Indian Institute of Information Technology Allahabad

Data Structures

Binary Search Tree

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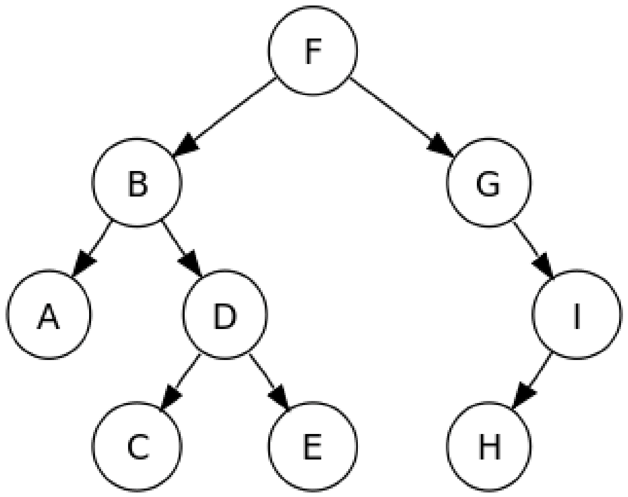
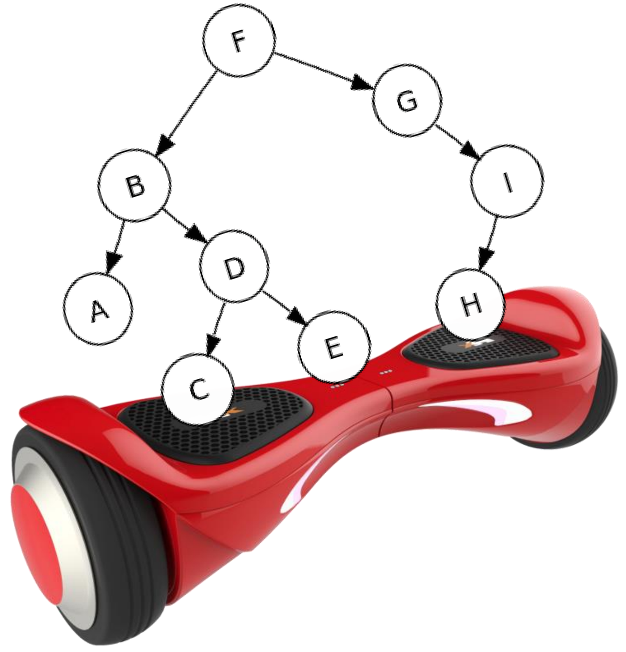
Tree

- Binary search trees
  - They are better when they're balanced.



this will lead us to…

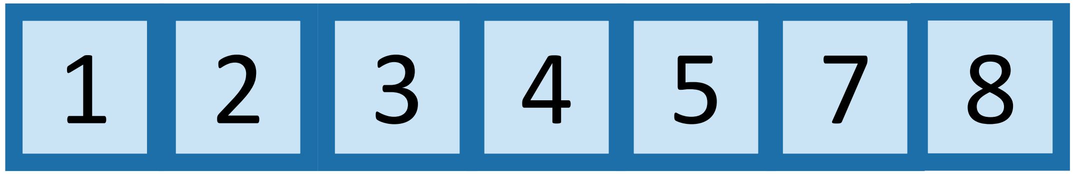
- Self-Balancing Binary Search Trees
  - **AVL Tree**
  - **2-3 Tree**
  - **Red-Black** trees.



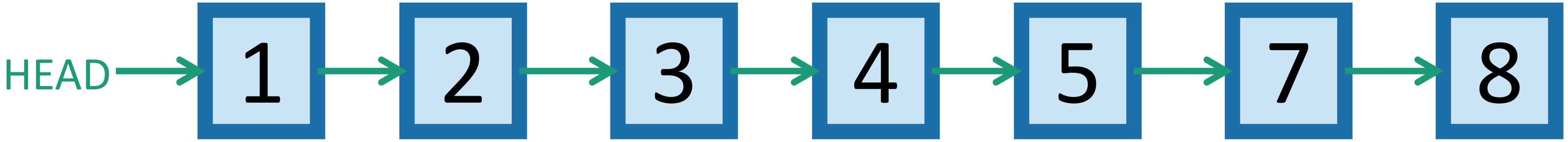
Some data structures

for storing objects like $\boxed{5}$ (aka, nodes with keys)

- (Sorted) arrays:



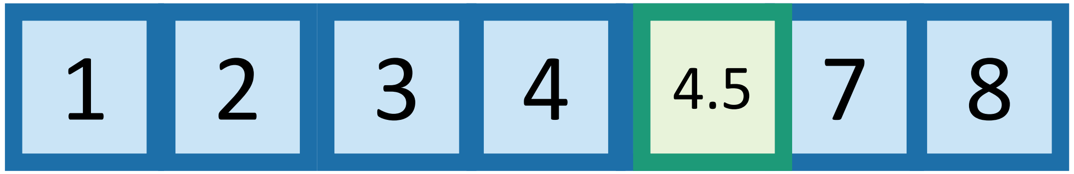
- (Sorted) linked lists:



- Some basic operations:
  - INSERT, DELETE, SEARCH

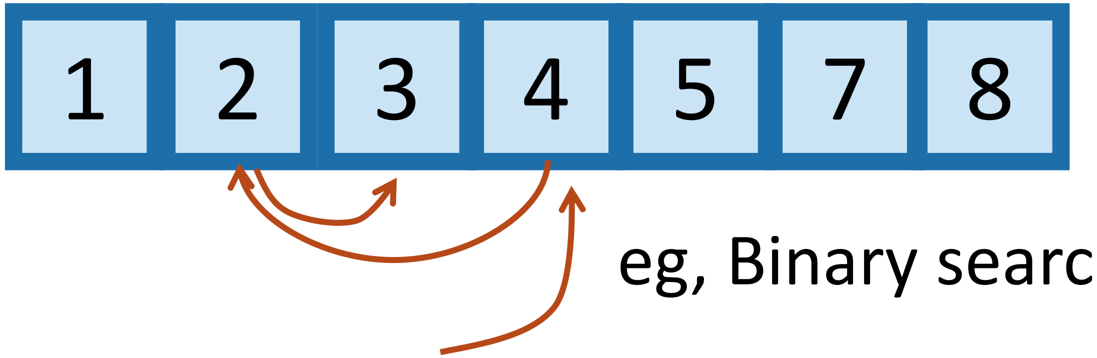
Sorted Arrays

- O(n) INSERT/DELETE:
  - First, find the relevant element (time O(log(n)) as below), and then move a bunch elements in the array:



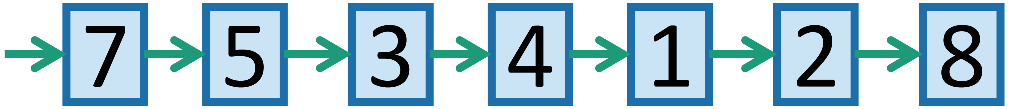
eg, insert 4.5

- O(log(n)) SEARCH:



