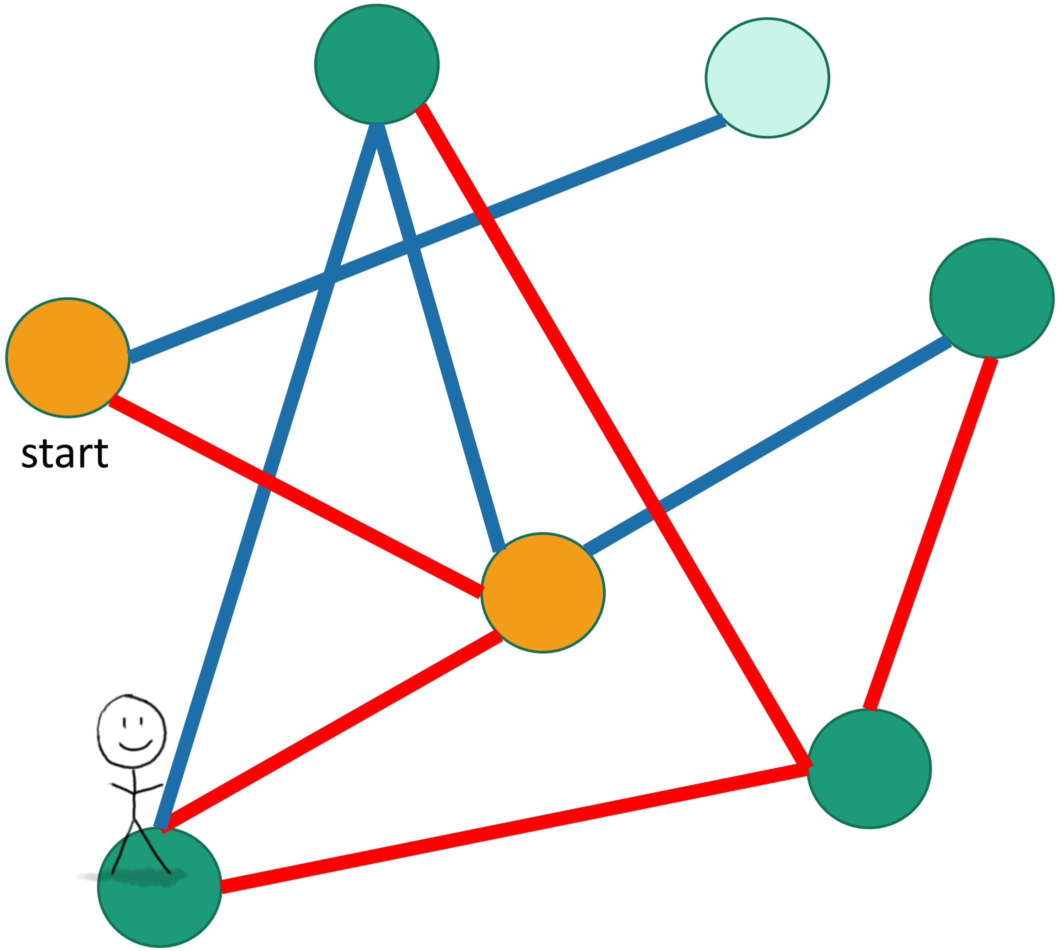

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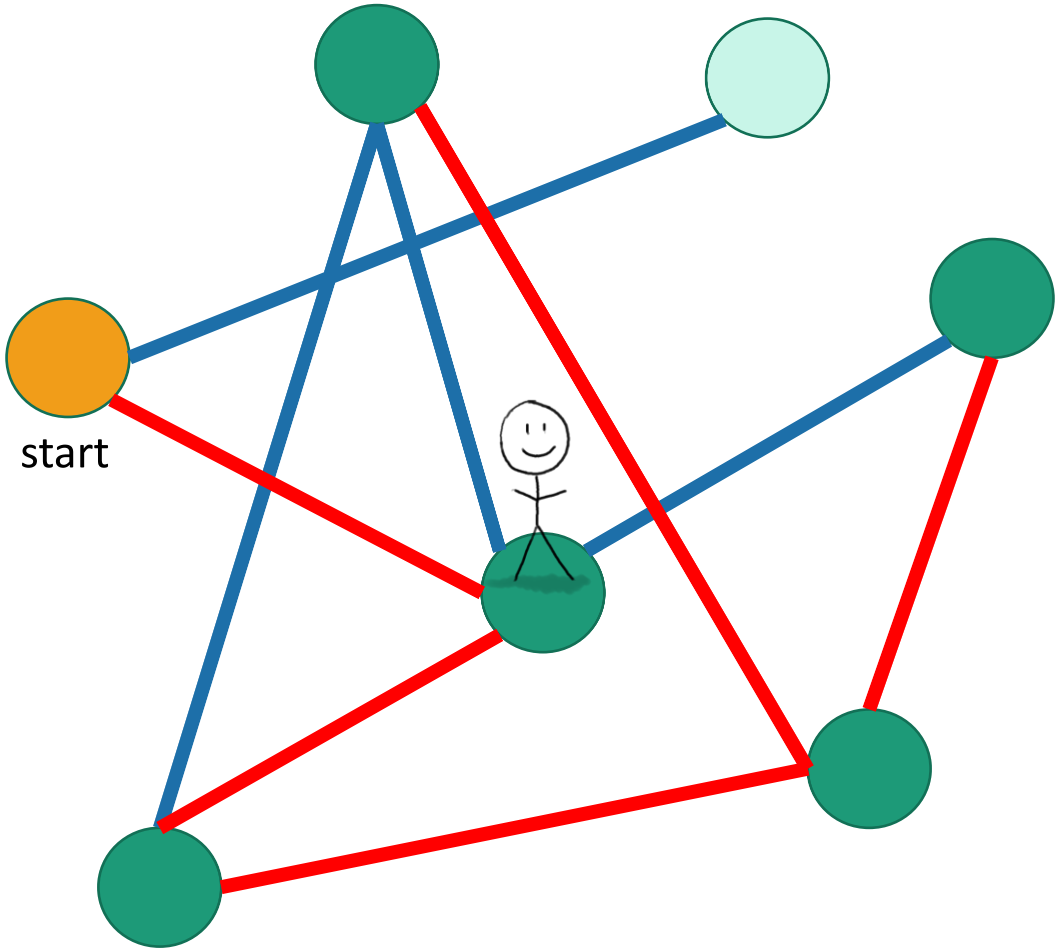

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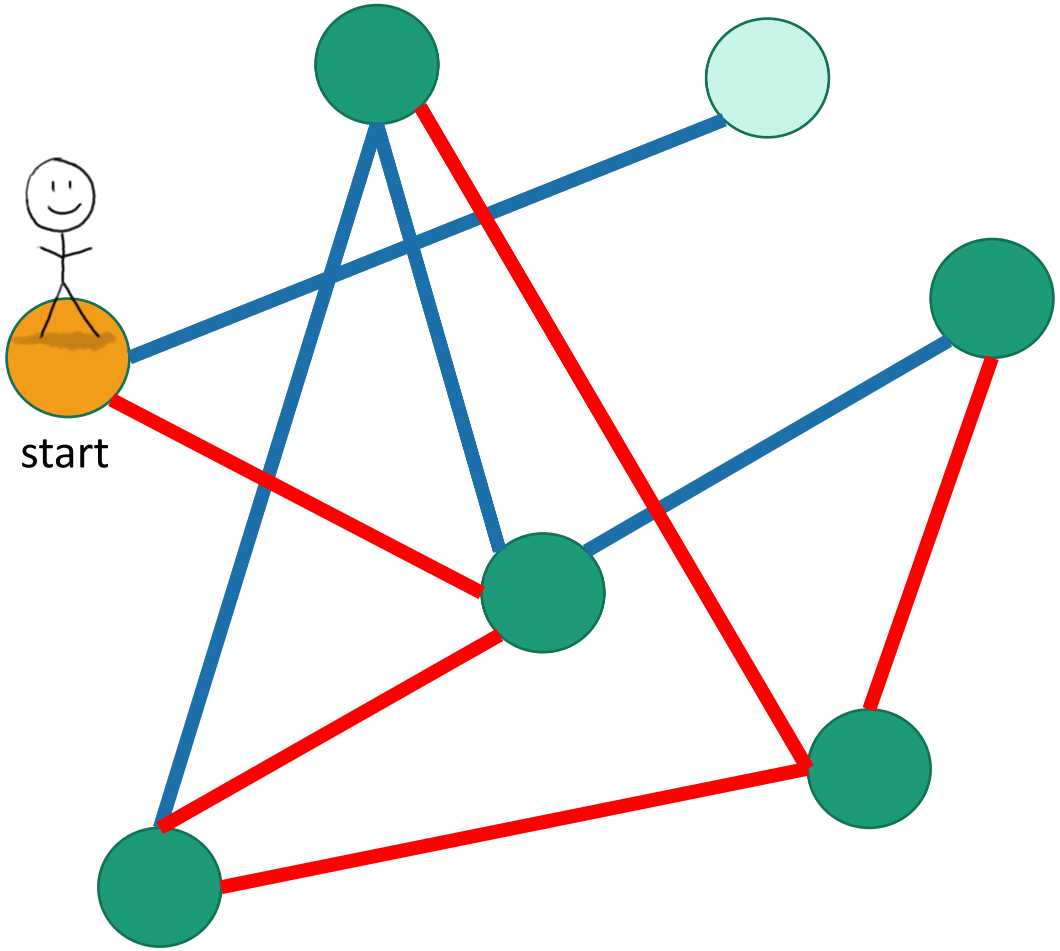

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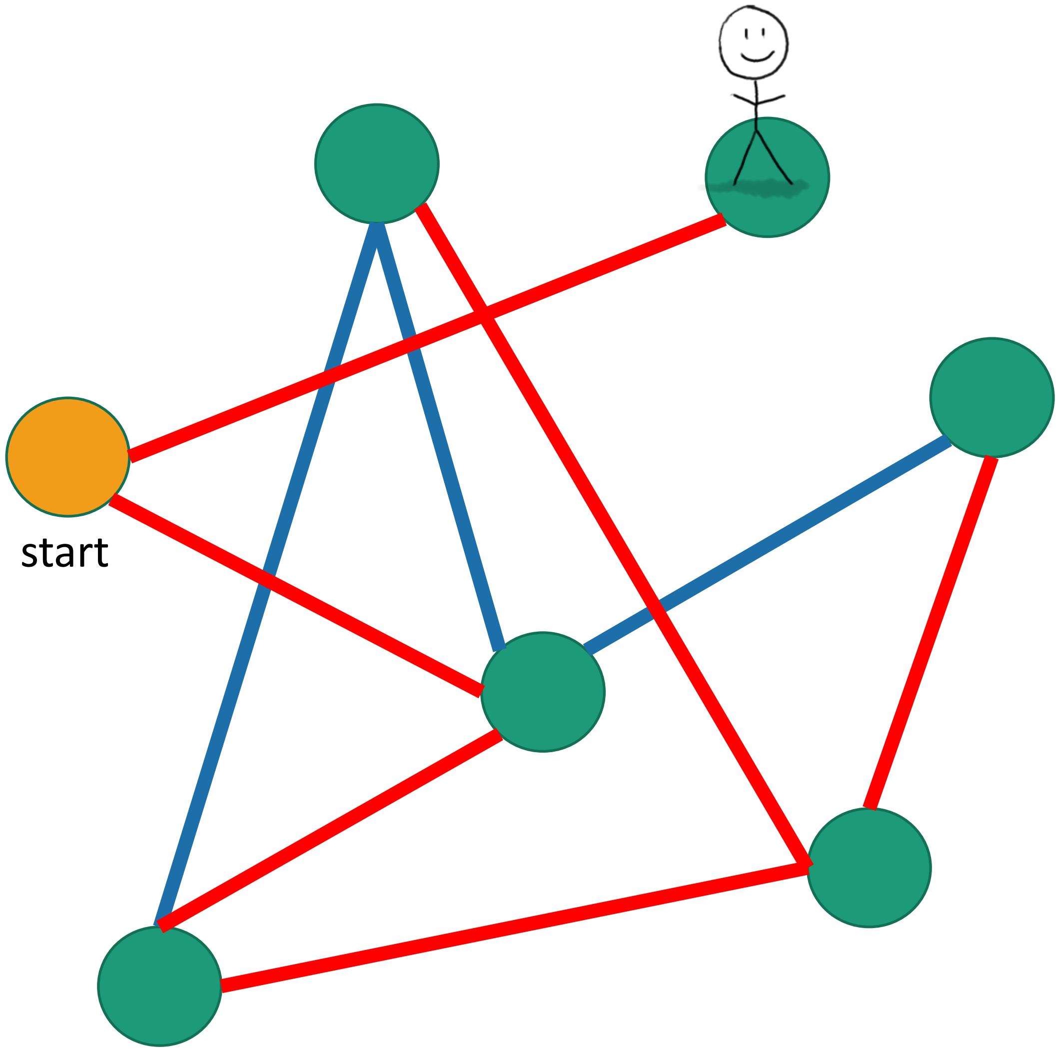

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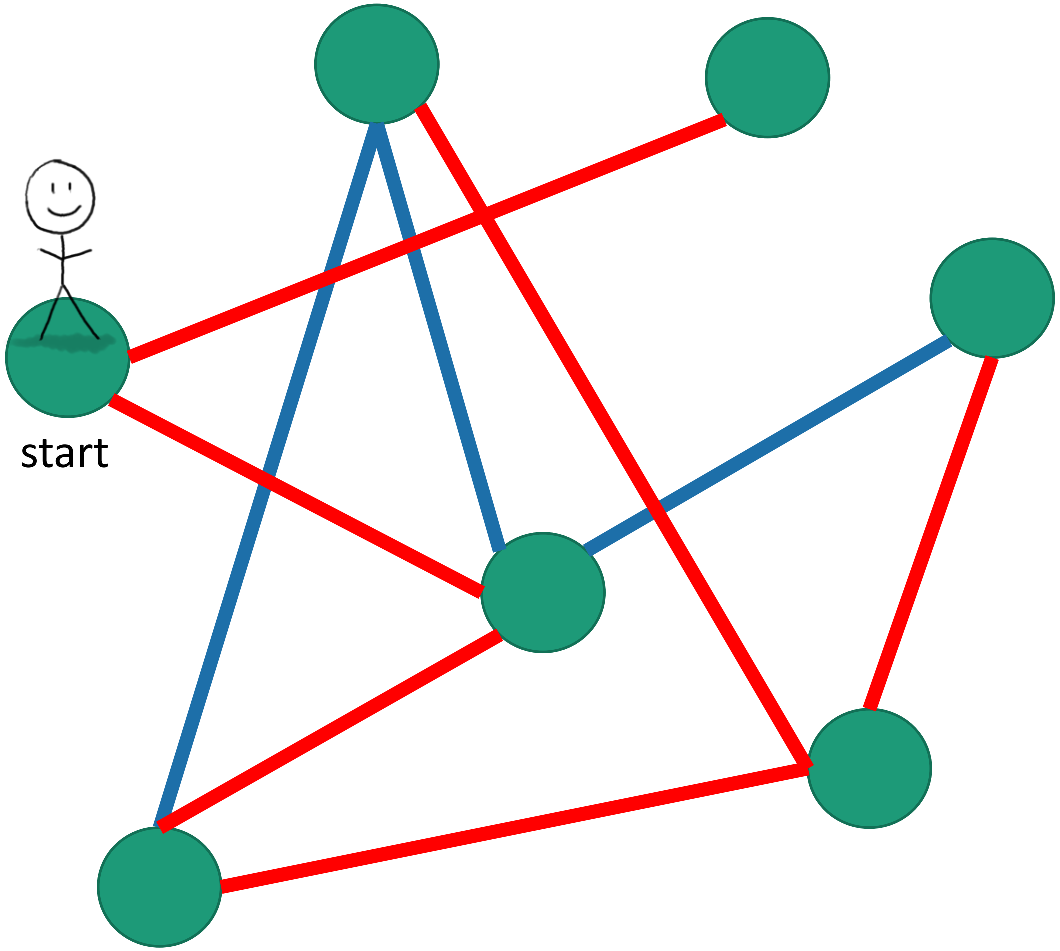

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Depth First Search
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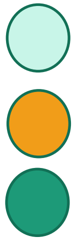

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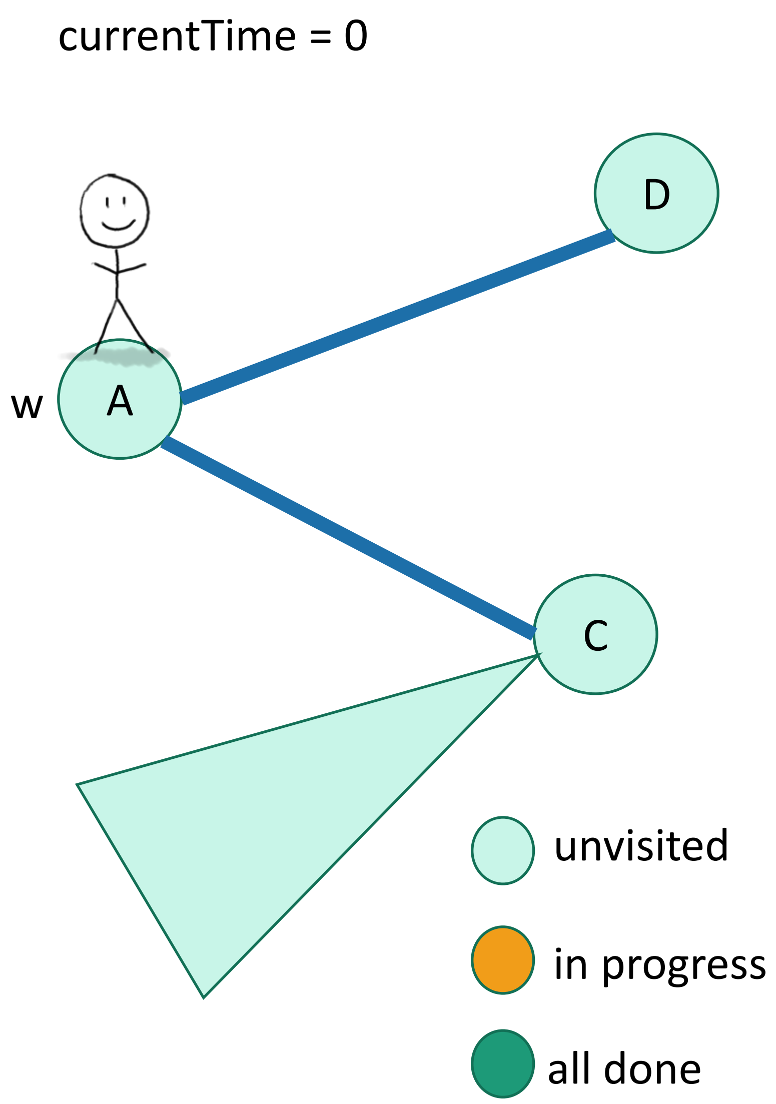
You might have seen other ways to implement DFS than what we are about to go through. This way has more bookkeeping – the bookkeeping will be useful later!

Depth First Search



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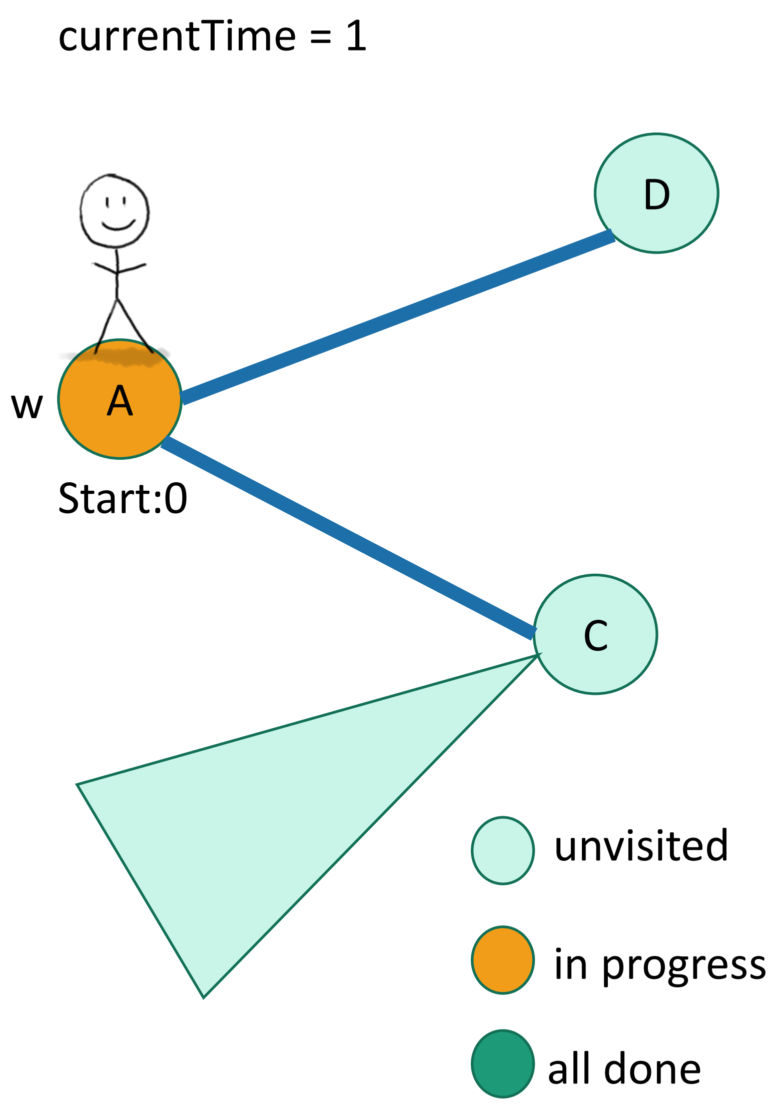