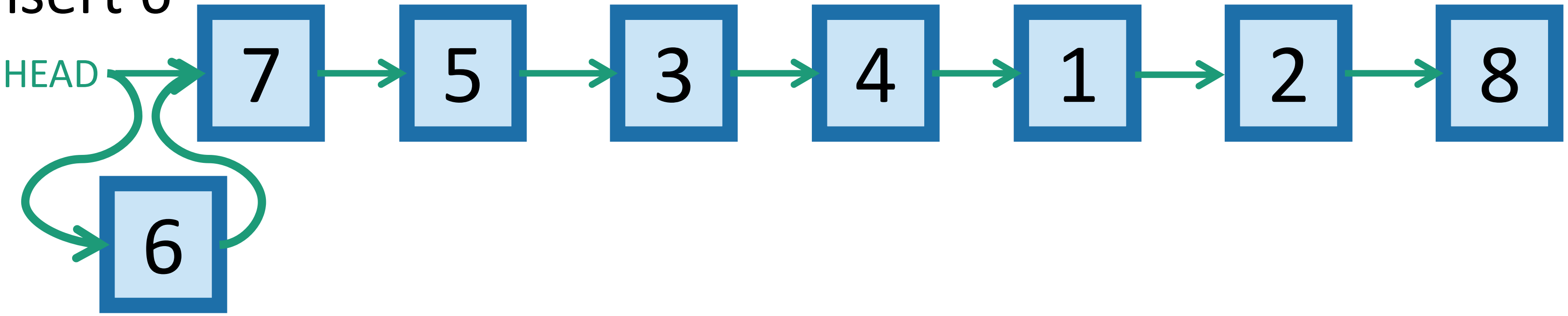
eg, Binary search to see if 3 is in A.

UNSorted linked lists

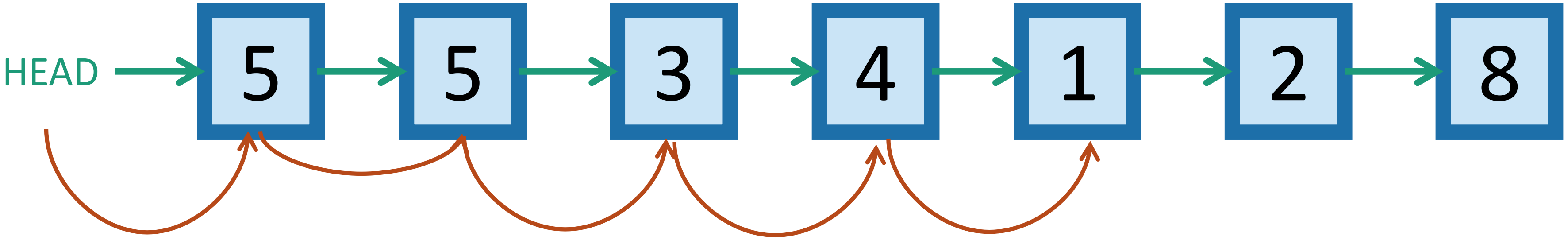


- O(1) INSERT:

eg, insert 6



- O(n) SEARCH/DELETE:



eg, search for 1 (and then you could delete it by manipulating pointers).

Motivation for Binary Search Trees

	Sorted Arrays	Linked Lists	Binary Search Trees*
Search	O(log(n))	O(n)	O(log(n))
Delete	O(n)	O(n)	O(log(n))
Insert	O(n)	O(1)	O(log(n))

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Motivation for Binary Search Trees

TODAY!

	Sorted Arrays	Linked Lists	Binary Search Trees*
Search	O(log(n))	O(n)	O(log(n))
Delete	O(n)	O(n)	O(log(n))
Insert	O(n)	O(1)	O(log(n))

Binary tree terminology

For today all keys are distinct.

Each node has at most two children.

The left child of 3 is 2

The right child of 3 is 4

The parent of 3 is 5

2 is a descendant of 5

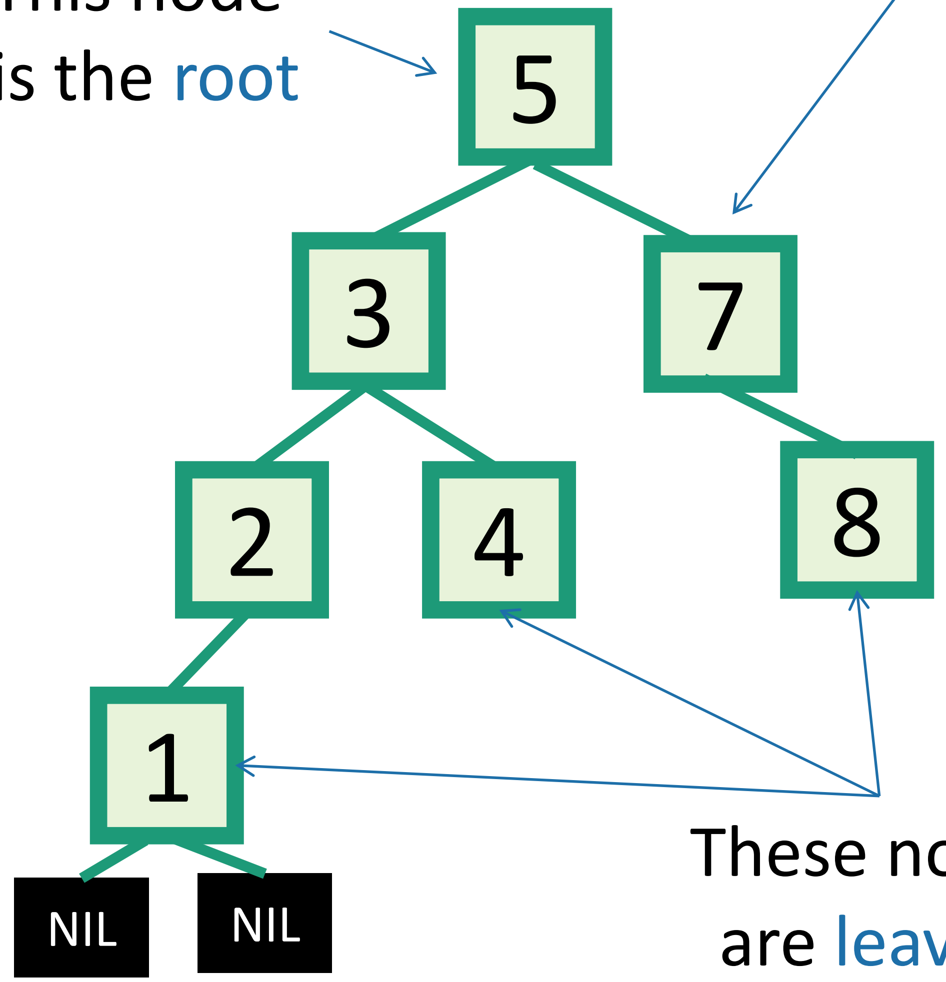
Each node has a pointer to its left child, right child, and parent.

Both children of 1 are NIL.
(Not usually drawn).

The height of this tree is 3. (Max number of edges from the root to a leaf).

This is a node.
It has a key (7).

This node is the root

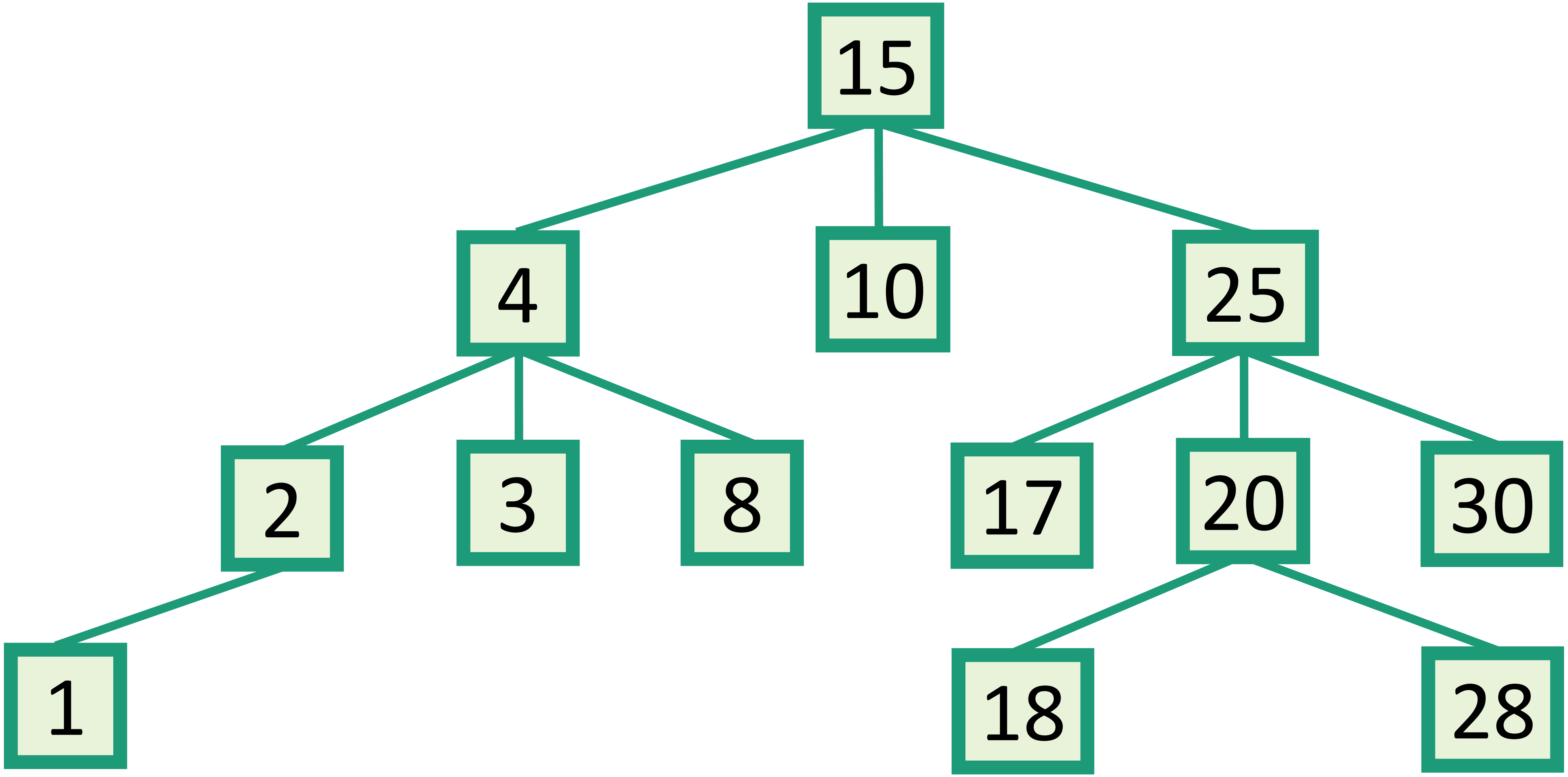


These nodes are leaves.

Definition: k-ary trees

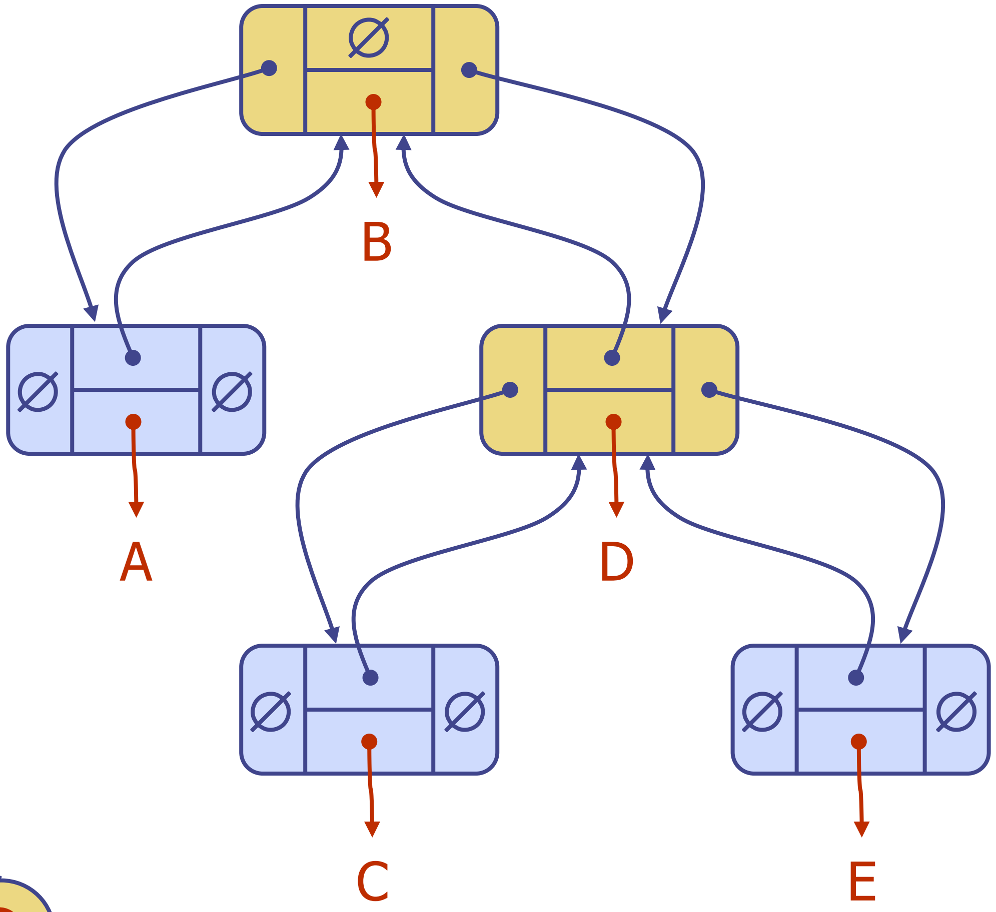
- Rooted tree where every vertex has no more than ‘k’ children
- Full k-ary if every internal vertex has exactly ‘k’ children (i.e., except leaf/external vertices).
- k=2 gives a binary tree
- k=3 gives a ternary tree