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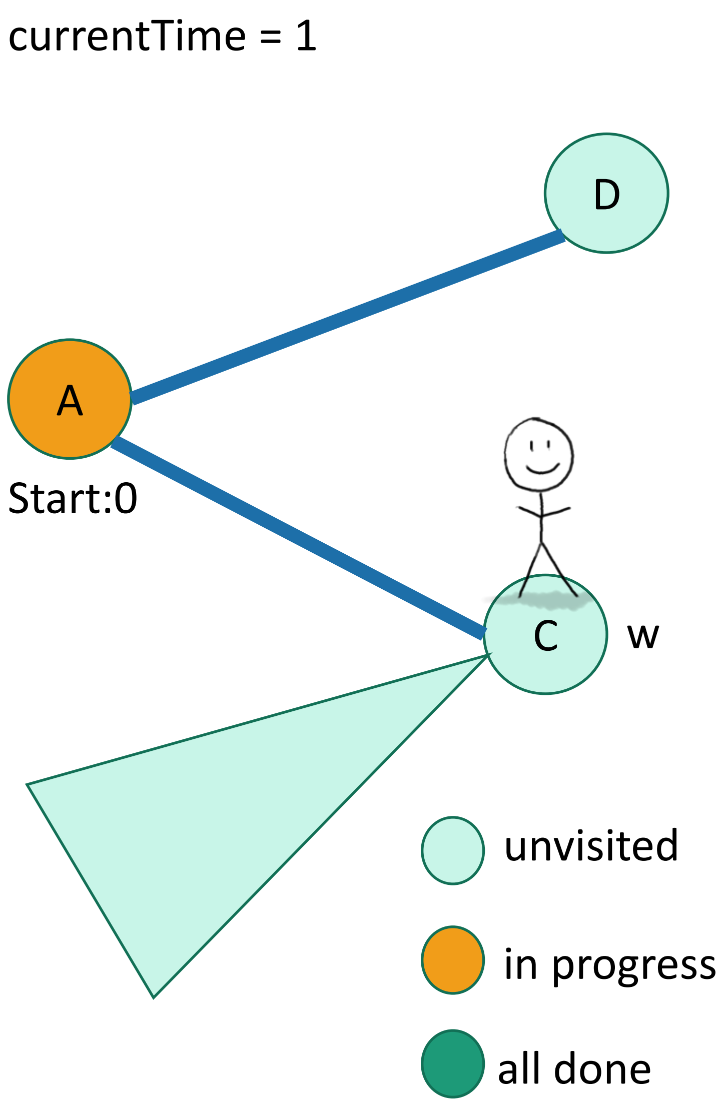

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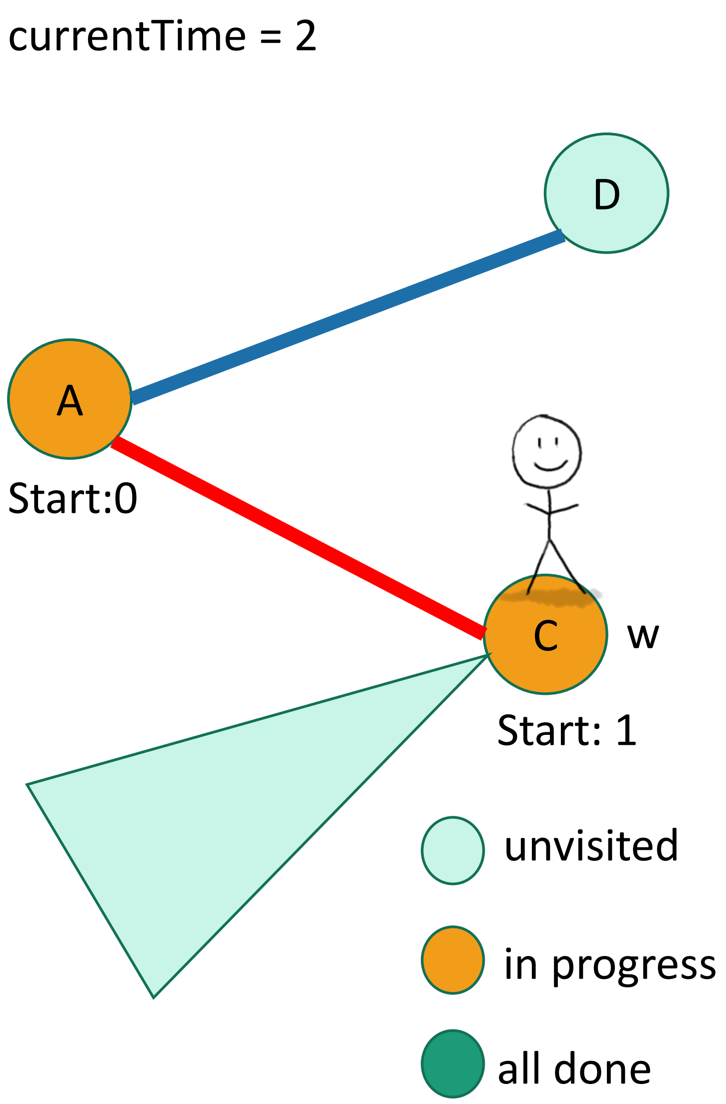

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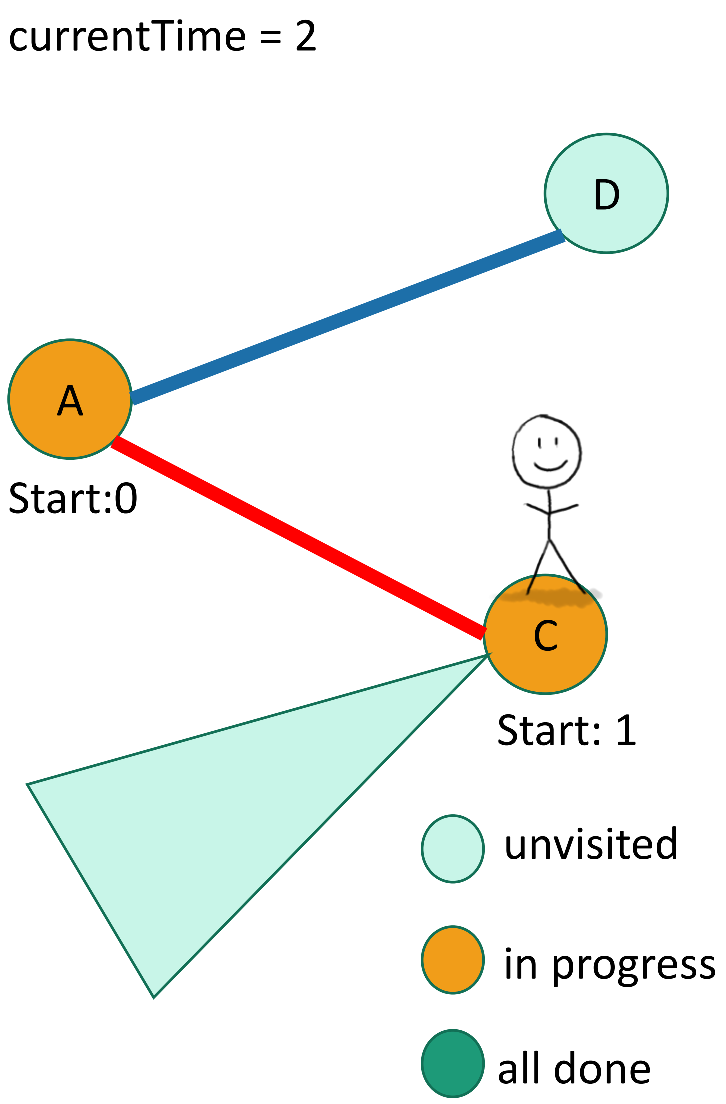

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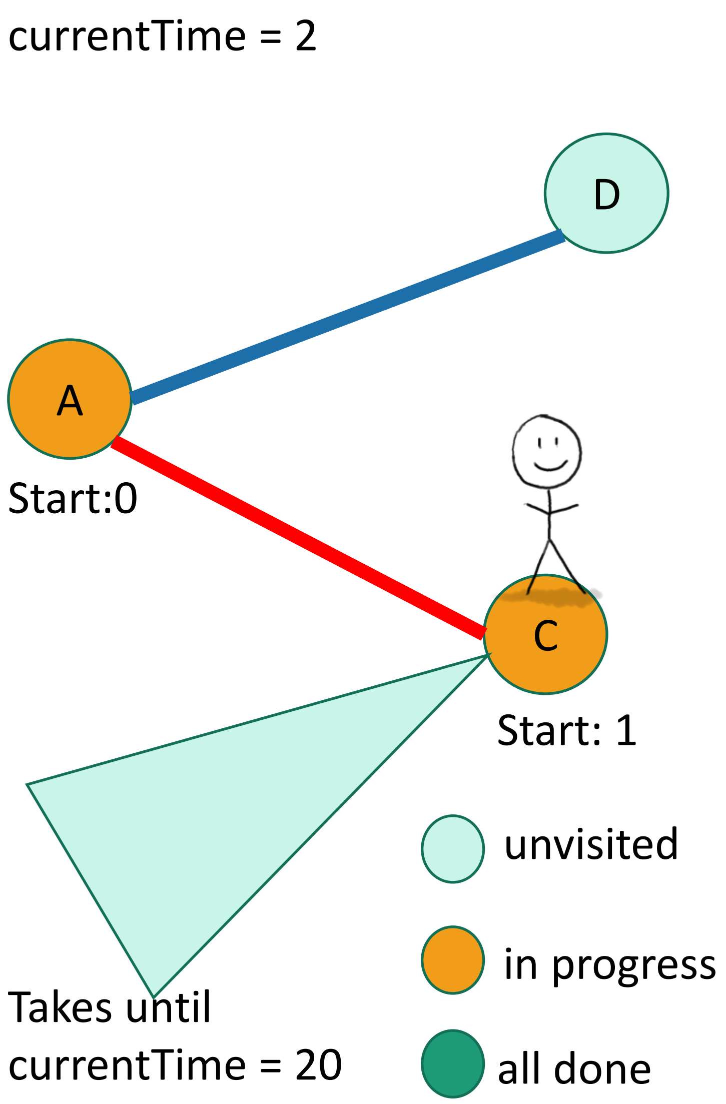

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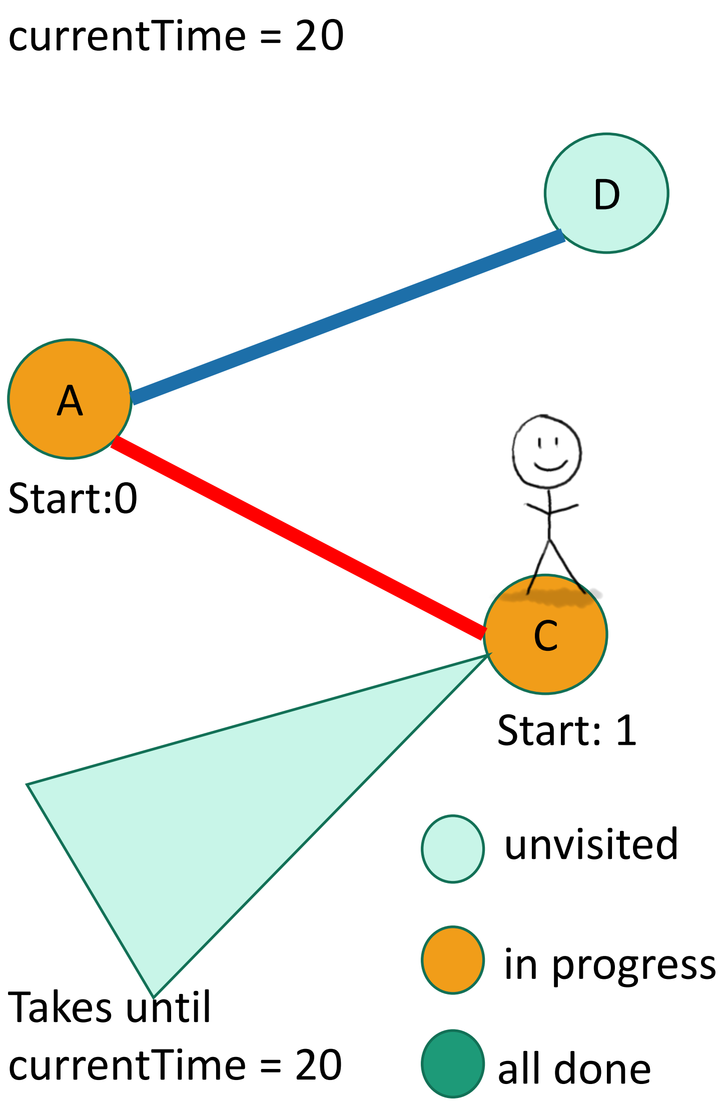

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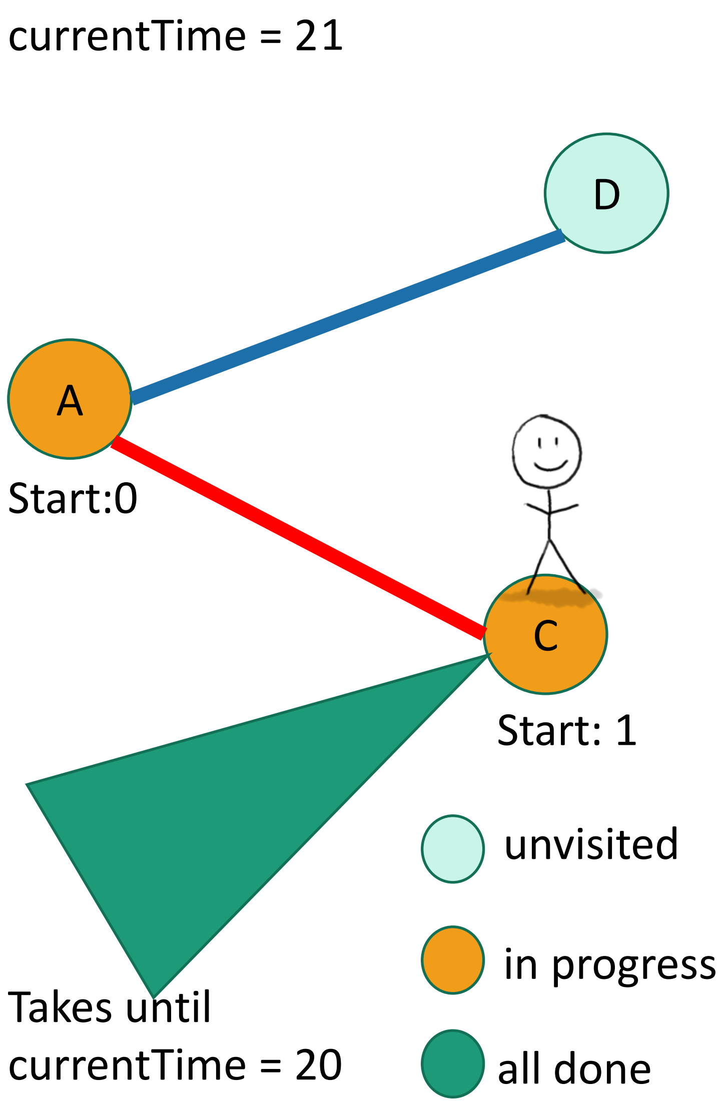

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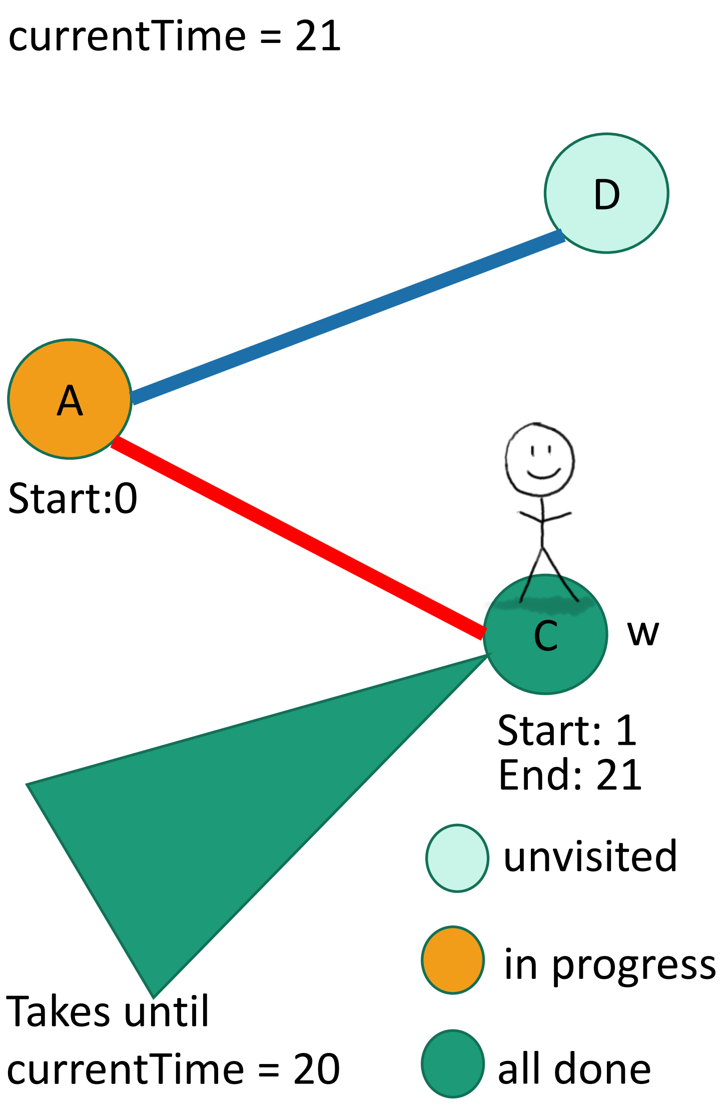

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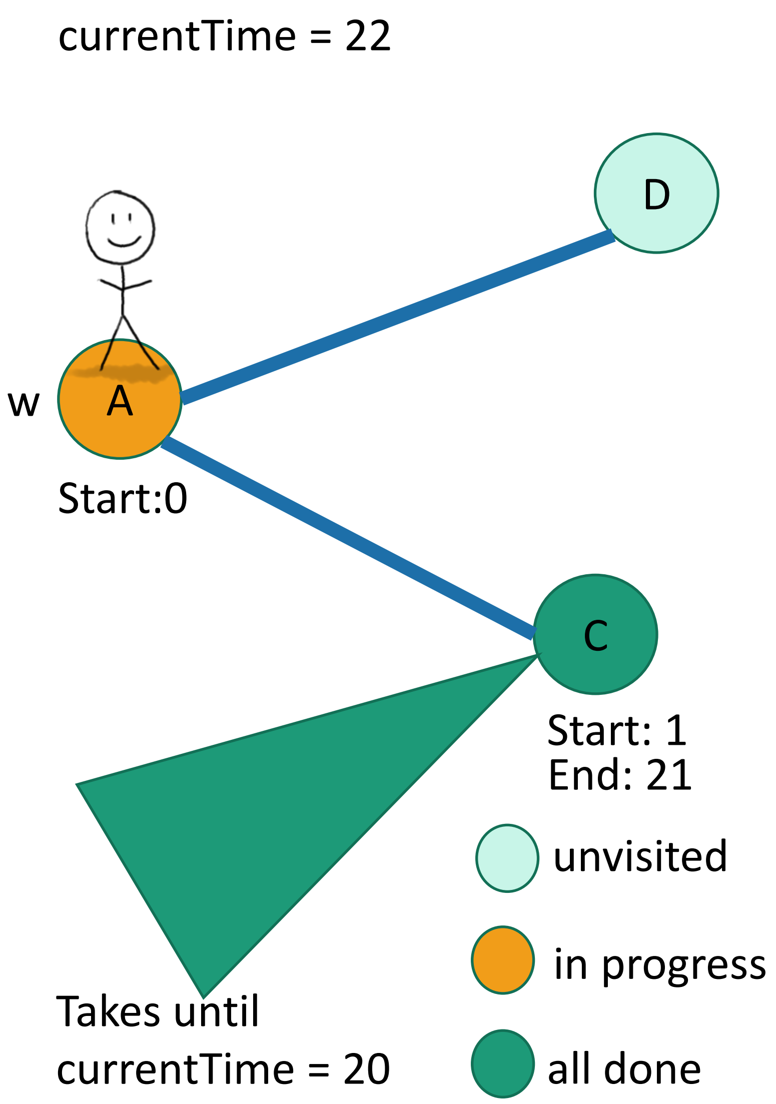

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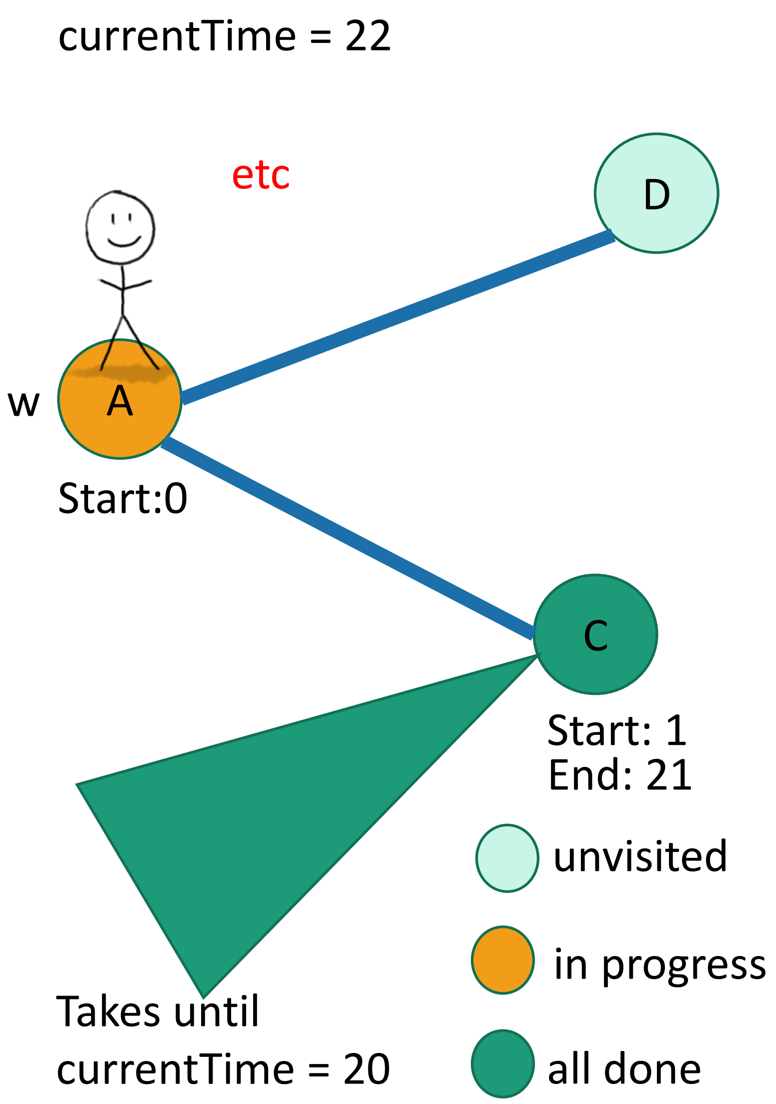