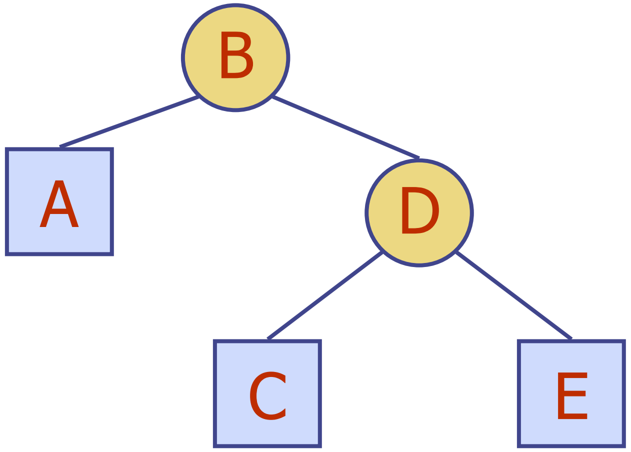
Example: 3-ary tree



Linked Structure for Binary Trees

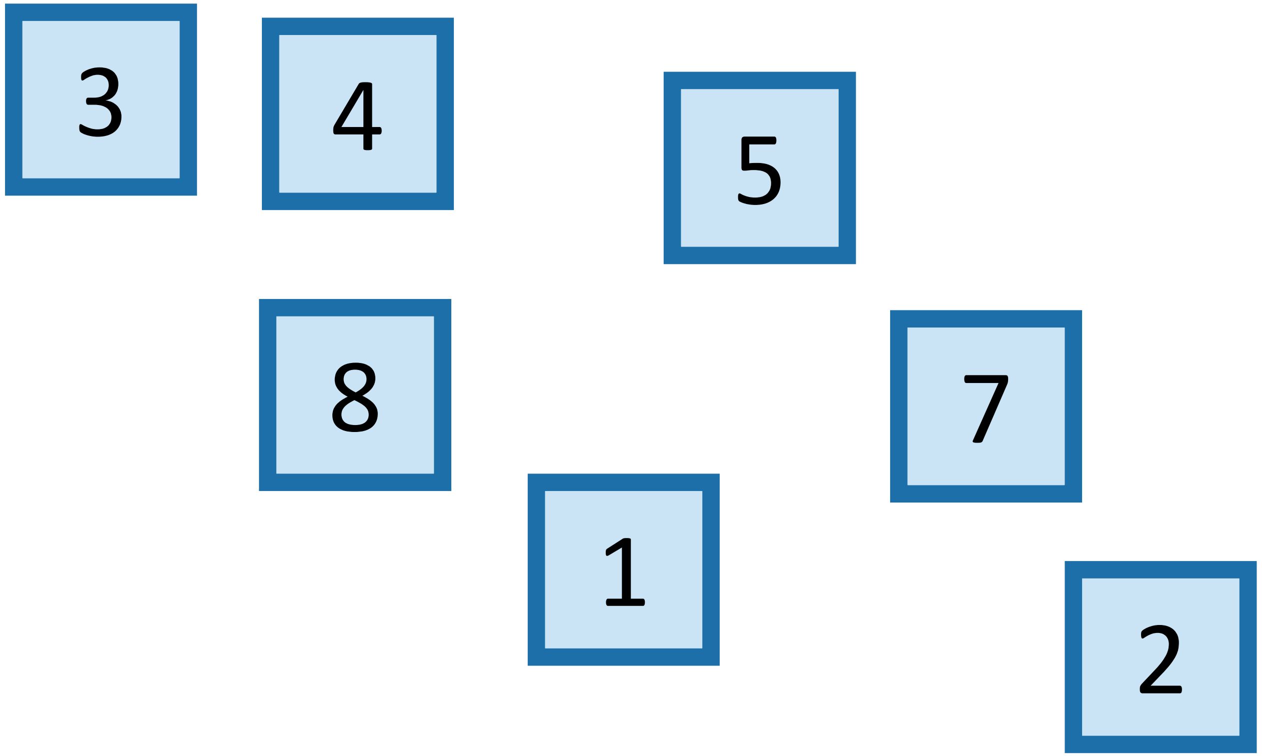
- A node is represented by a structure storing
  - Element
  - Parent node pointer
  - Left child node pointer
  - Right child node pointer
- Node structure implement the Position ADT





Binary Search Trees

- A BST is a binary tree so that:
  - Every LEFT descendant of a node has key less than that node.
  - Every RIGHT descendant of a node has key larger than that node.
- Example of building a binary search tree:



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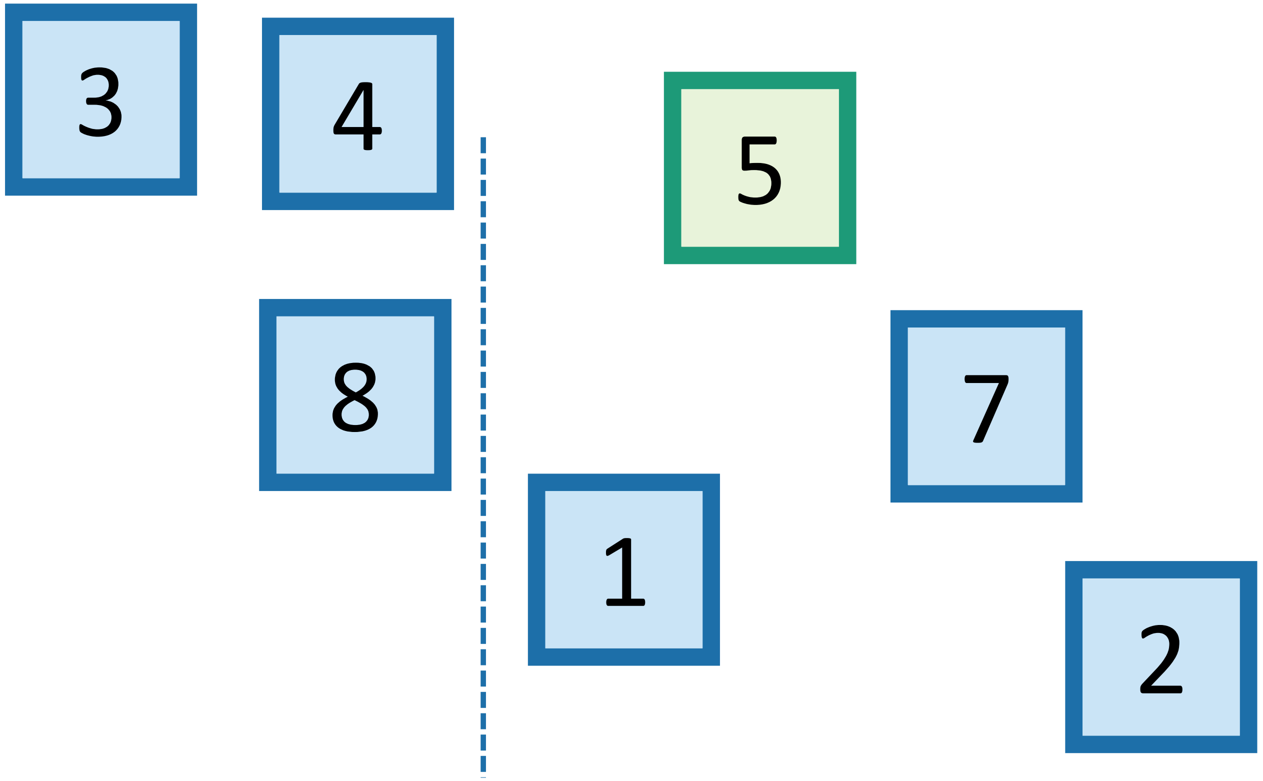