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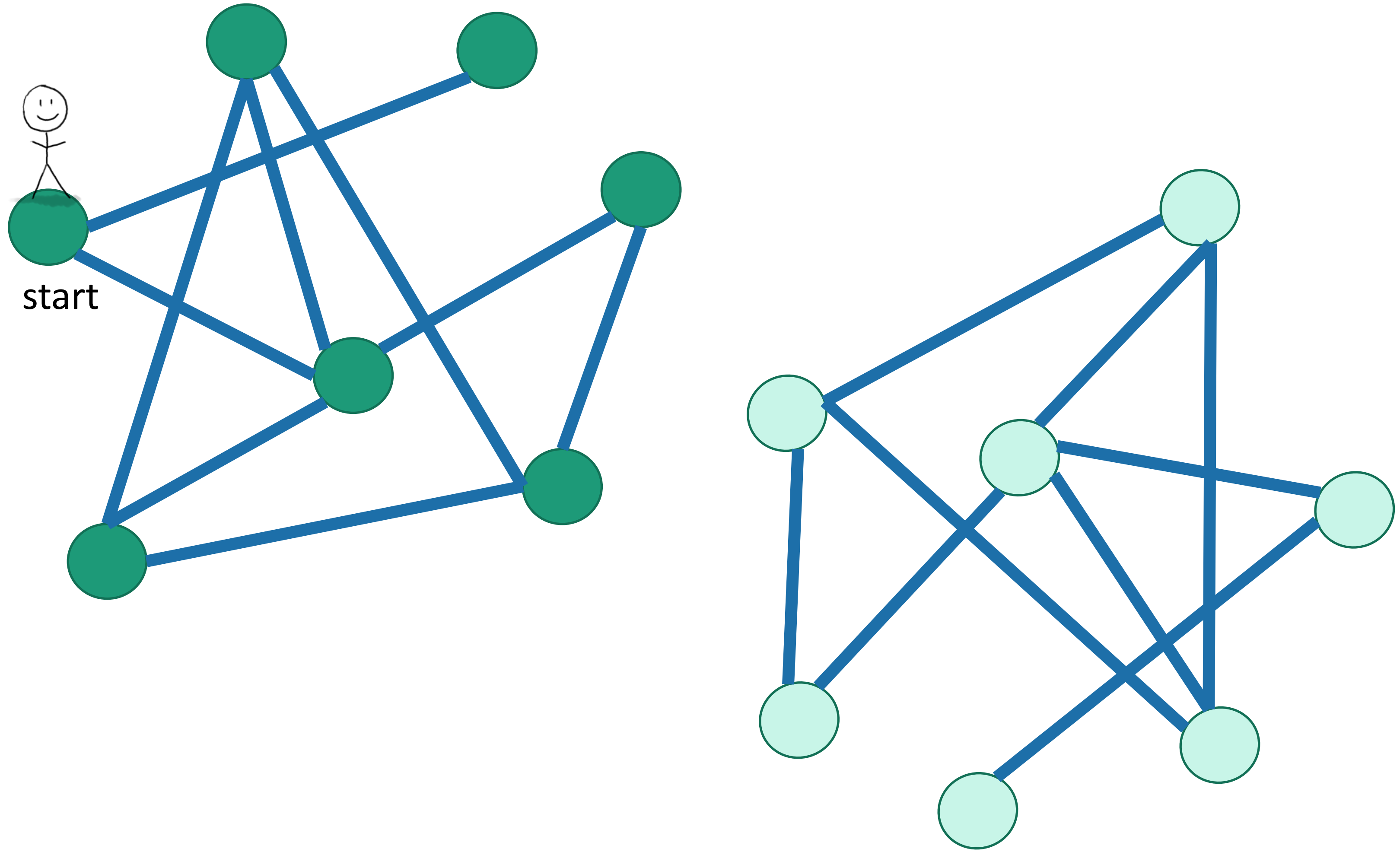
Fun exercise

- Write pseudocode for an iterative version of DFS.

DFS finds all the nodes reachable from the starting point



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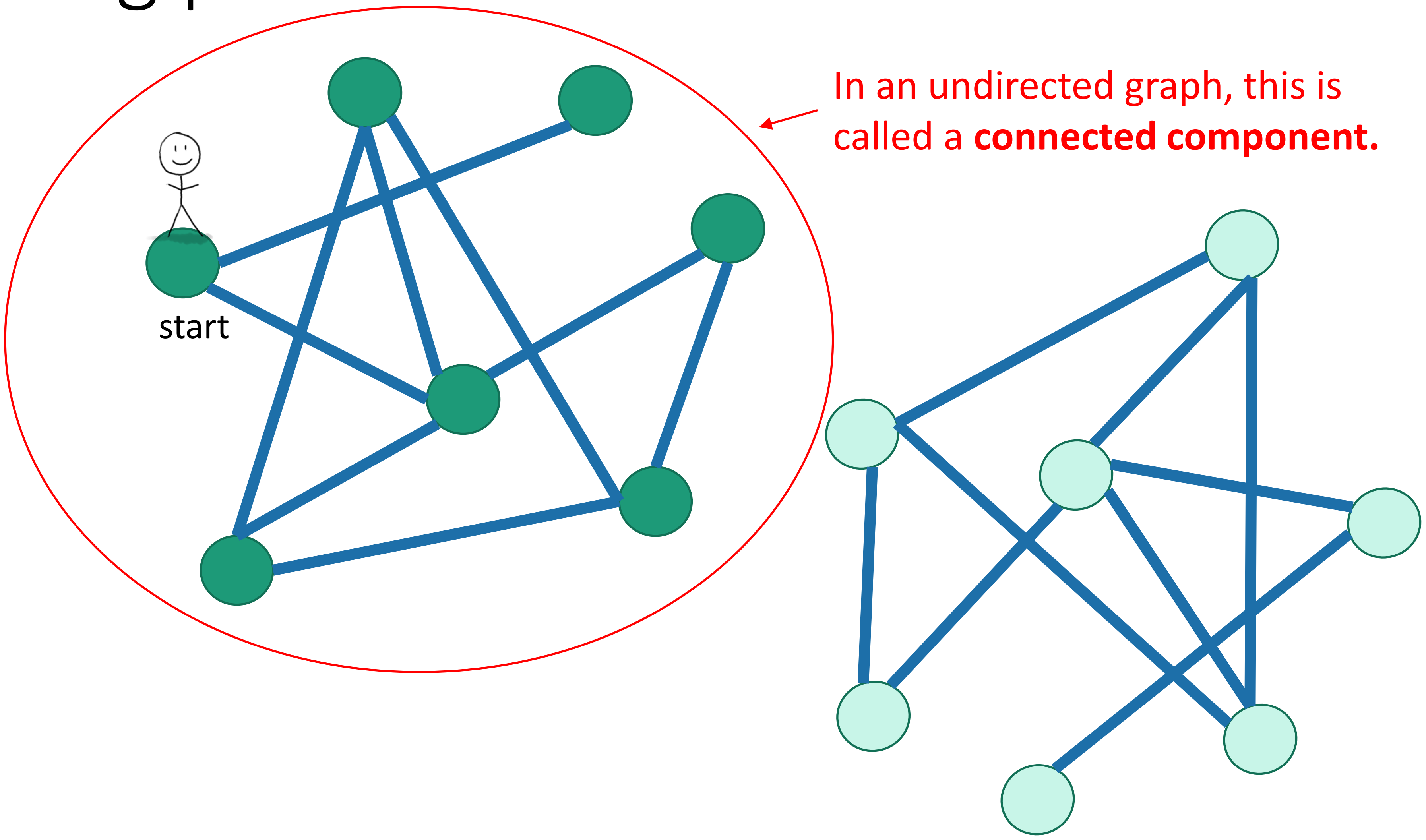

In an undirected graph, this is called a **connected component.**

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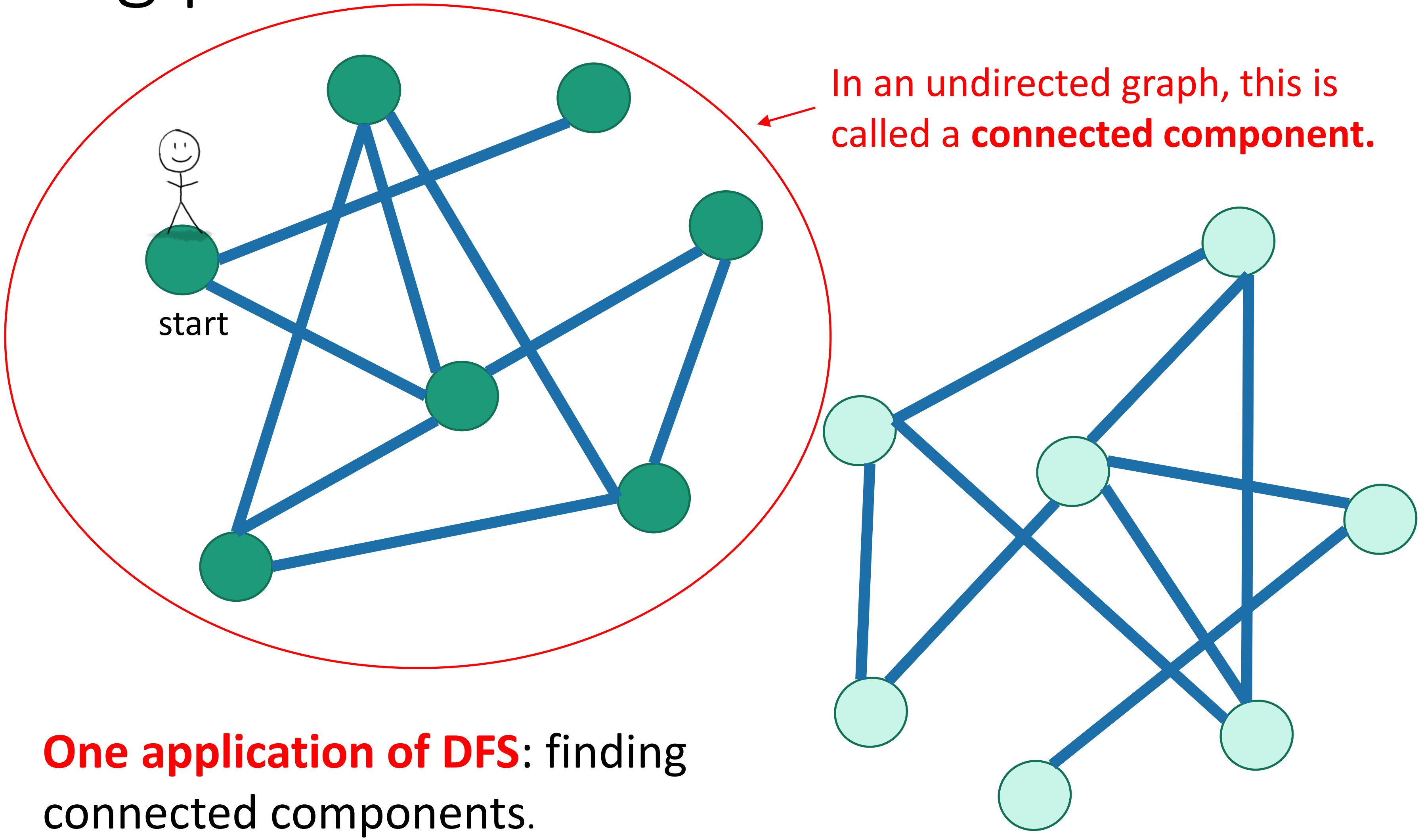
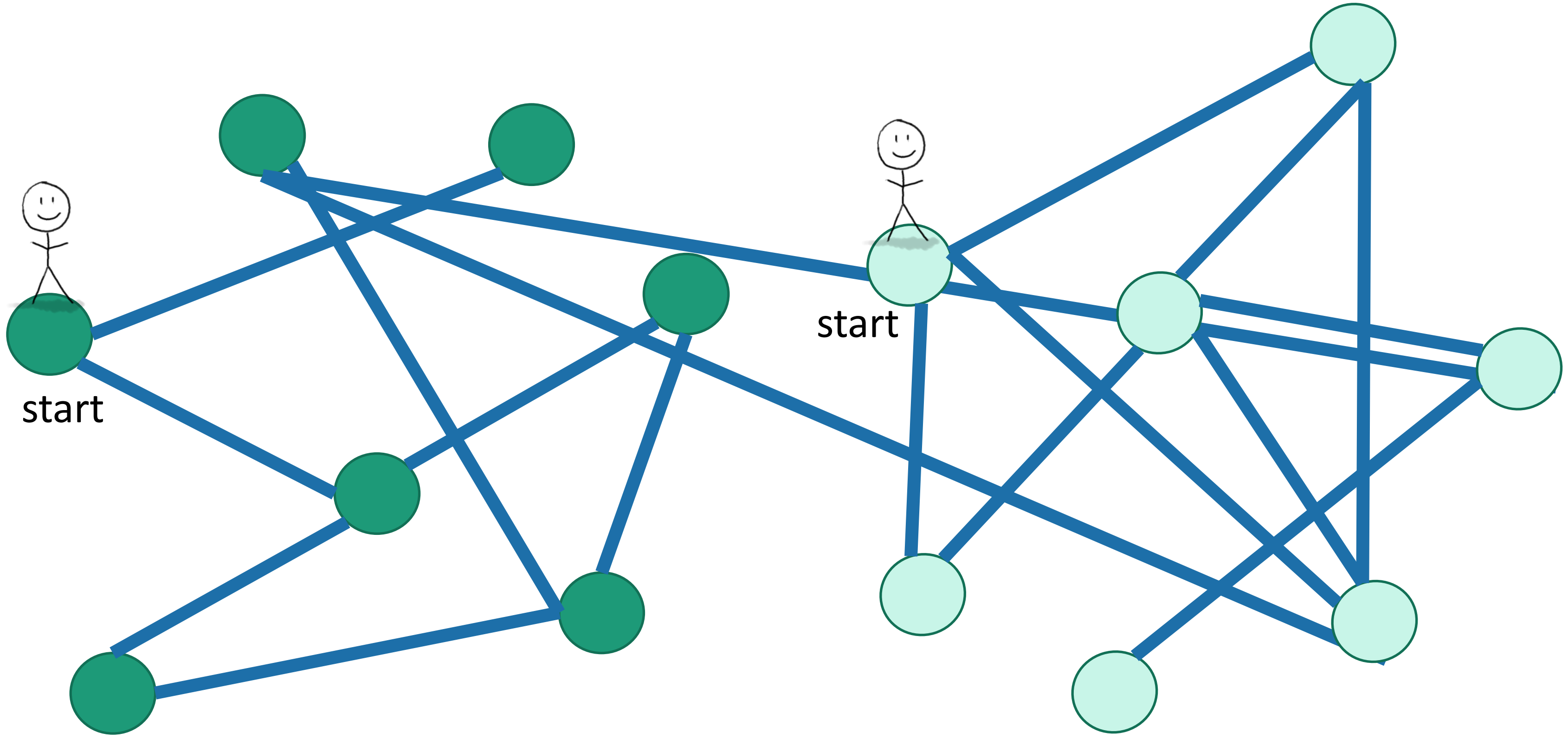


One application of DFS: finding connected components.

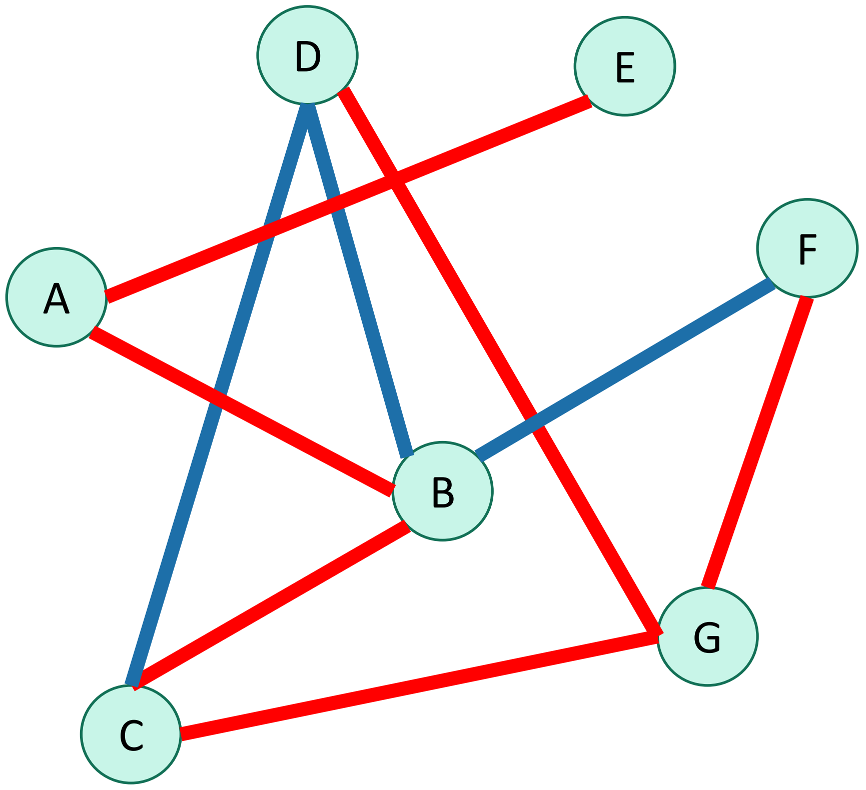
To explore the whole graph

- Do it repeatedly!



Why is it called depth-first?

- We are implicitly building a tree:



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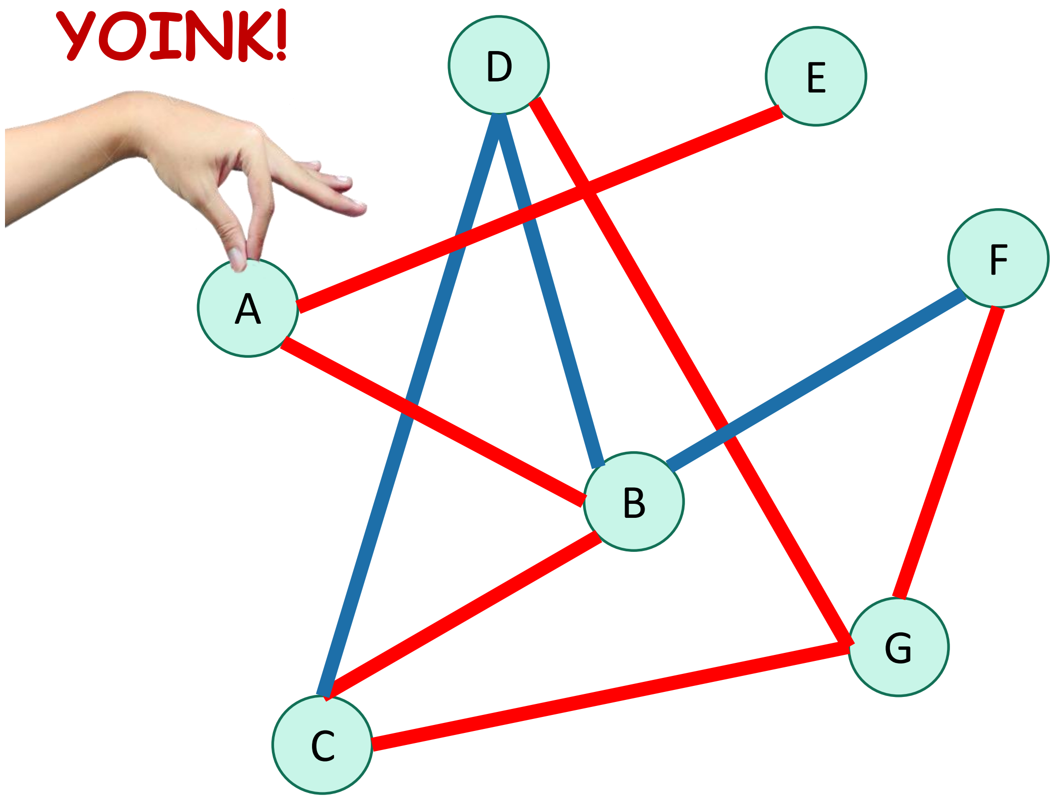

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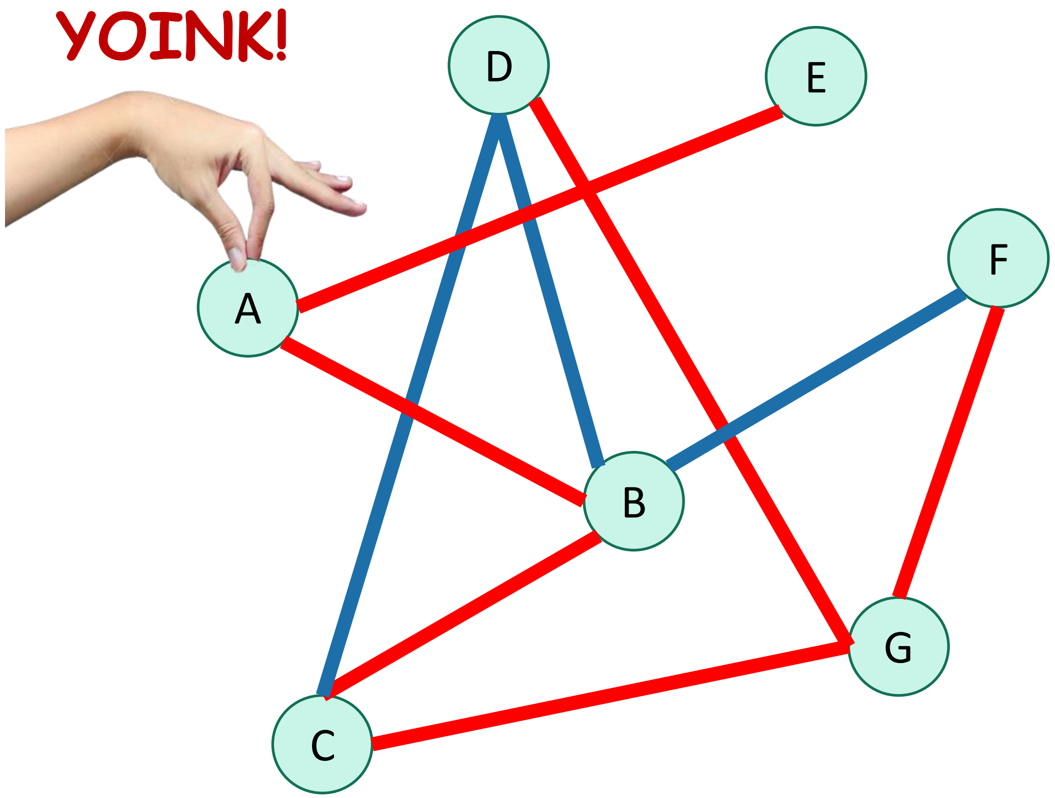