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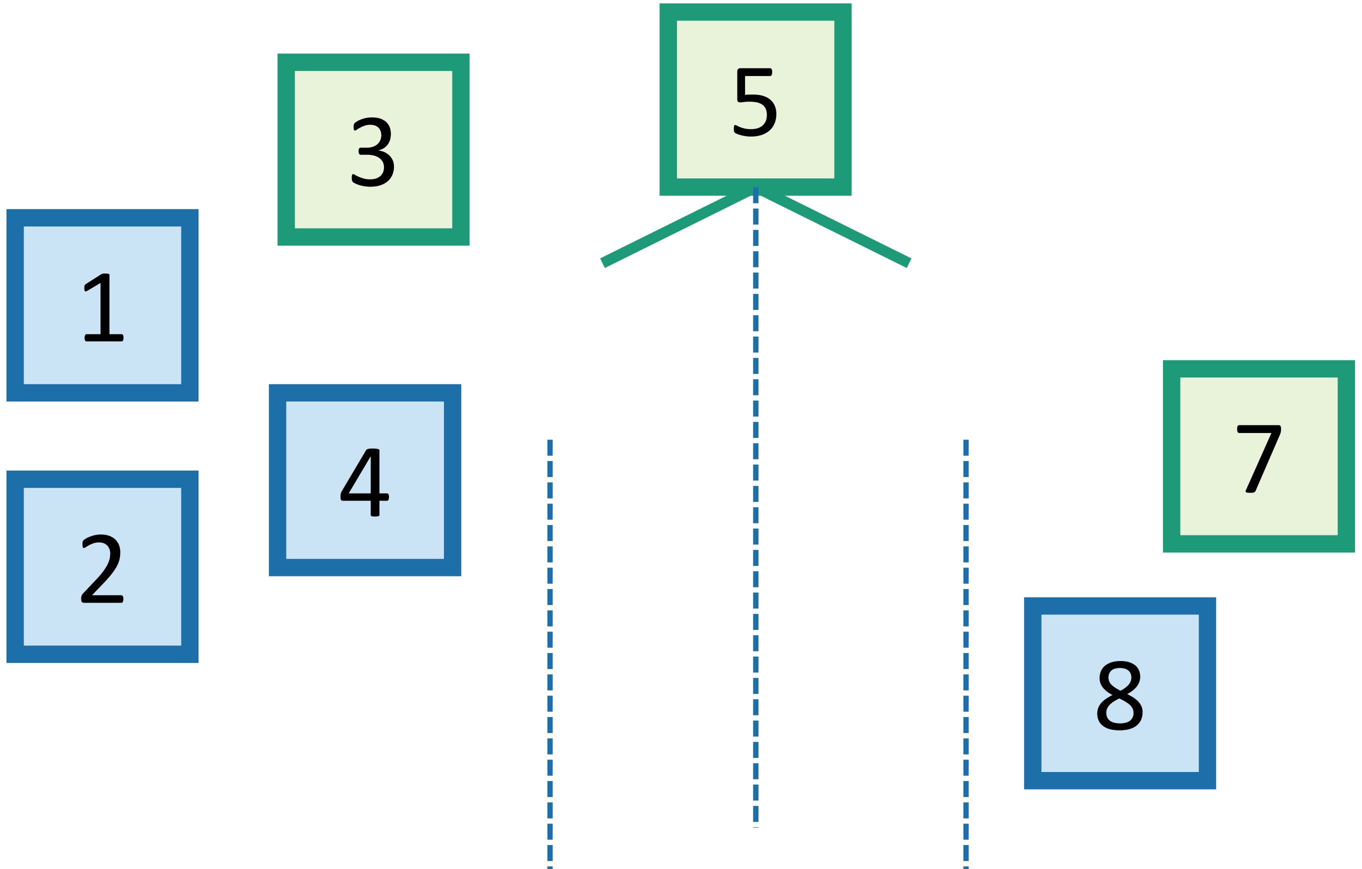

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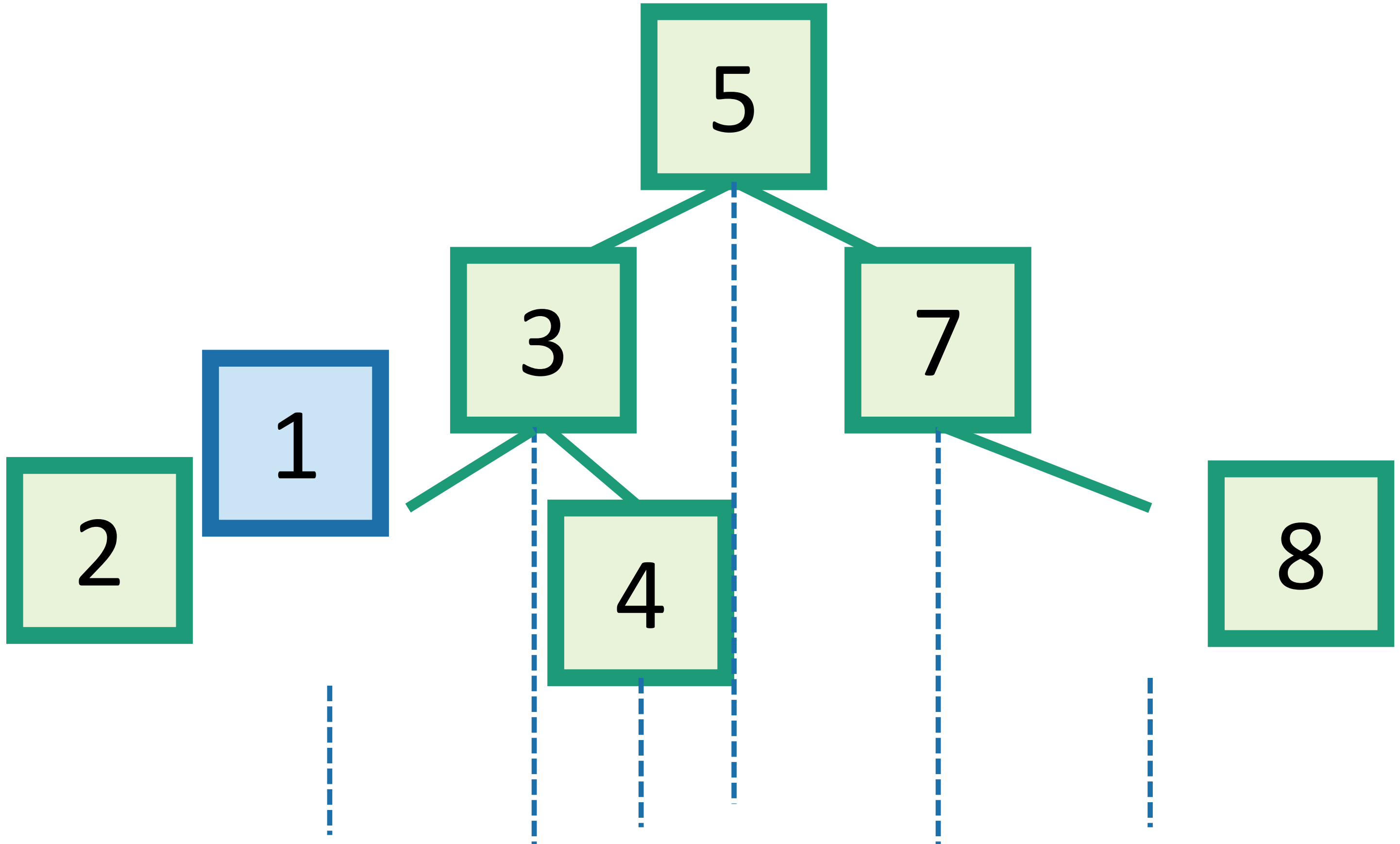