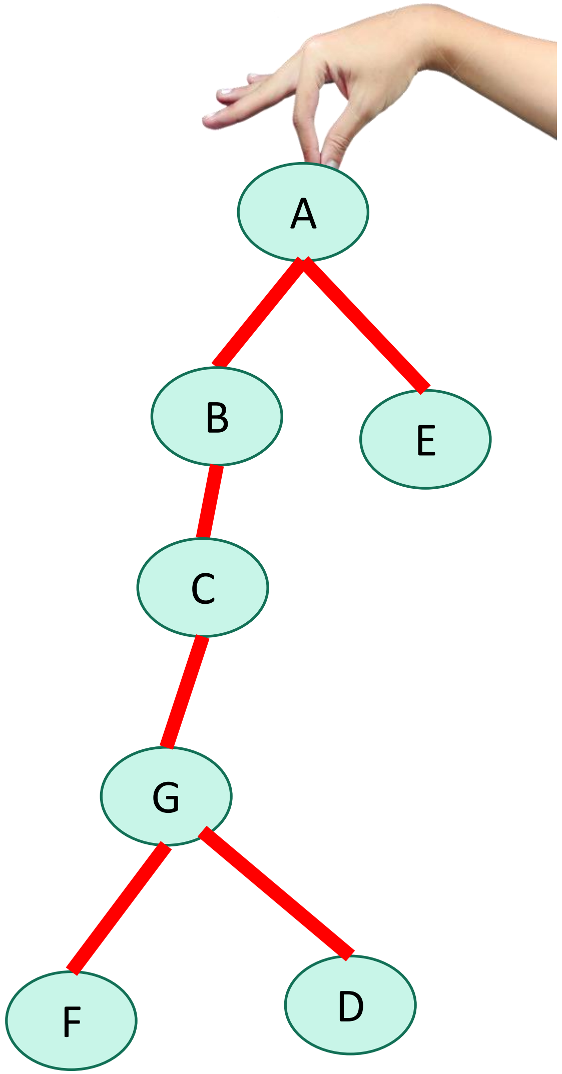

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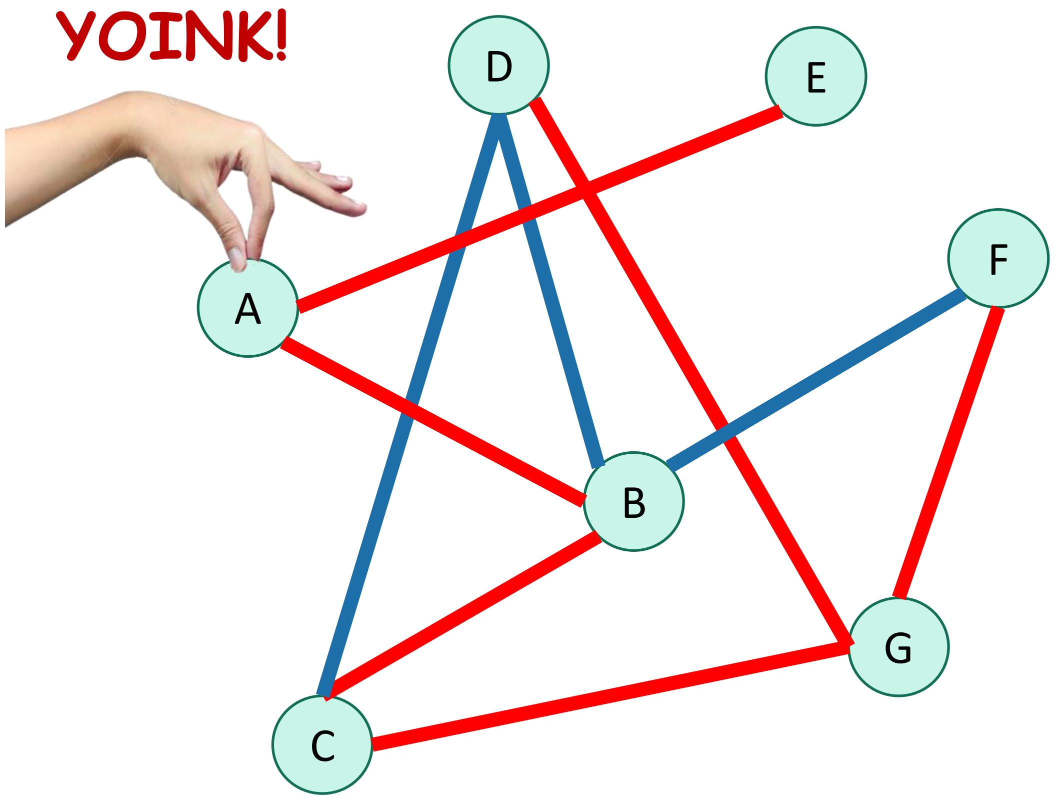

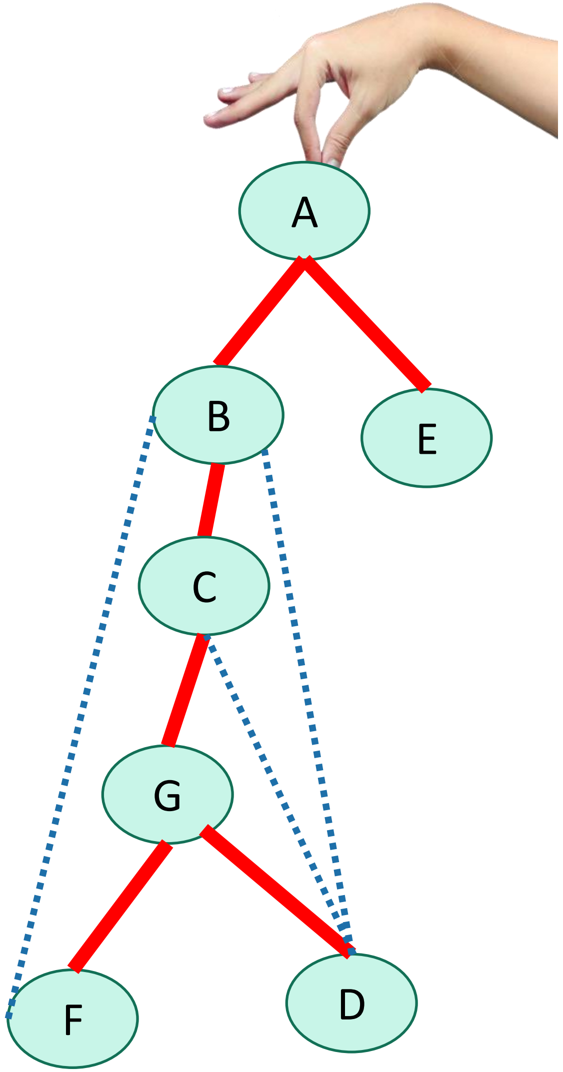

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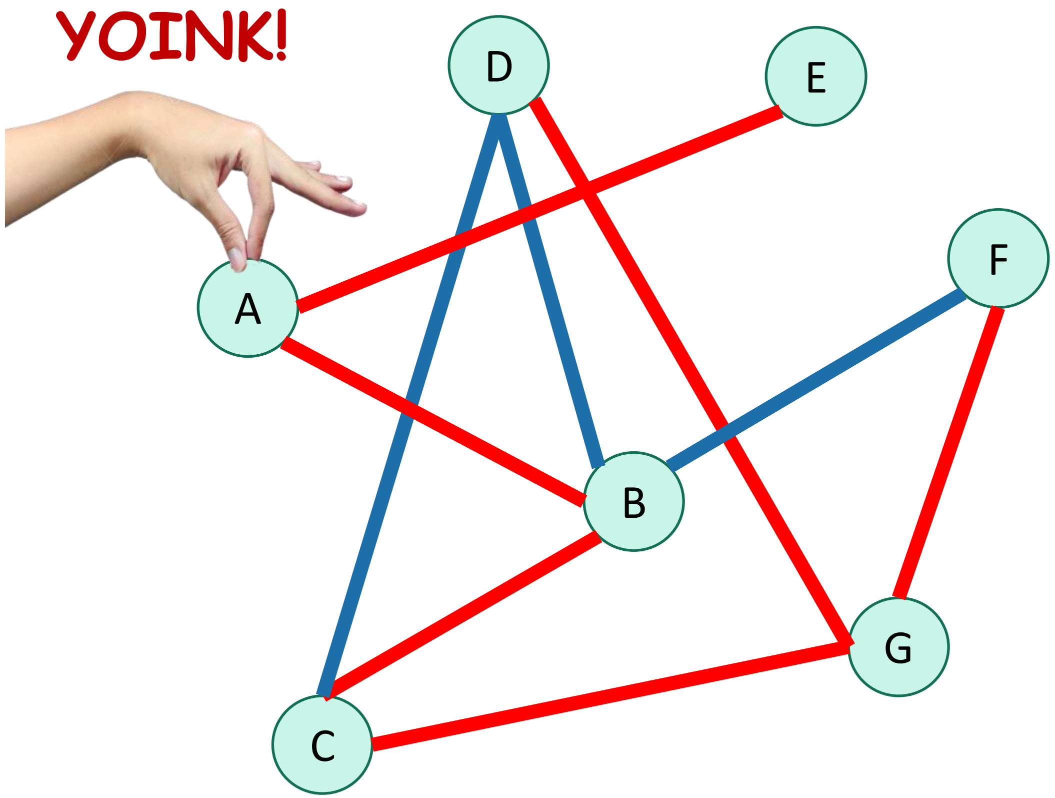

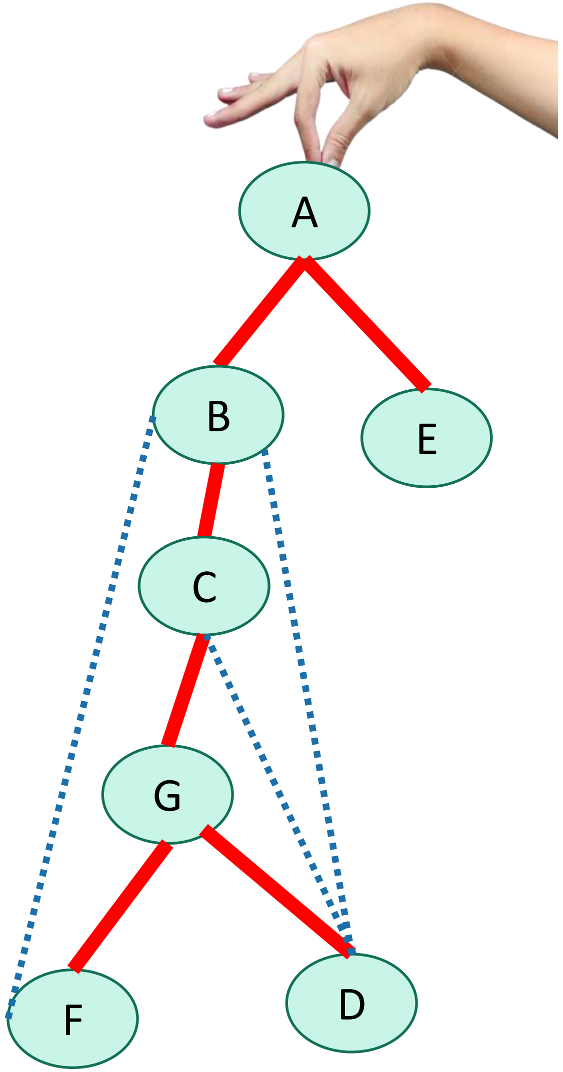

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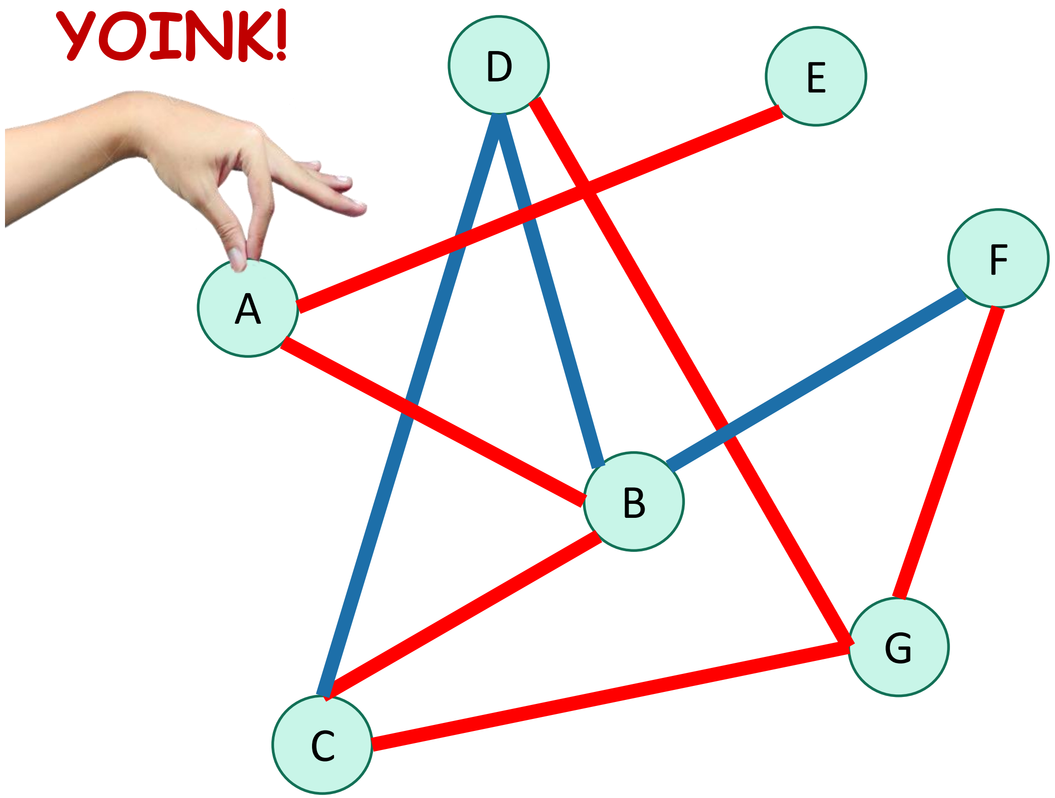

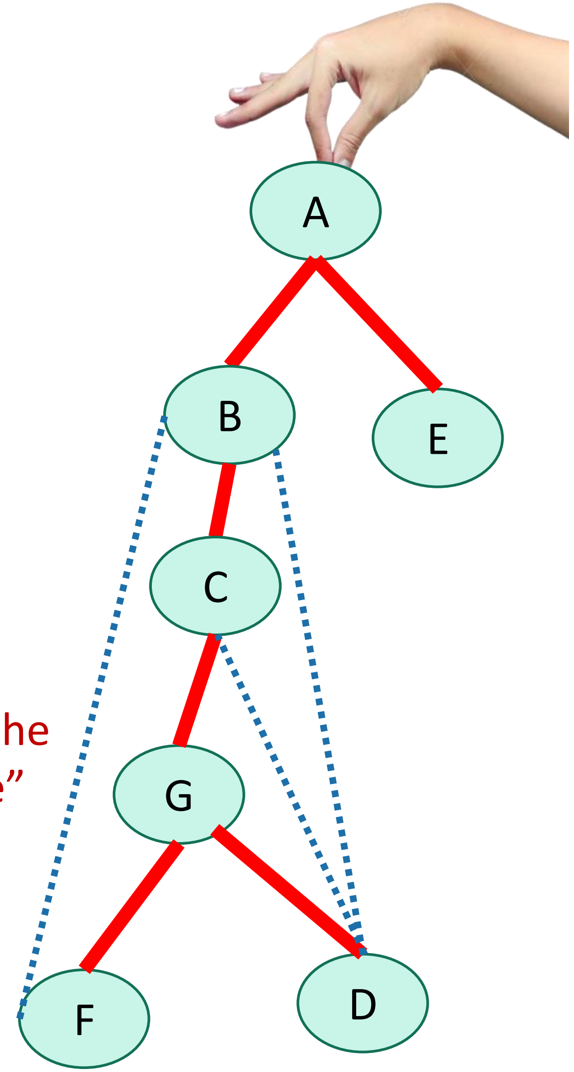

Call this the "DFS tree"

- First, we go as deep as we can.

Running time

To explore **just the connected component** we started in

- We look at each edge at most twice.
  - Once from each of its endpoints
- And basically we don’t do anything else.
- So…

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$$O(m)$$

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To explore just the connected component we started in

- Assume we are using the linked-list format for G.
- Say C = (V', E') is a connected component.
- We visit each vertex in C exactly once.
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In a connected graph, $|V'| \leq |E'| + 1$.

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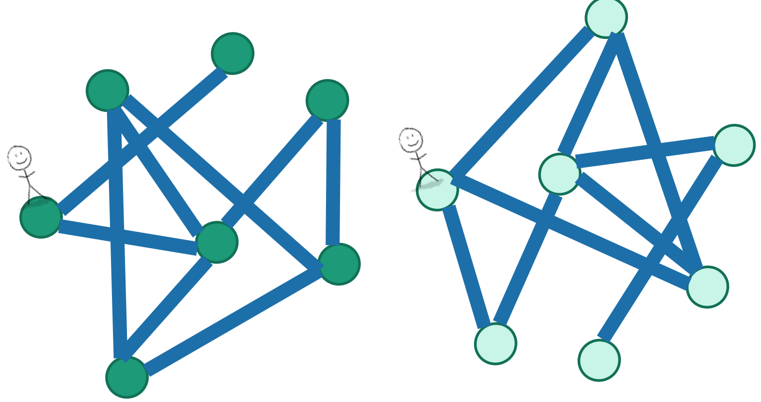

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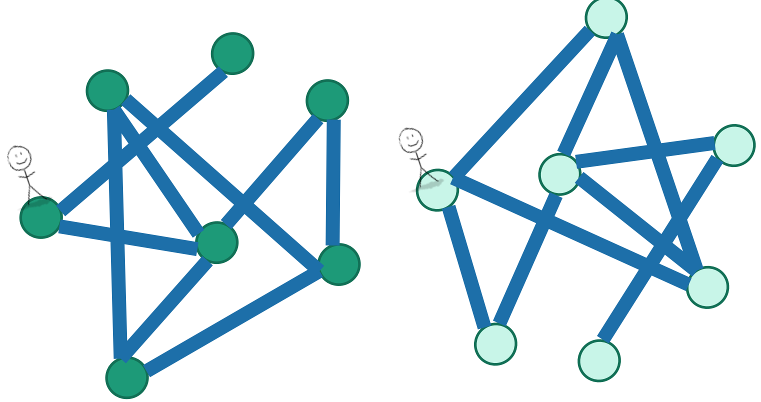
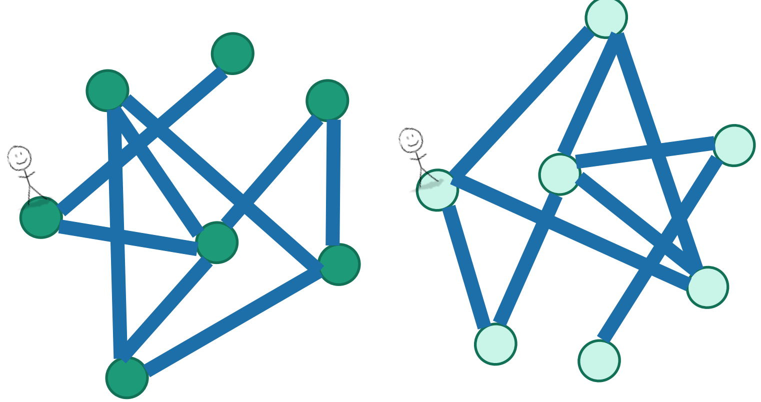


- Explore the connected components one-by-one.
- This takes time O(n + m)
  - Same computation as before:

$$\sum_{w \in V} \left(O(\deg(w)) + O(1) \right) = O(|E| + |V|) = O(n + m)$$

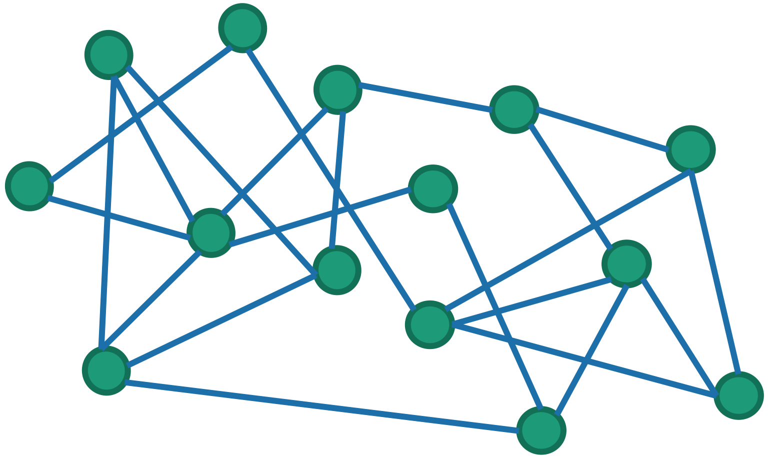
Running time

To explore the whole graph



- Explore the connected components one-by-one.
- This takes time O(n + m)
  - Same computation as before:

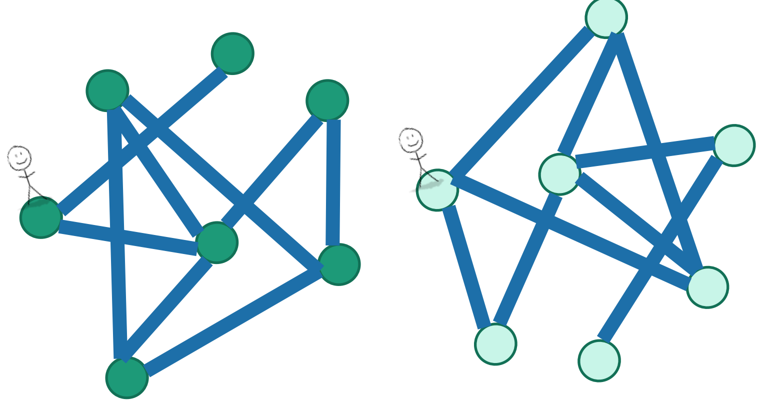
$$\sum_{w \in V} (O(\deg(w)) + O(1)) = O(|E| + |V|) = O(n + m)$$