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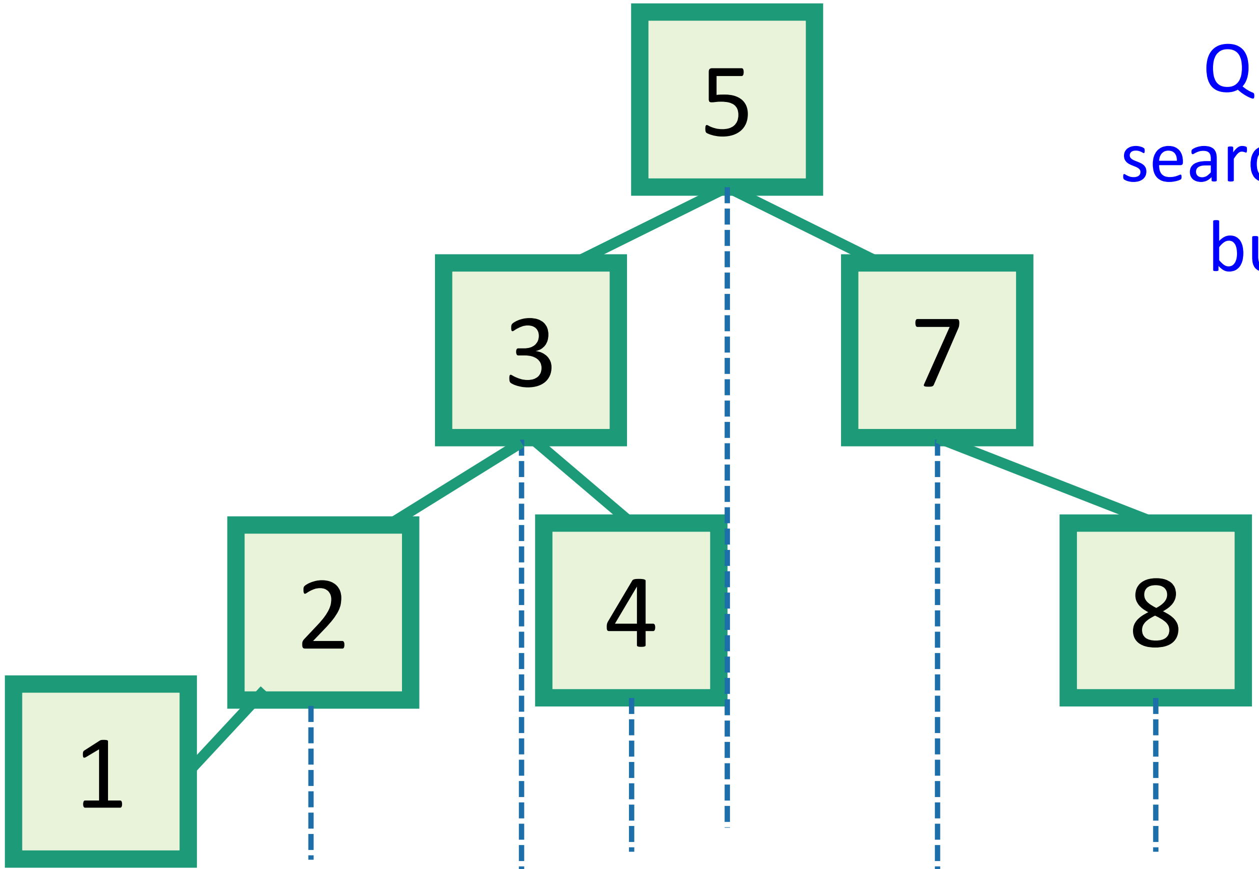
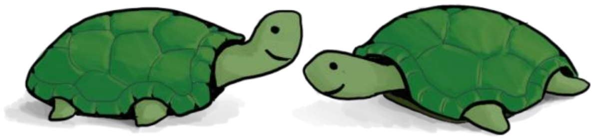


Q: Is this the only binary search tree I could possibly build with these values?

A: **No.** I made choices about which nodes to choose when. Any choices would have been fine.

Binary Search Trees

Which of these is a BST?



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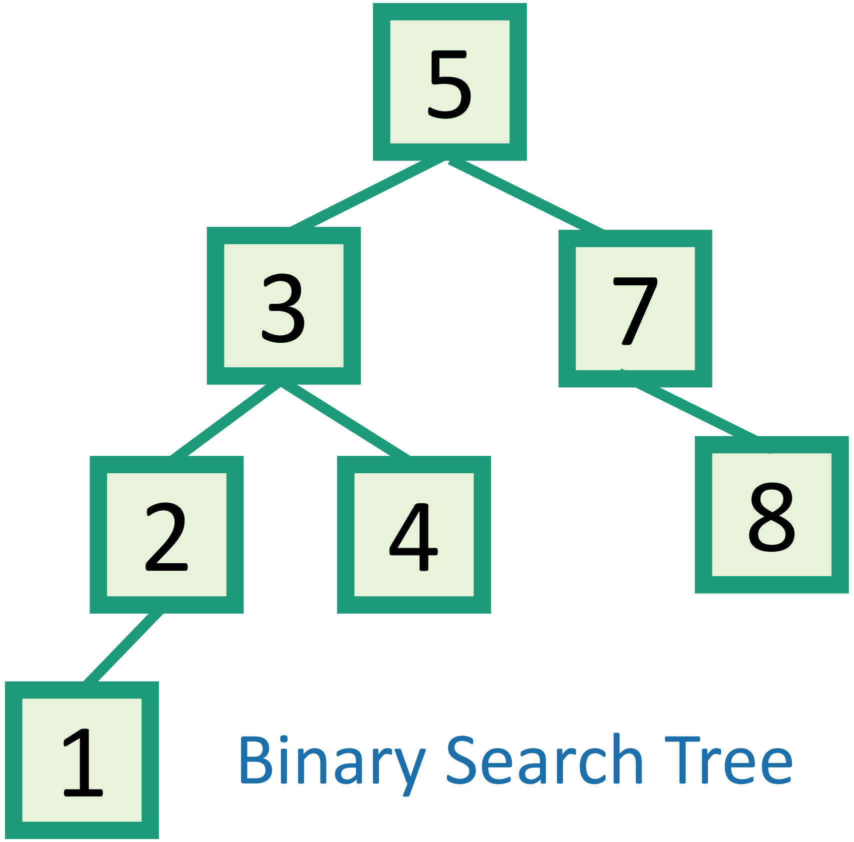
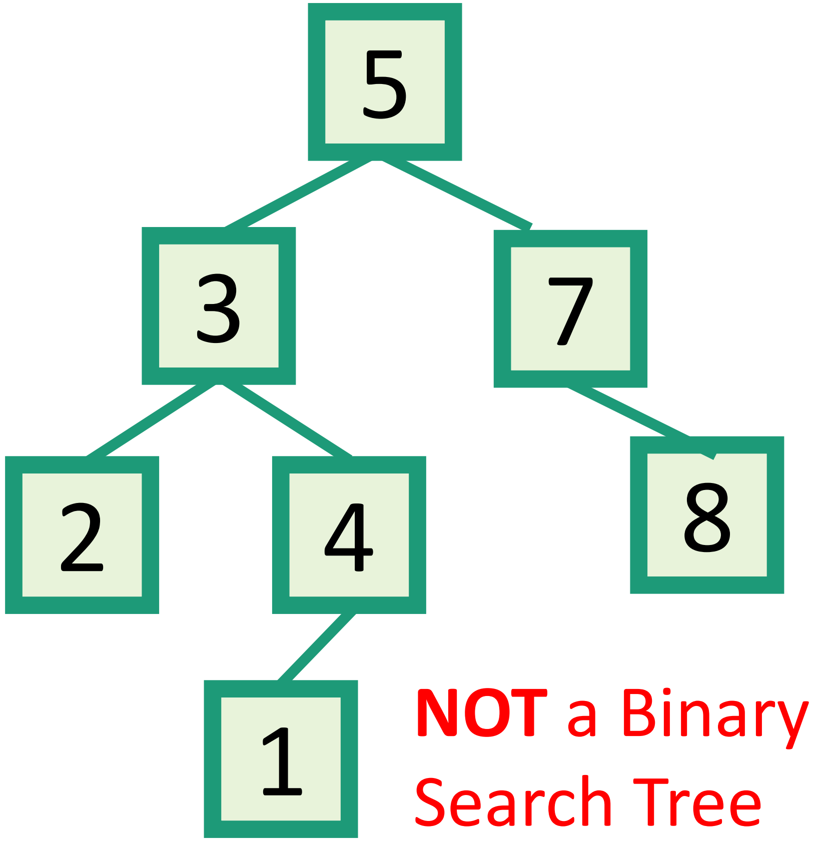


Binary Search Tree

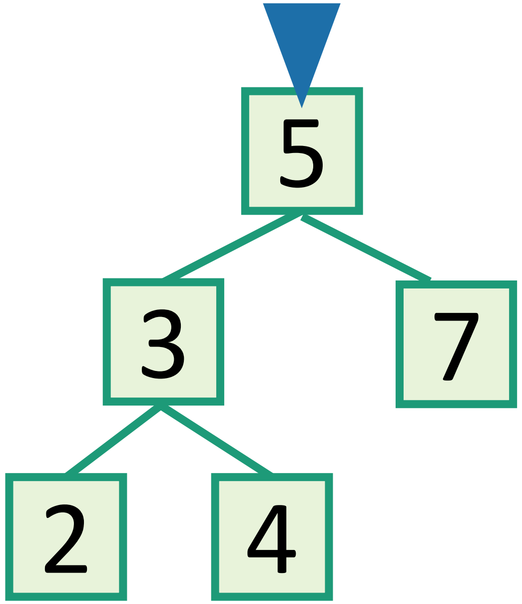


NOT a Binary Search Tree

Aside: In-Order Traversal of BSTs

- Output all the elements in sorted order!

- inOrderTraversal(x):
  - if x!= NIL:
    - inOrderTraversal(x.left)
    - print(x.key)
    - inOrderTraversal(x.right)



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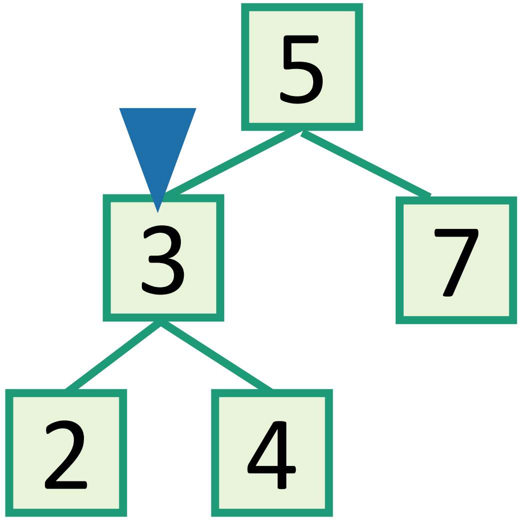

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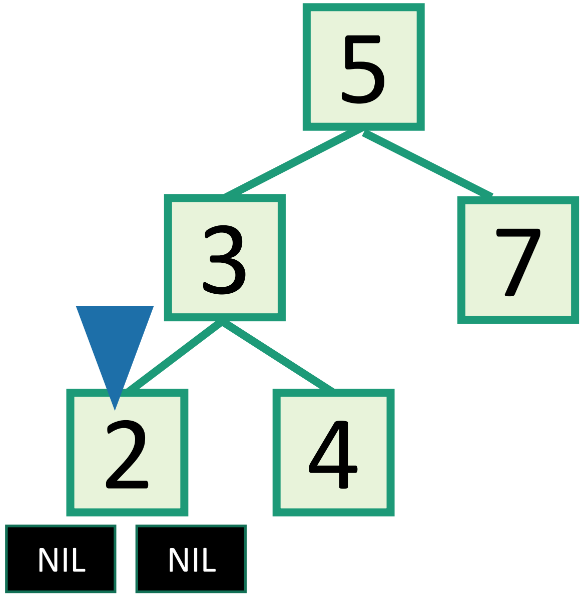

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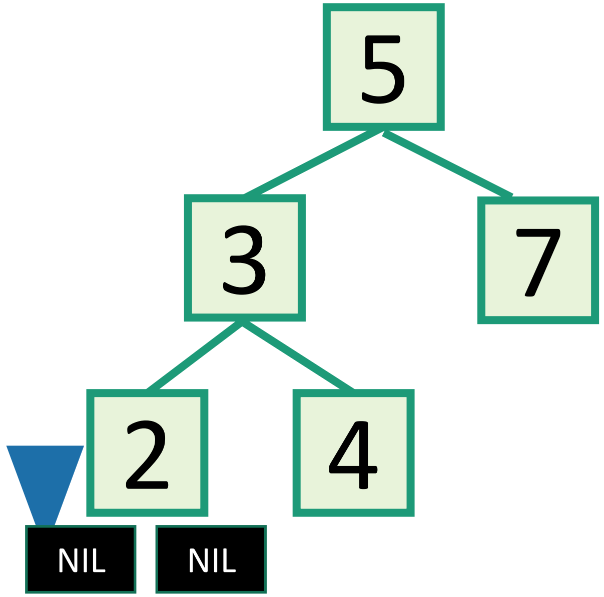