Here the running time is O(m) like before

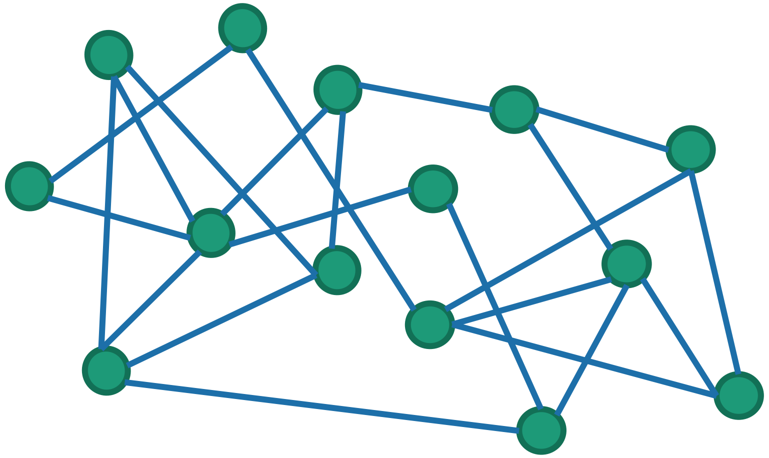
Running time

To explore the whole graph



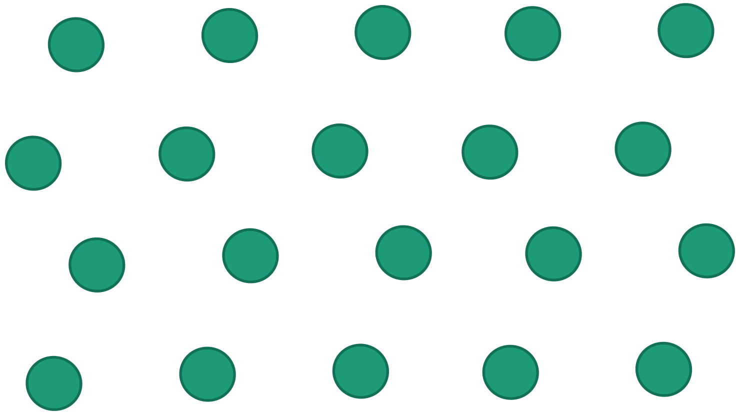
- Explore the connected components one-by-one.
- This takes time O(n + m)
  - Same computation as before:

$$\sum_{w \in V} (O(\deg(w)) + O(1)) = O(|E| + |V|) = O(n + m)$$



Here the running time is O(m) like before

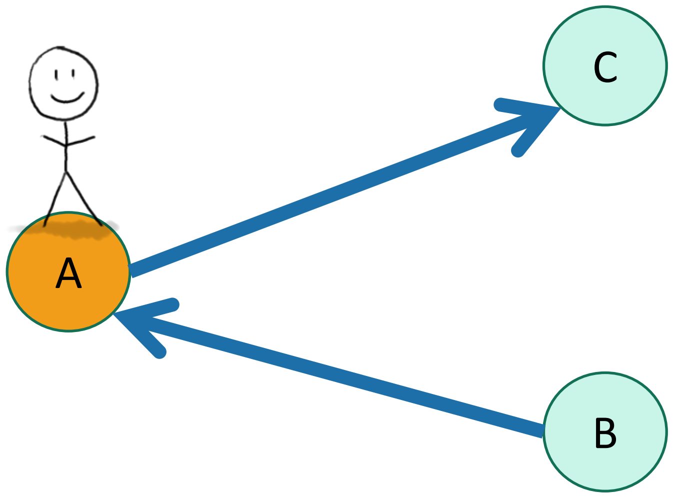
or



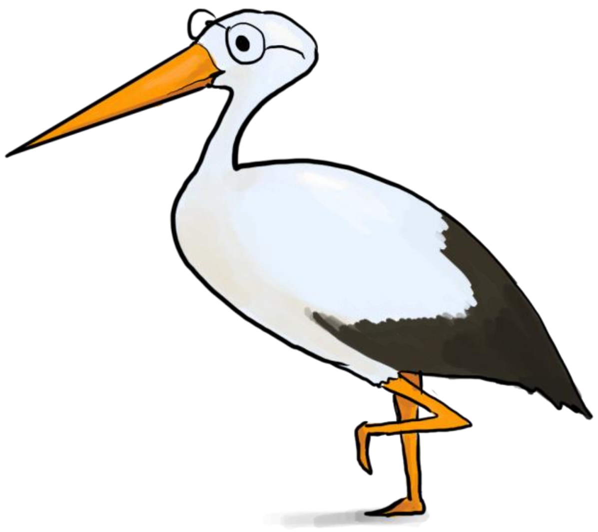
Here m=0 but it still takes time O(n) to explore the graph.

You check:

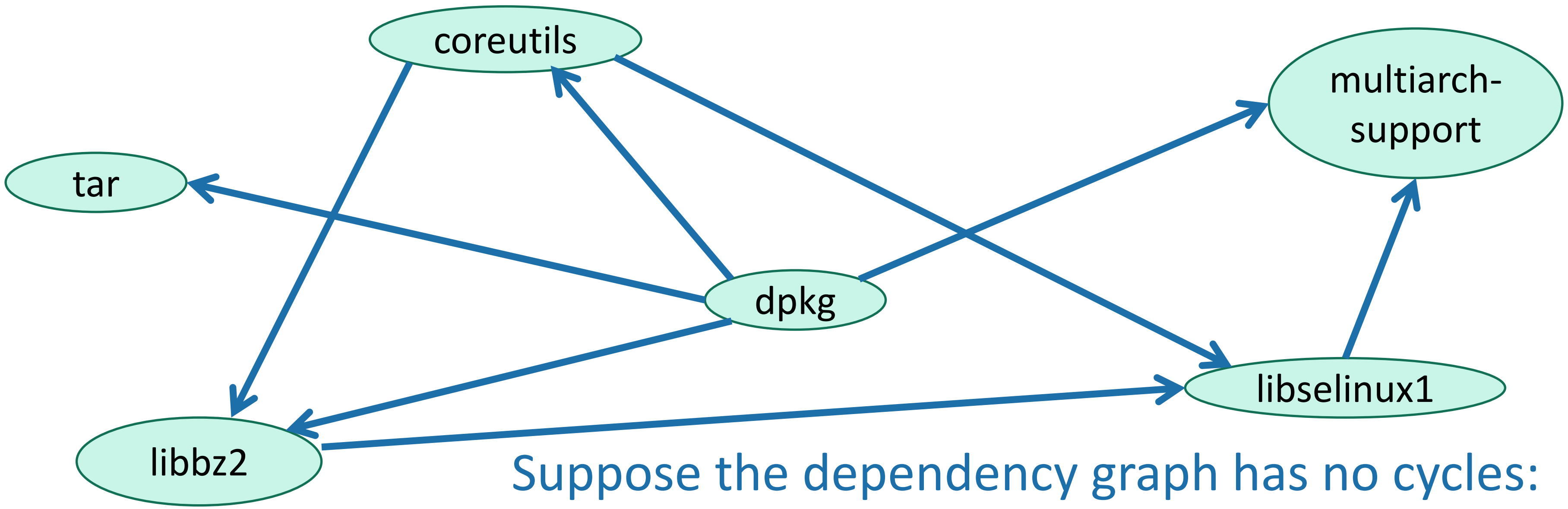
DFS works fine on directed graphs too!



Only walk to C, not to B.



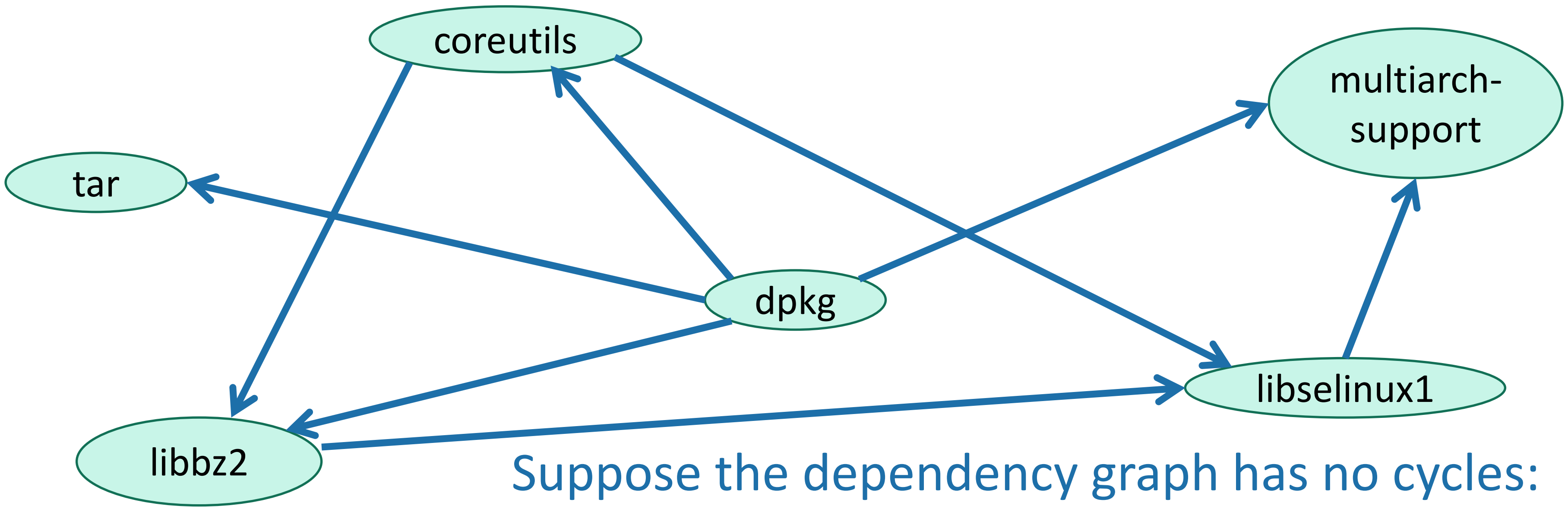
Application of DFS: topological sorting



Suppose the dependency graph has no cycles:
it is a **Directed Acyclic Graph (DAG)**

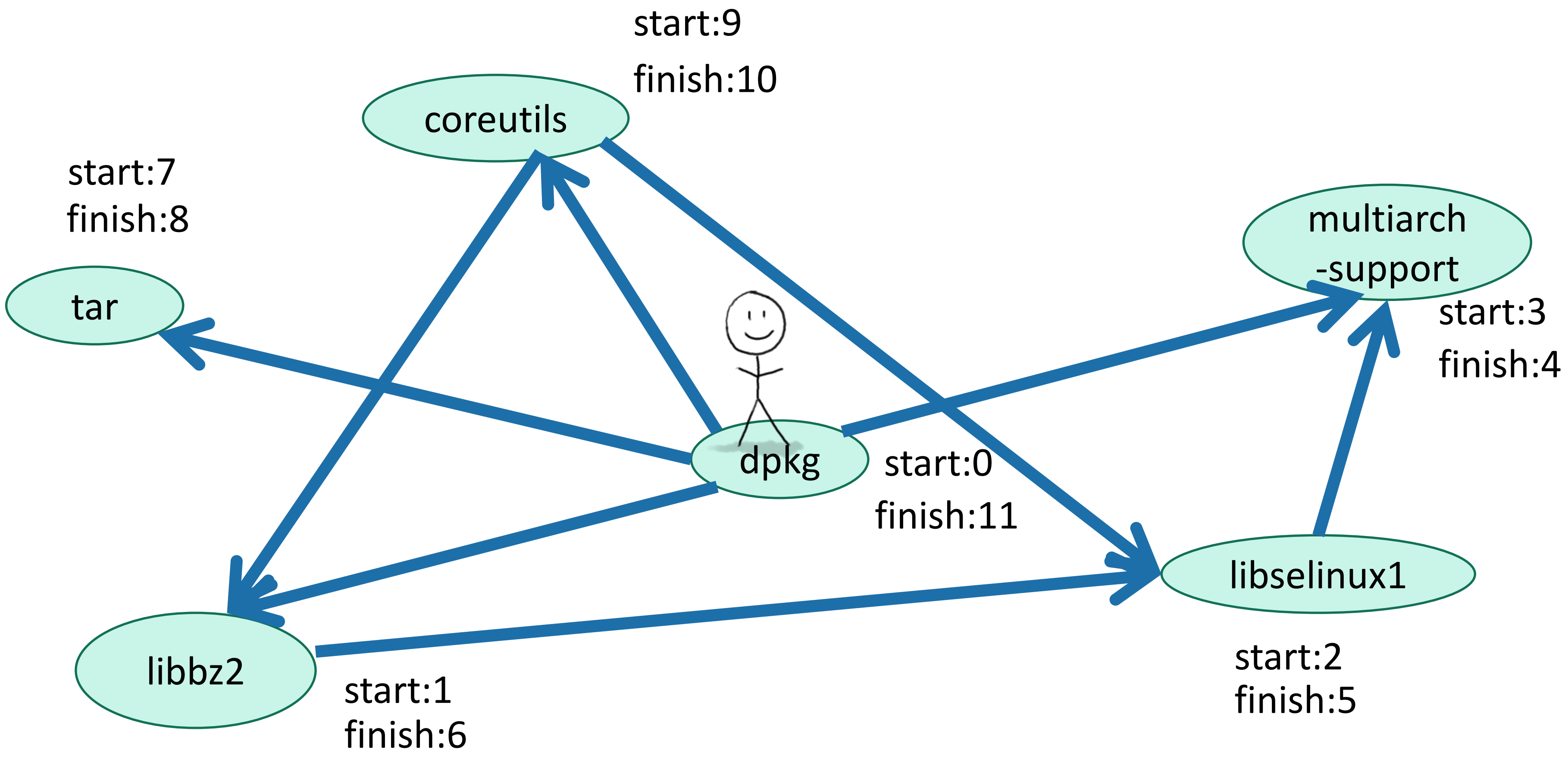
Application of DFS: topological sorting

- Find an ordering of vertices so that all of the dependency requirements are met.
  - Aka, if v comes before w in the ordering, there is not an edge from w to v.



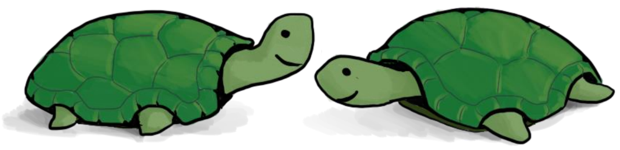
Suppose the dependency graph has no cycles: it is a **Directed Acyclic Graph (DAG)**

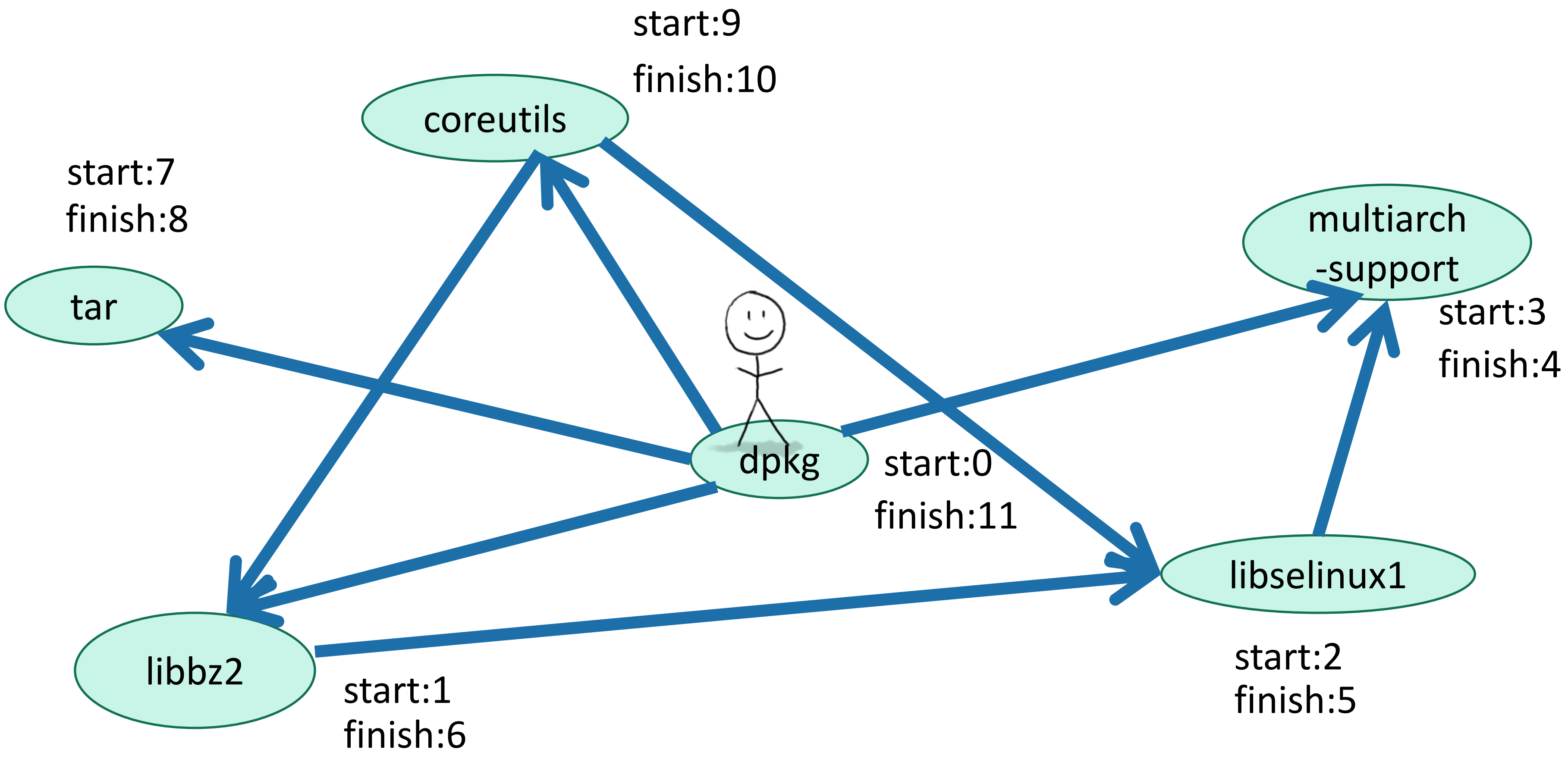
Let’s do DFS


start:9
finish:10
coreutils
start:7
finish:8
tar
multiarch
-support
start:3
finish:4
dpkg
start:0
finish:11
libselinux1
start:2
finish:5
libbz2
start:1
finish:6

Let’s do DFS

What do you notice about the finish times? Any ideas for how we should do topological sort?

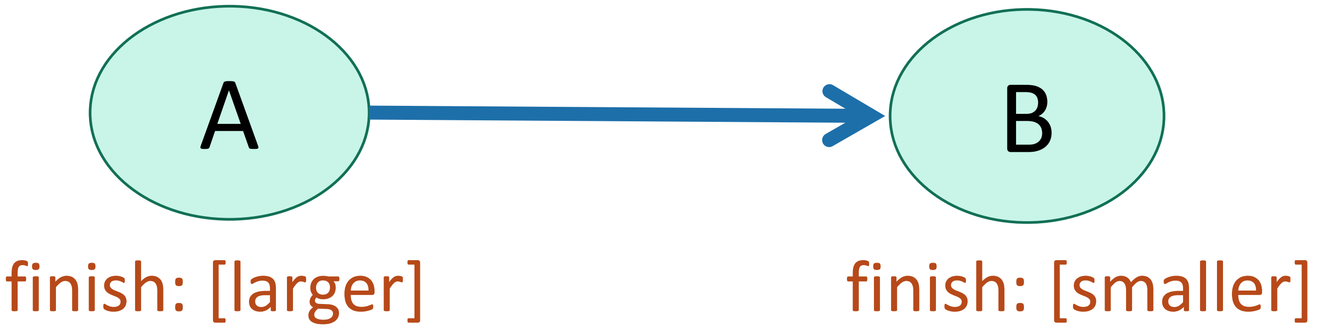




Suppose the underlying graph has no cycles

Finish times seem useful

Claim: In general, we'll always have:



Finish times seem useful

Suppose the underlying graph has no cycles

Claim: In general, we'll always have:

finish: [larger]

finish: [smaller]

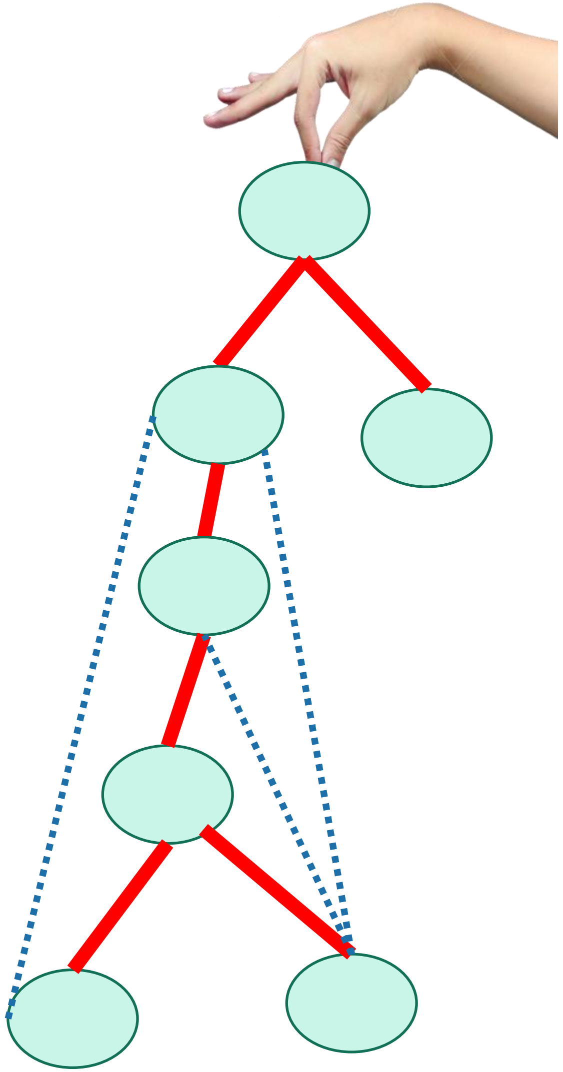
To understand why, let's go back to that DFS tree.

A more general statement

(this holds even if there are cycles)

This is called the “parentheses theorem”

(check this statement carefully!)



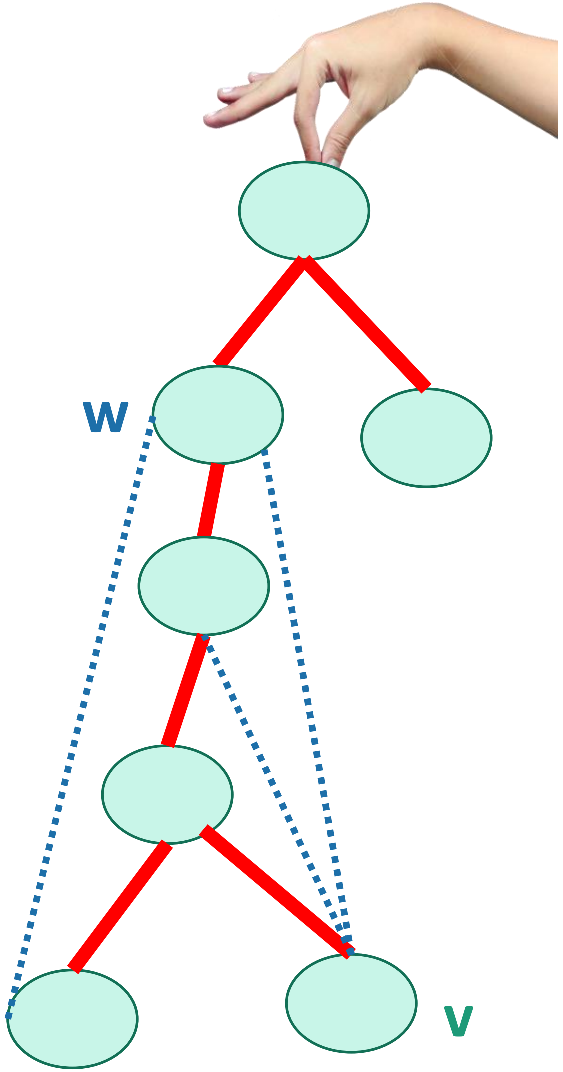
A more general statement

(this holds even if there are cycles)

This is called the “parentheses theorem”

(check this statement carefully!)

- If v is a descendant of w in this tree:



A more general statement

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This is called the "parentheses theorem"

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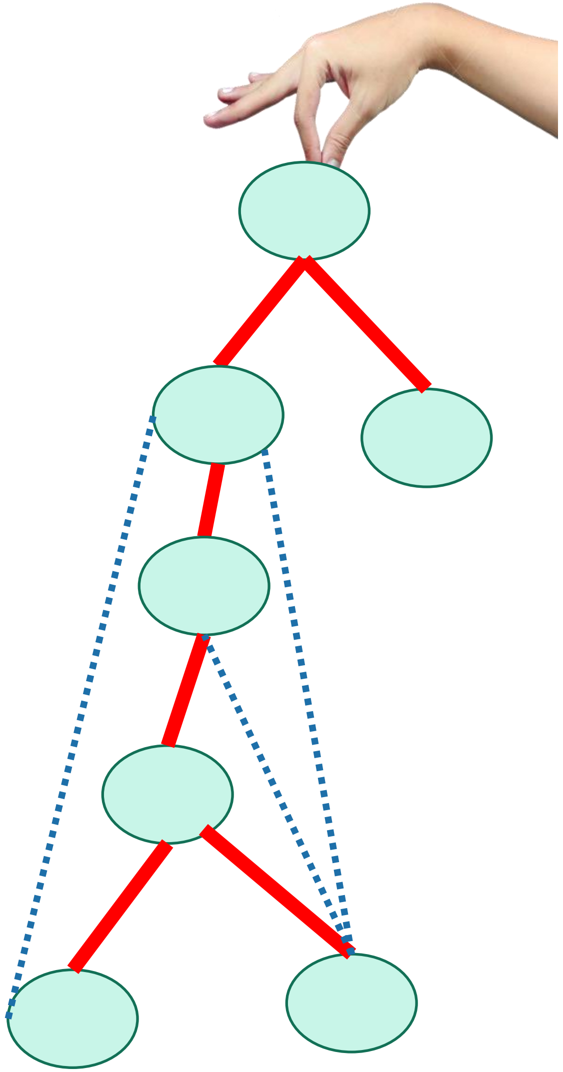
- If v is a descendant of w in this tree:

- If w is a descendant of v in this tree:



A more general statement

(this holds even if there are cycles)

This is called the “parentheses theorem”

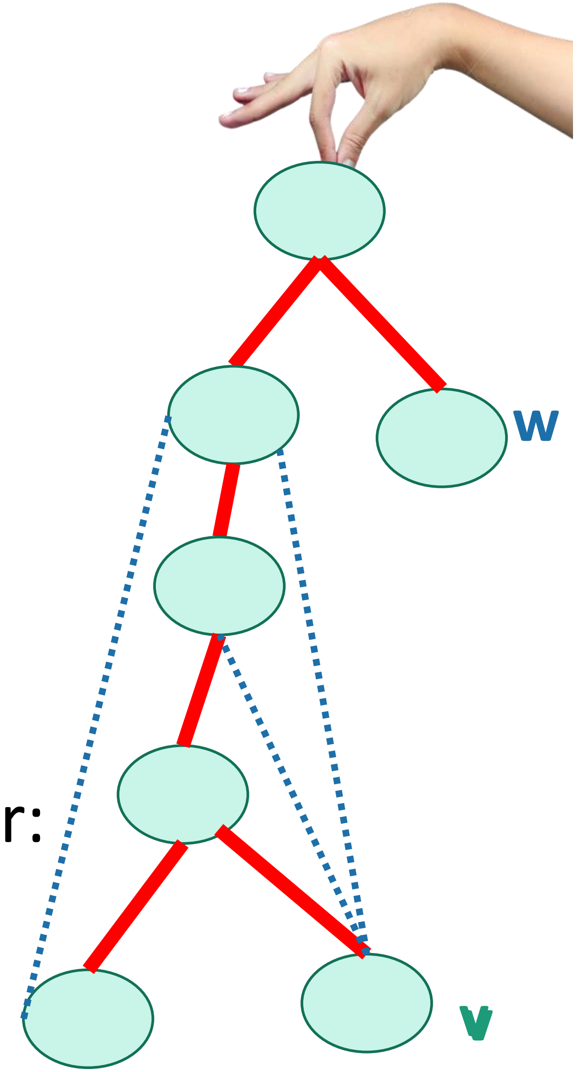
(check this statement carefully!)

- If v is a descendant of w in this tree:

- If w is a descendant of v in this tree:

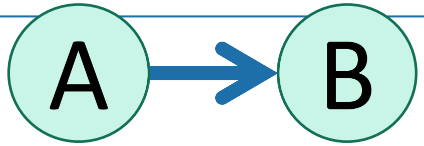
- If neither are descendants of each other:

(or the other way around)



Theorem

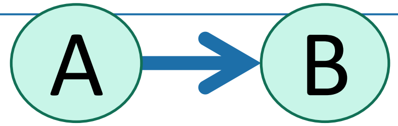
- If we run DFS on a directed acyclic graph,

If 

Then B.finishTime < A.finishTime

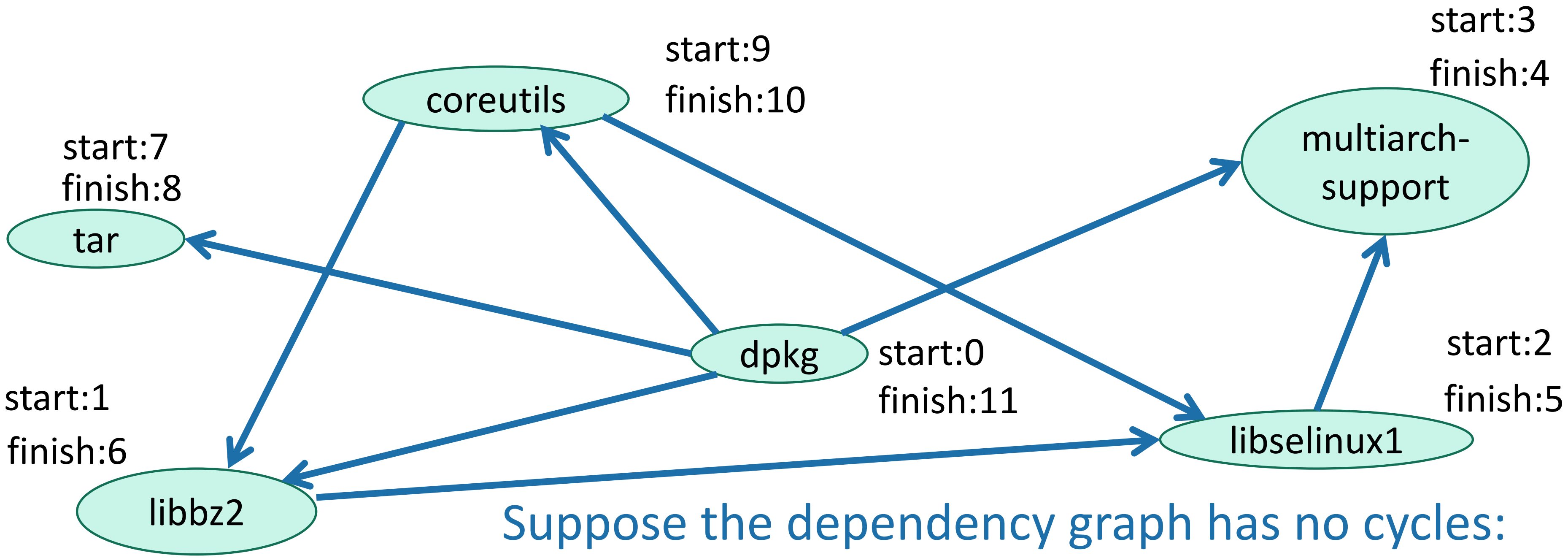
Back to topological sorting

If



Then B.finishTime < A.finishTime

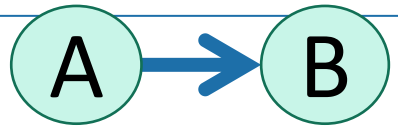
- In what order should I install packages?



Suppose the dependency graph has no cycles: it is a **Directed Acyclic Graph (DAG)**

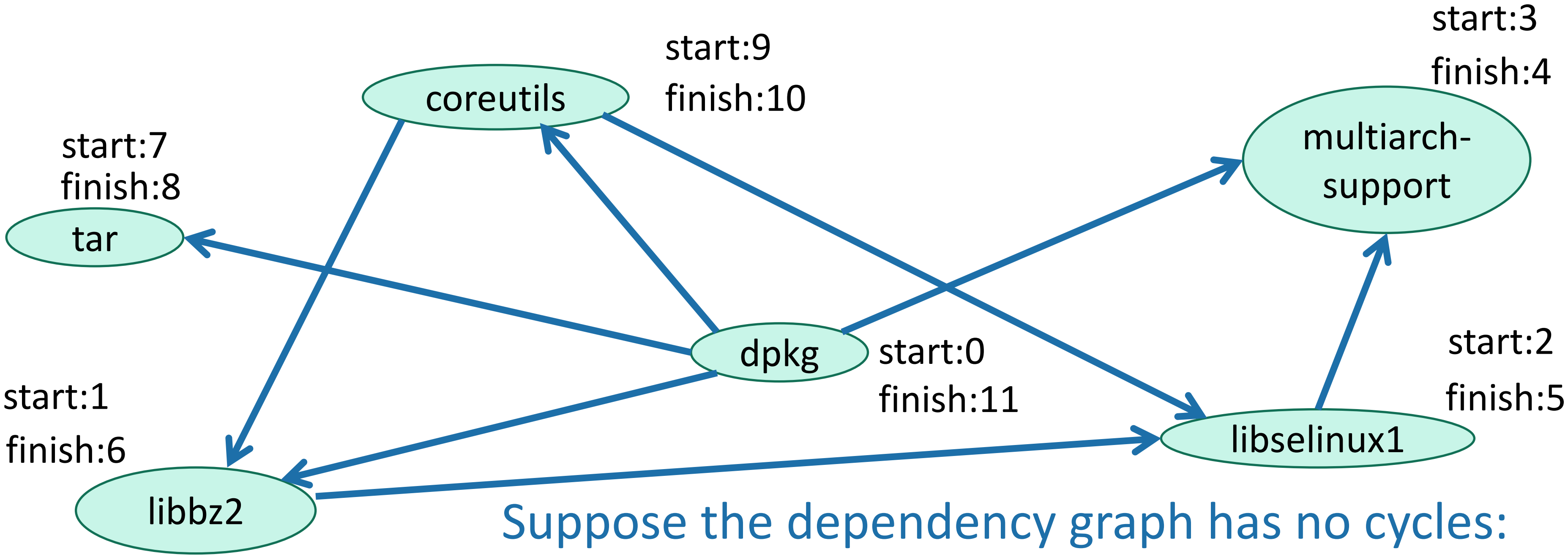
Back to topological sorting

If



Then B.finishTime < A.finishTime

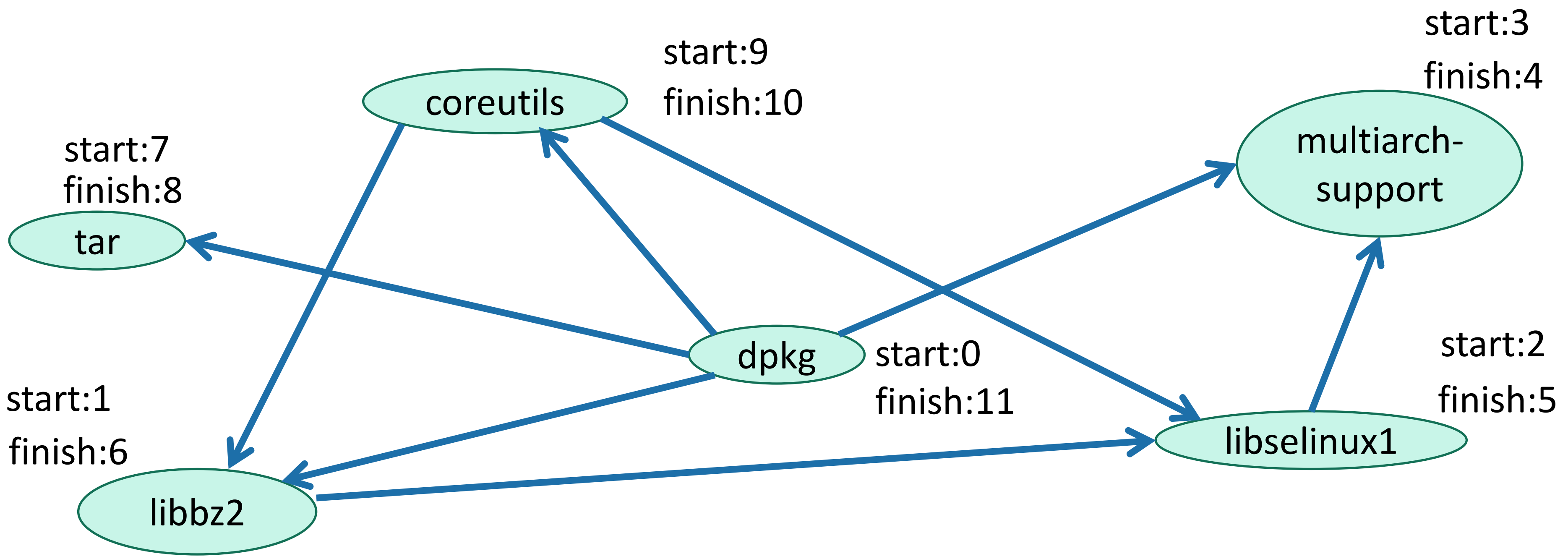
- In what order should I install packages?
- In reverse order of finishing time in DFS!



Suppose the dependency graph has no cycles:
it is a **Directed Acyclic Graph (DAG)**

Topological Sorting (on a DAG)

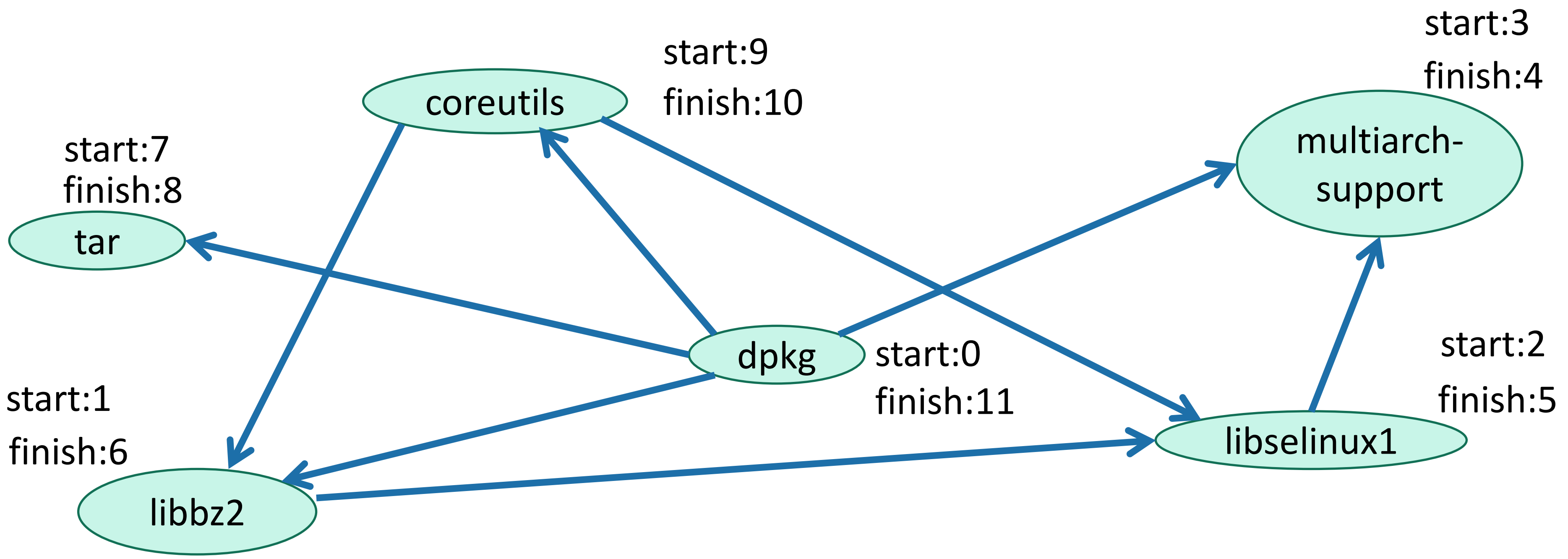
- Do DFS
- When you mark a vertex as **all done**, put it at the **beginning** of the list.



Topological Sorting (on a DAG)

- Do DFS
- When you mark a vertex as **all done,** put it at the **beginning** of the list.

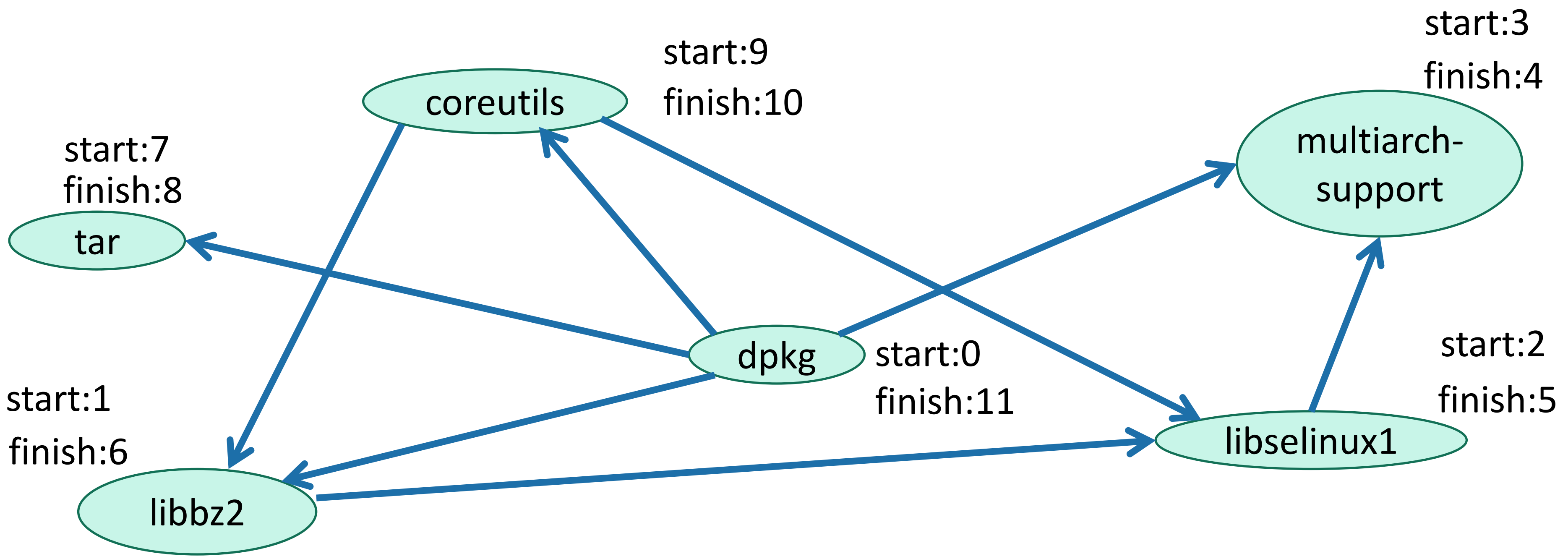
- `multiarch_support`



Topological Sorting (on a DAG)

- Do DFS
- When you mark a vertex as **all done**, put it at the **beginning** of the list.

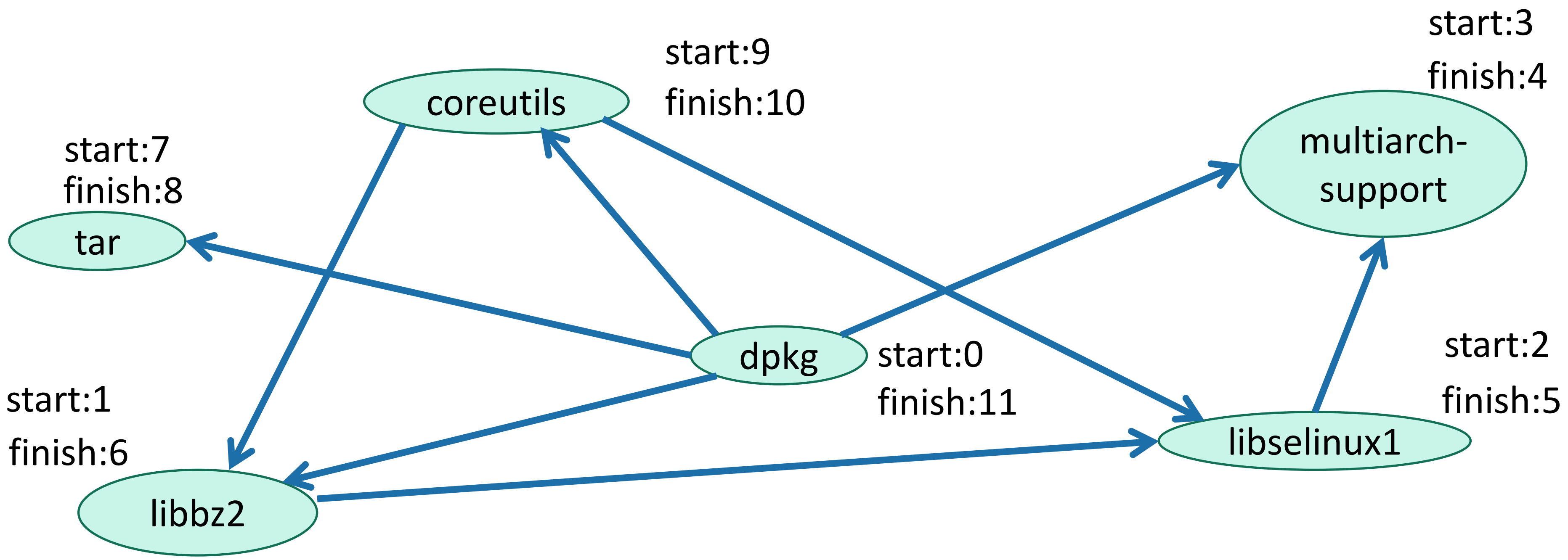
- `libselinux1`
- `multiarch_support`



Topological Sorting (on a DAG)

- Do DFS
- When you mark a vertex as **all done**, put it at the beginning of the list.

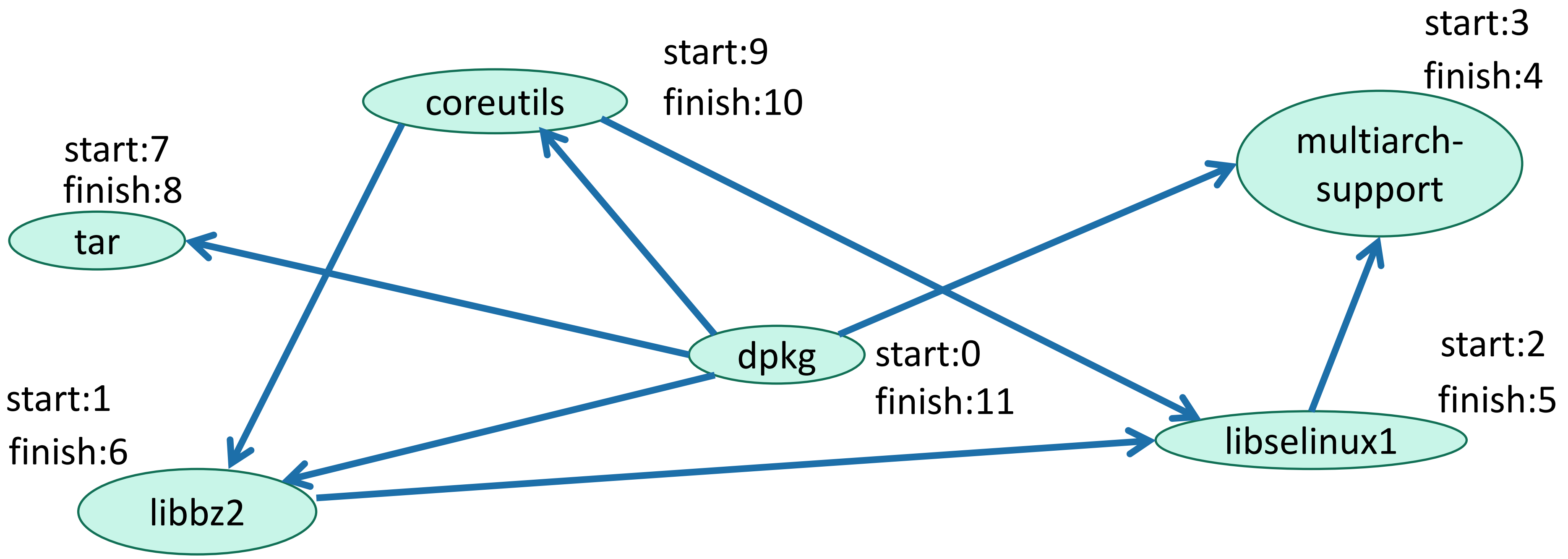
- `libbz2`
- `libselinux1`
- `multiarch_support`



Topological Sorting (on a DAG)

- Do DFS
- When you mark a vertex as **all done**, put it at the **beginning** of the list.

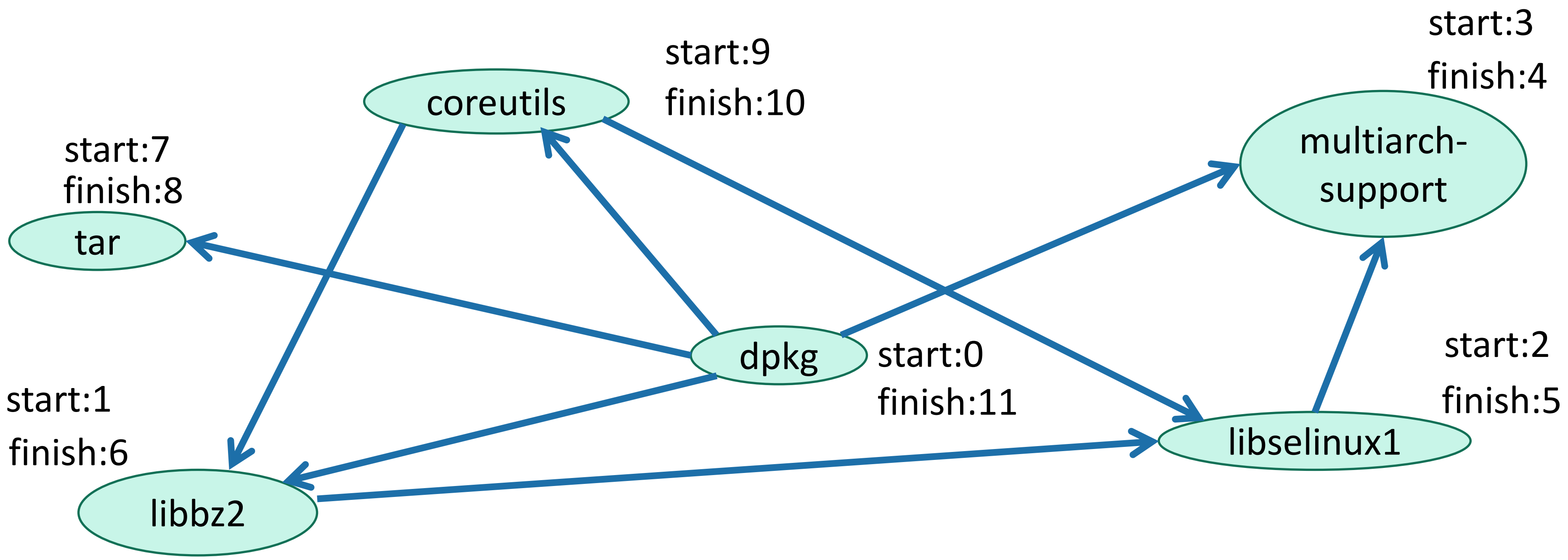
- `tar`
- `libbz2`
- `libselinux1`
- `multiarch_support`



Topological Sorting (on a DAG)

- Do DFS
- When you mark a vertex as **all done**, put it at the beginning of the list.

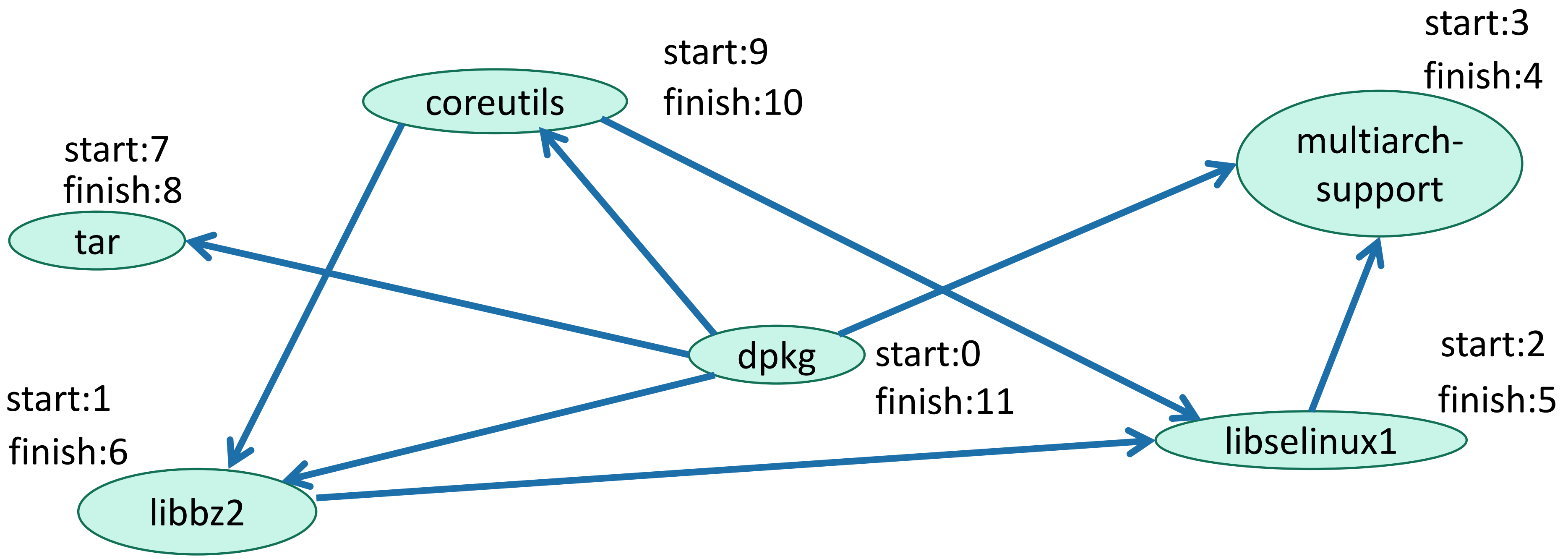
- `coreutils`
- `tar`
- `libbz2`
- `libselinux1`
- `multiarch_support`



Topological Sorting (on a DAG)

- Do DFS
- When you mark a vertex as **all done**, put it at the **beginning** of the list.

- `dpkg`
- `coreutils`
- `tar`
- `libbz2`
- `libselinux1`
- `multiarch_support`

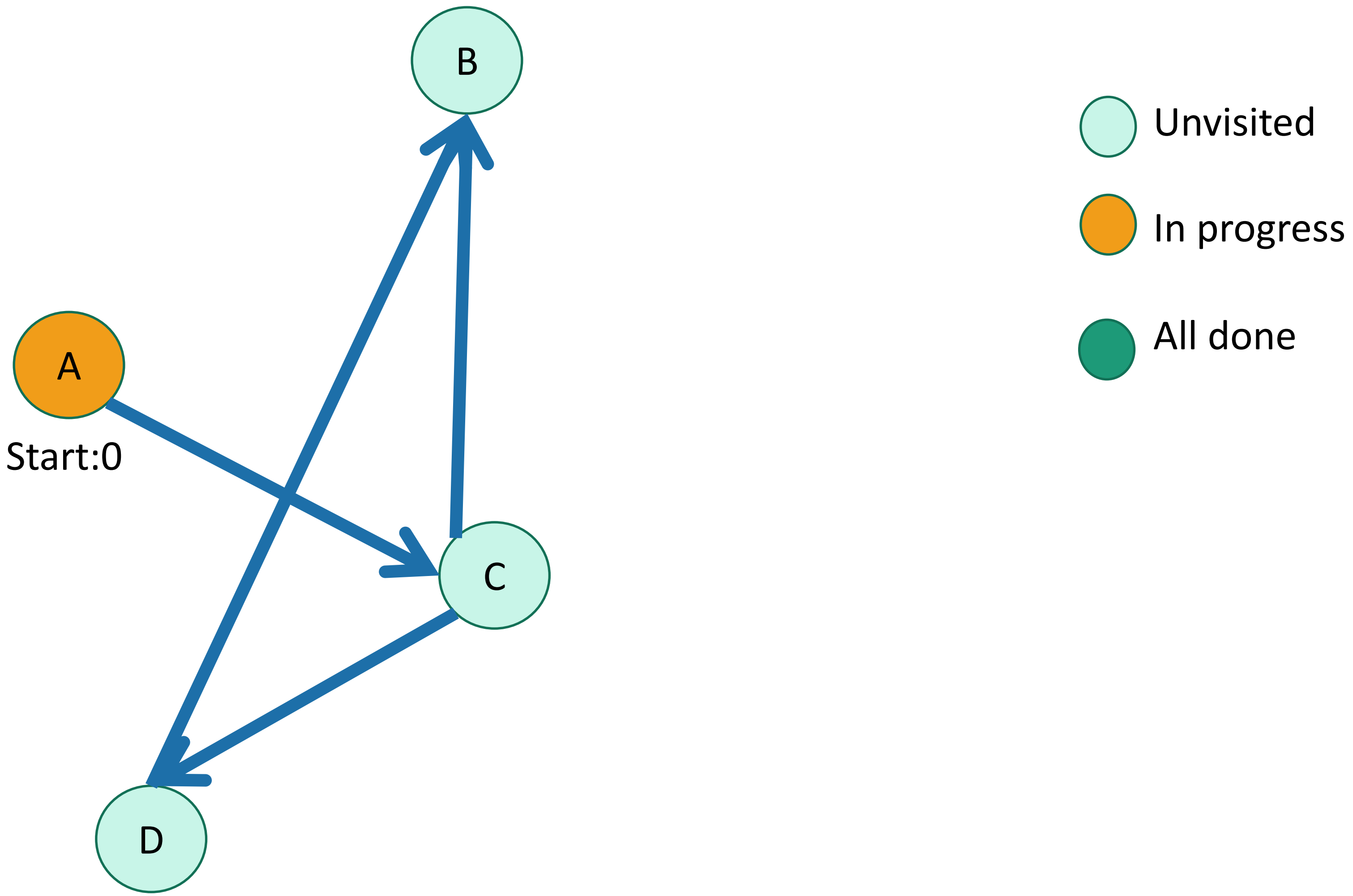


What did we just learn?

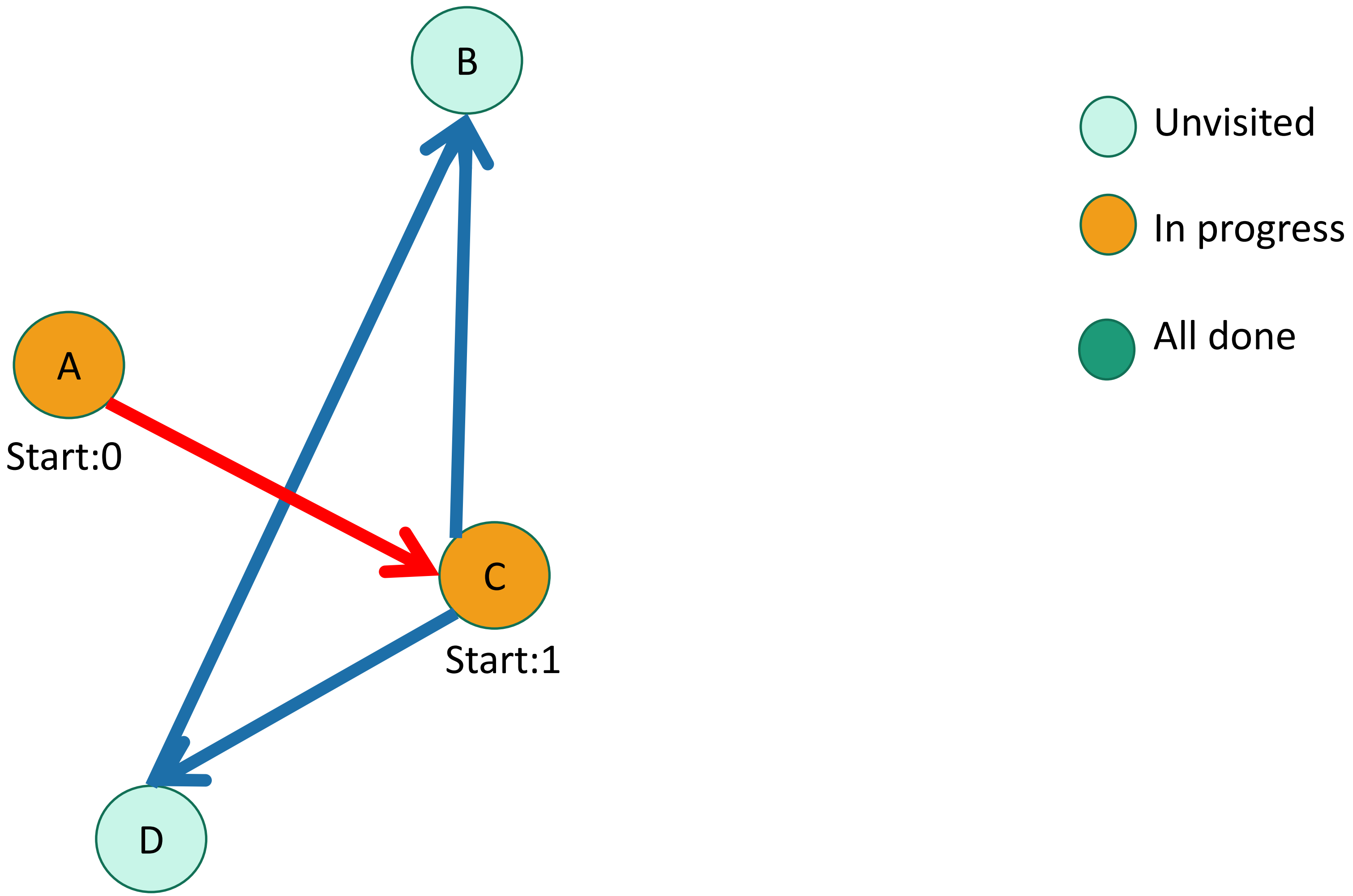
- DFS can help you solve the **topological sorting problem**
  - That’s the fancy name for the problem of finding an ordering that respects all the dependencies

- Thinking about the DFS tree is helpful.

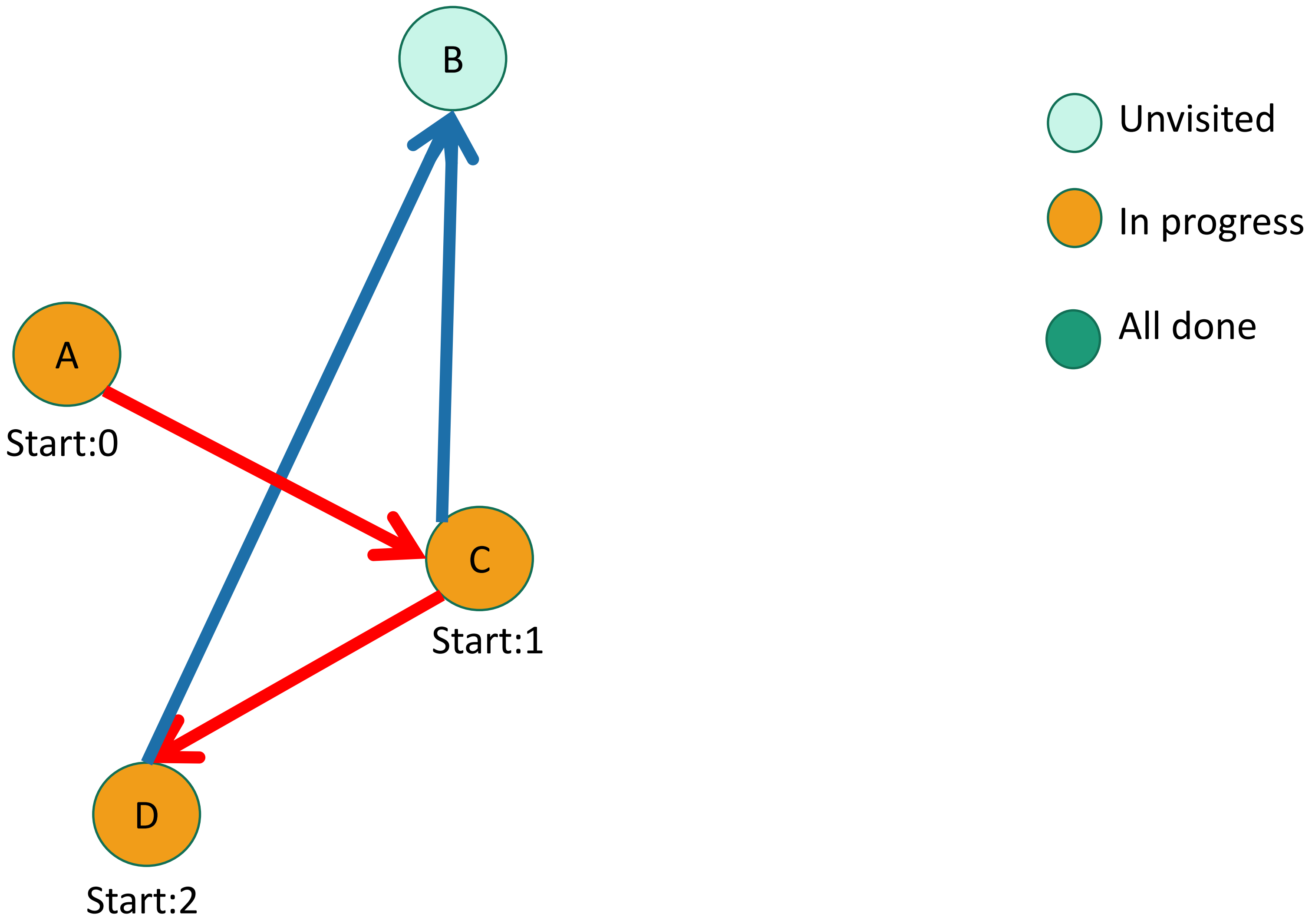
Example:



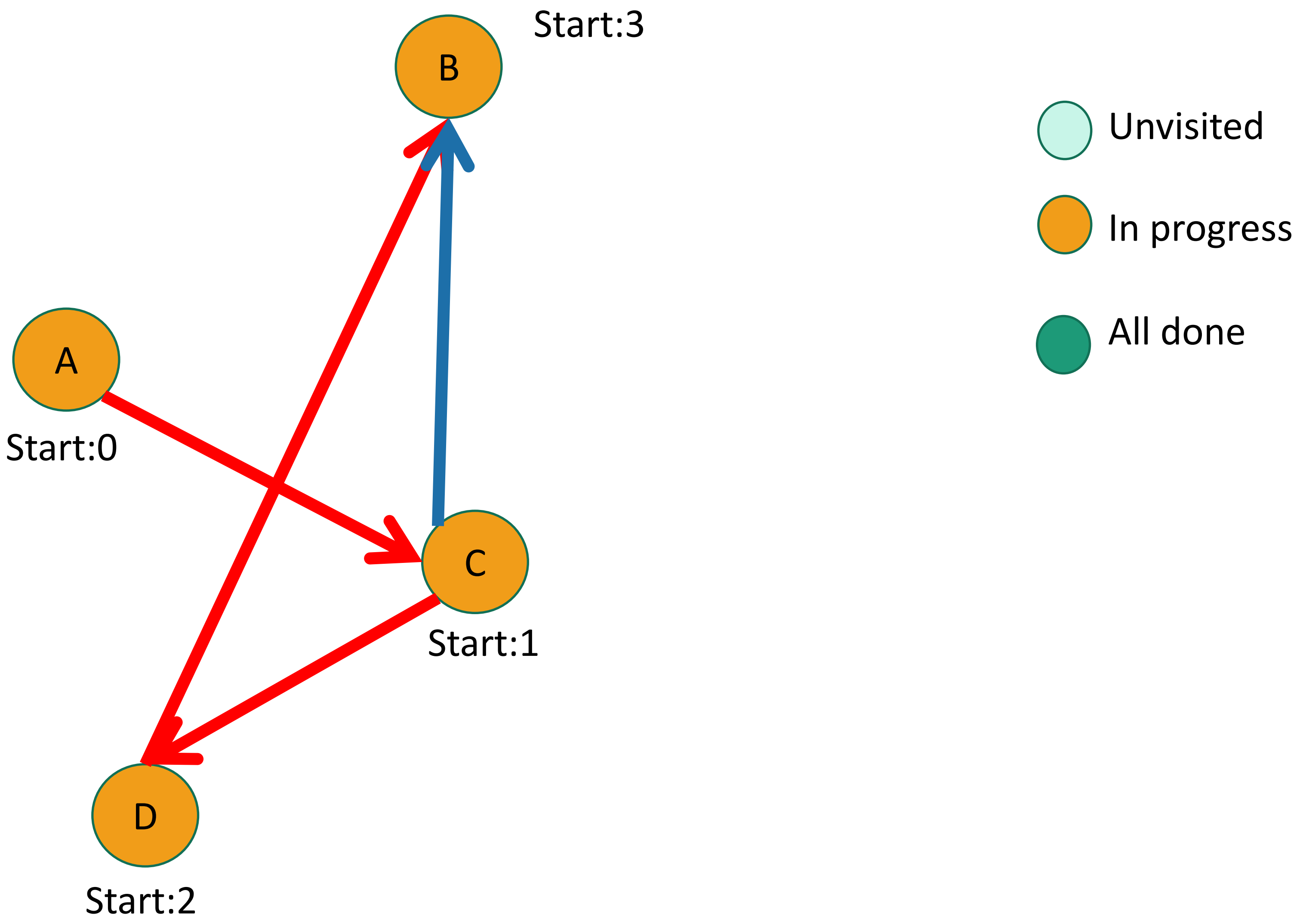
Example



Example

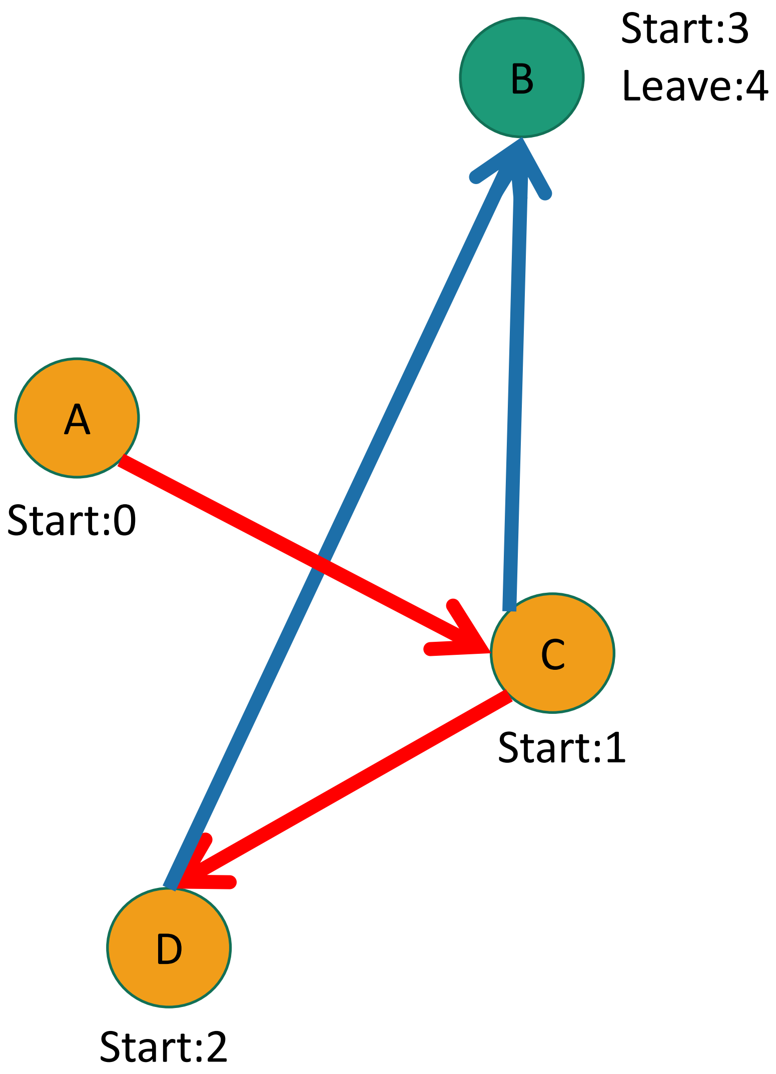


Example

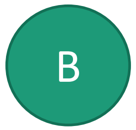


Start:3
B
A
Start:0
C
Start:1
D
Start:2
Unvisited
In progress
All done

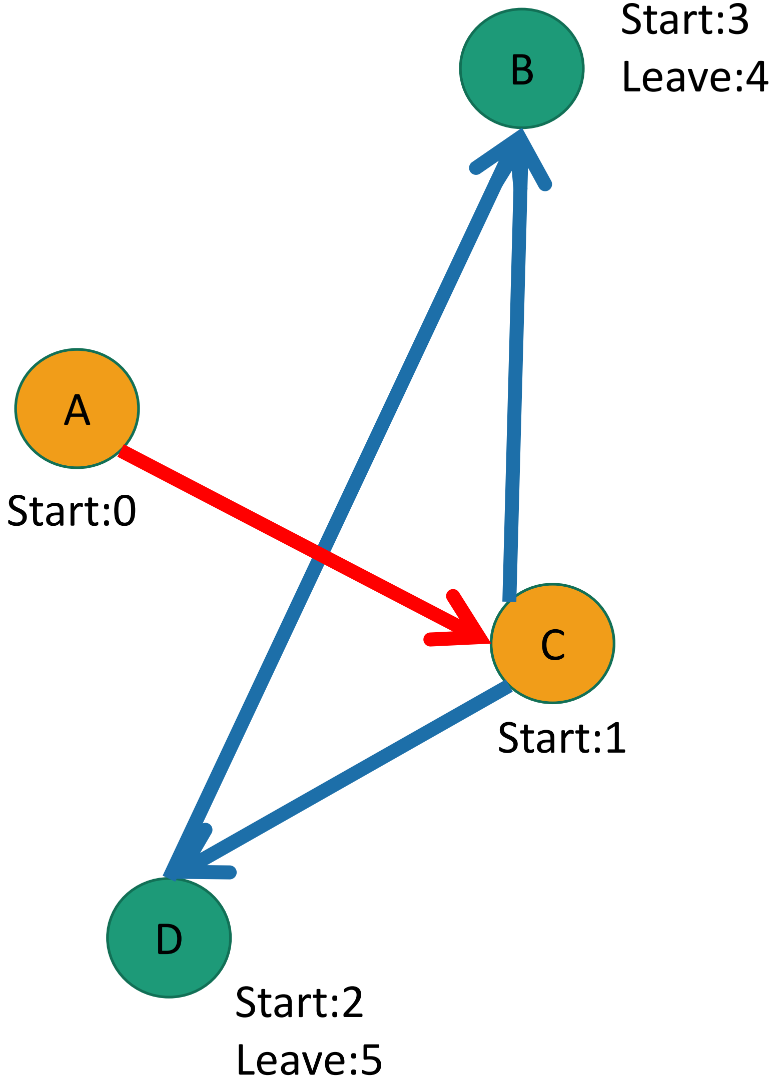
Example


B
Start:3
Leave:4
A
Start:0
C
Start:1
D
Start:2


Unvisited
In progress
All done

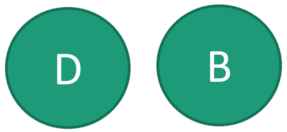

B

Example



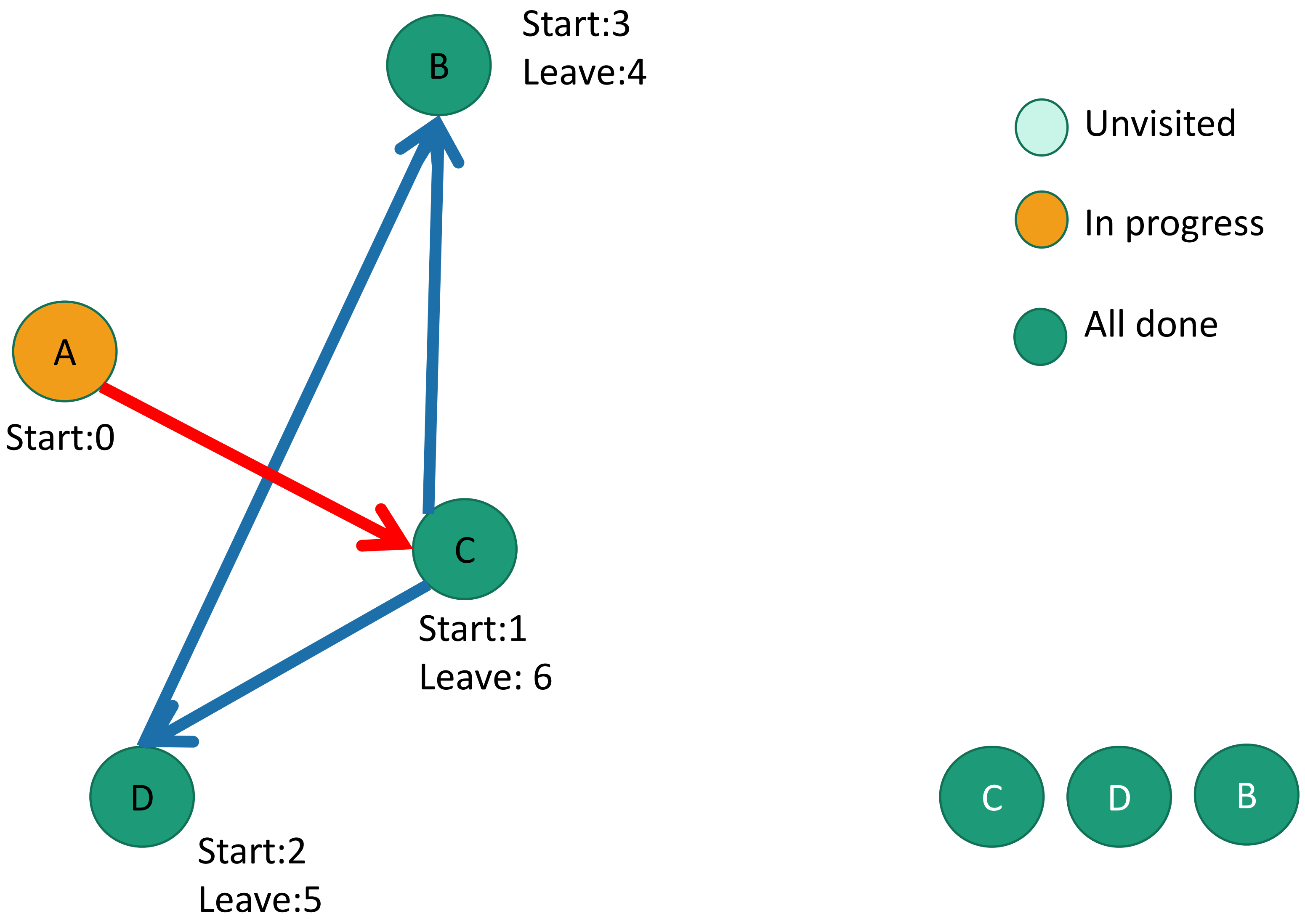
Start:3
Leave:4
B
A
Start:0
C
Start:1
D
Start:2
Leave:5

Unvisited
In progress
All done



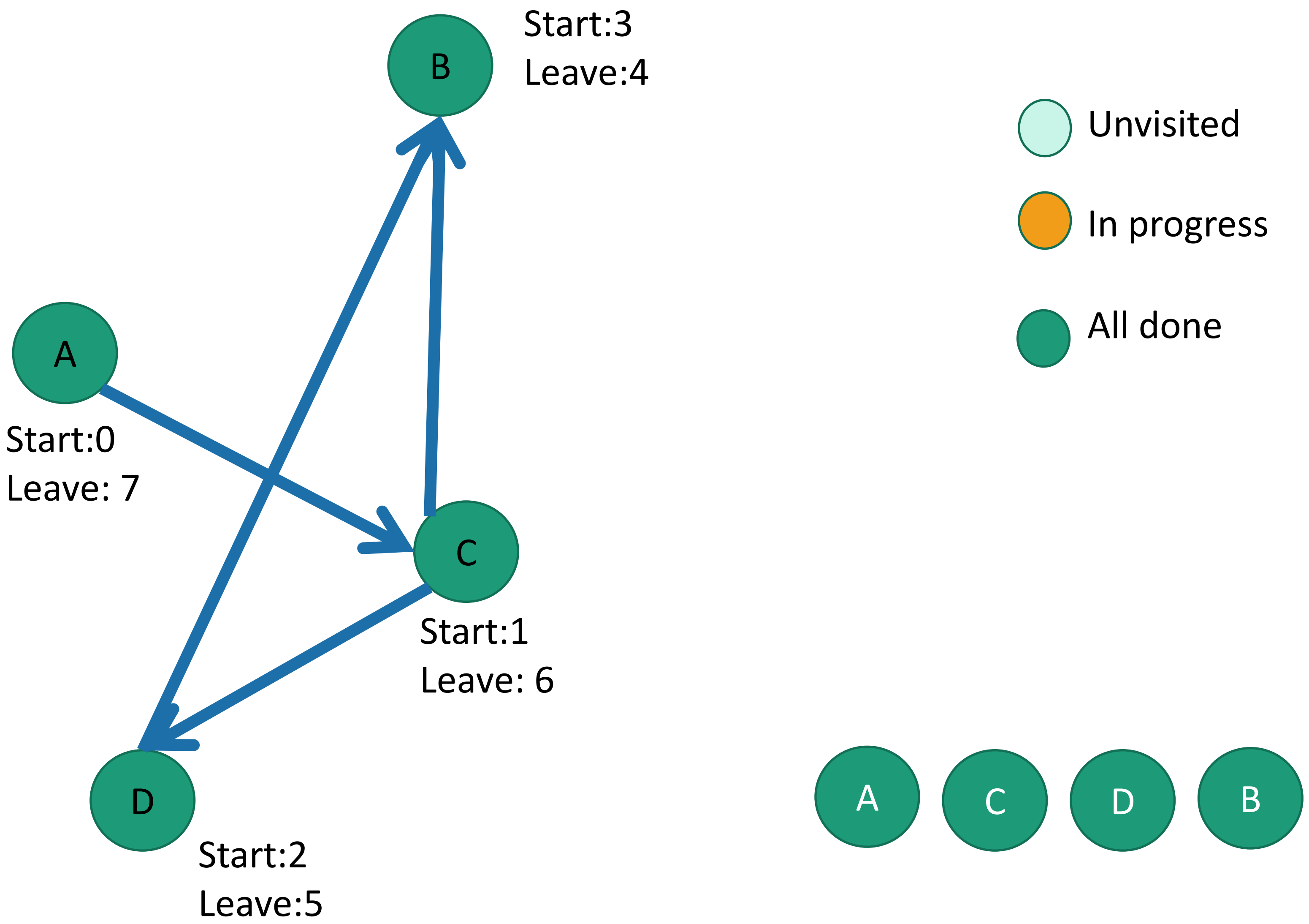
D
B

Example

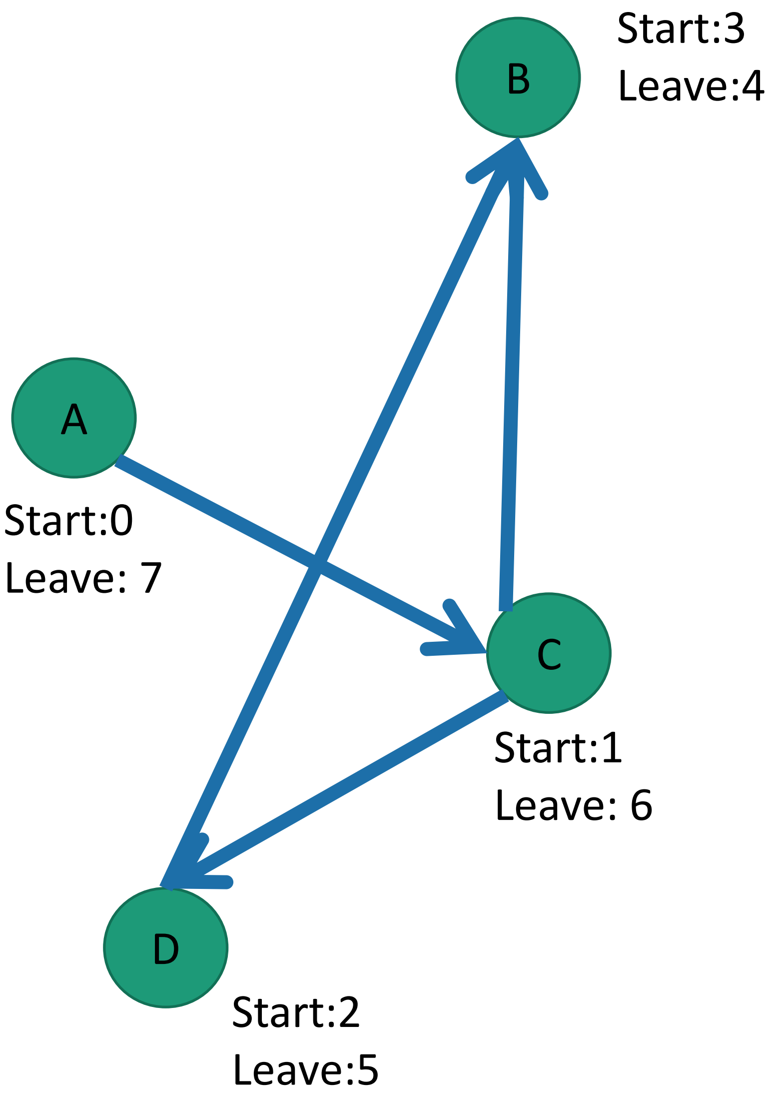


B
Start:3
Leave:4
A
Start:0
C
Start:1
Leave: 6
D
Start:2
Leave:5
Unvisited
In progress
All done
C
D
B

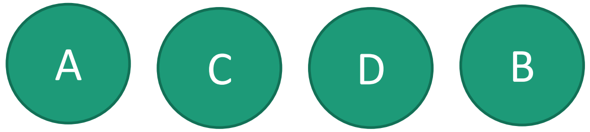
Example


B
Start:3
Leave:4
A
Start:0
Leave: 7
C
Start:1
Leave: 6
D
Start:2
Leave:5
Unvisited
In progress
All done
A
C
D
B

Example

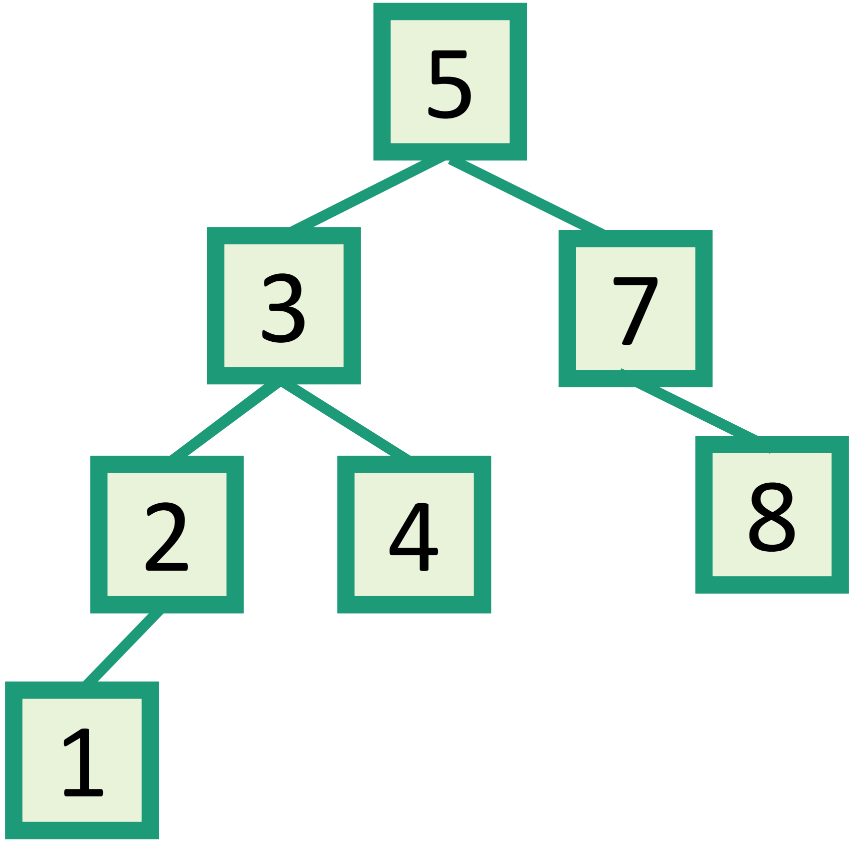


Do them in this order:



Another use of DFS that we've already seen

- In-order enumeration of binary search trees



Do DFS and print a node's label when you are done with the left child and before you begin the right child.

Acknowledgement

- Stanford University

Thank You