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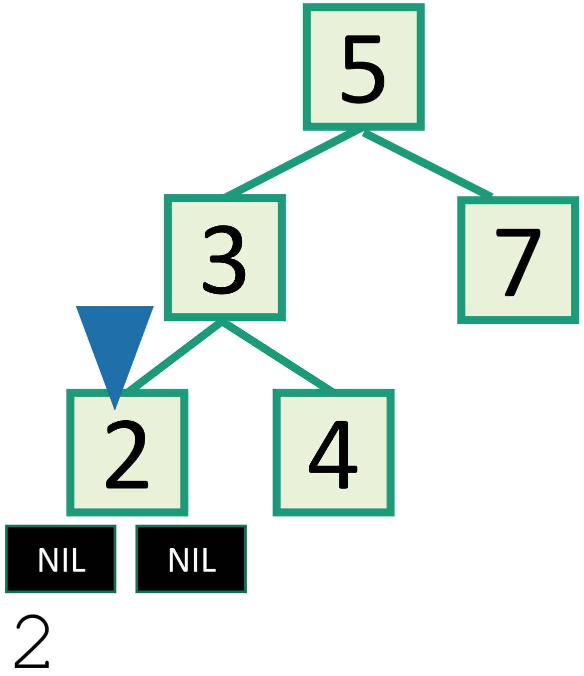

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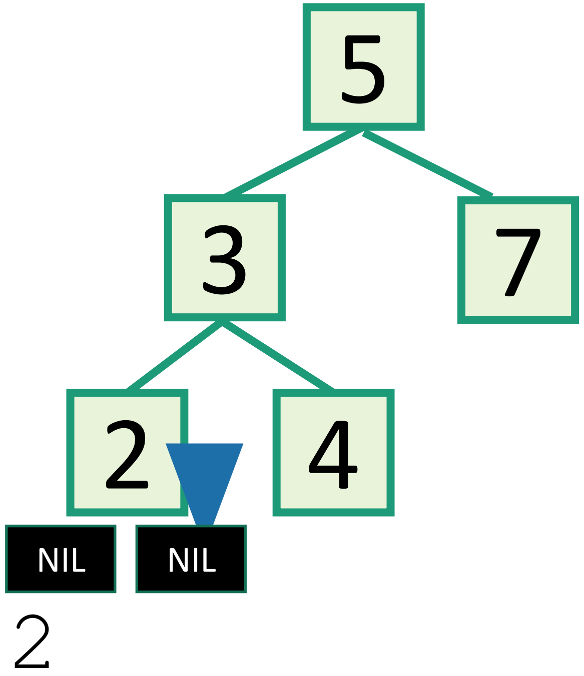

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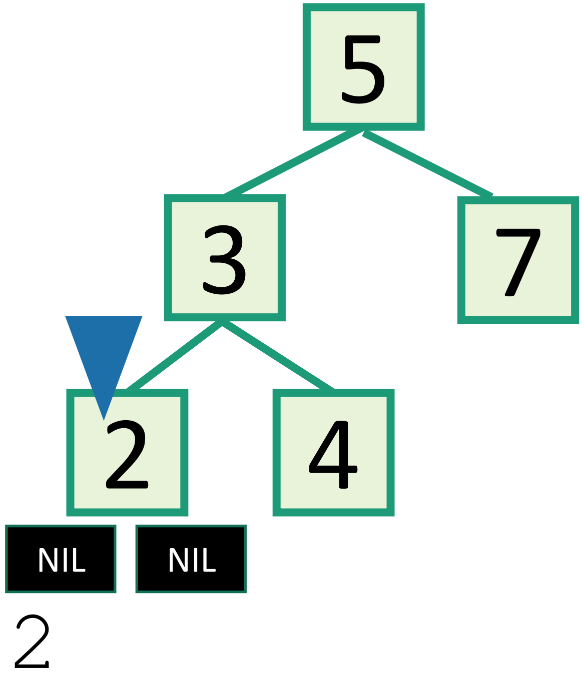

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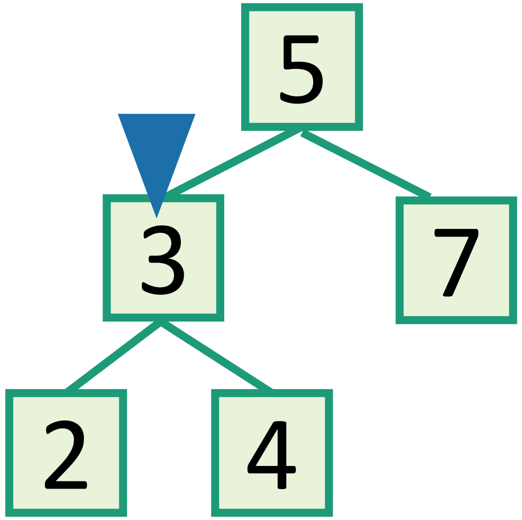

2 3

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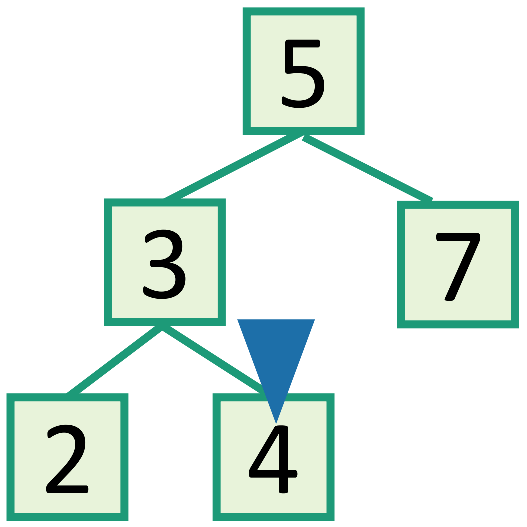

2 3 4

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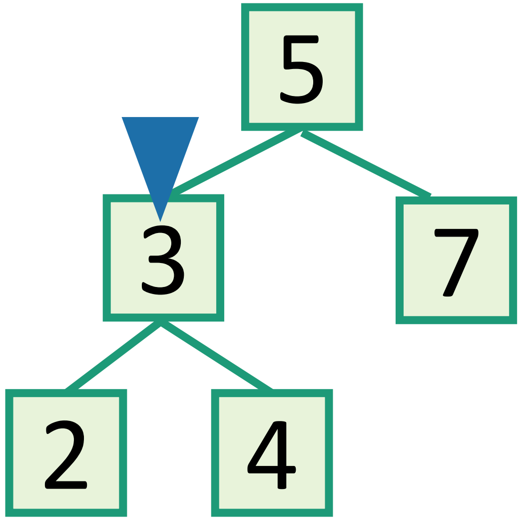

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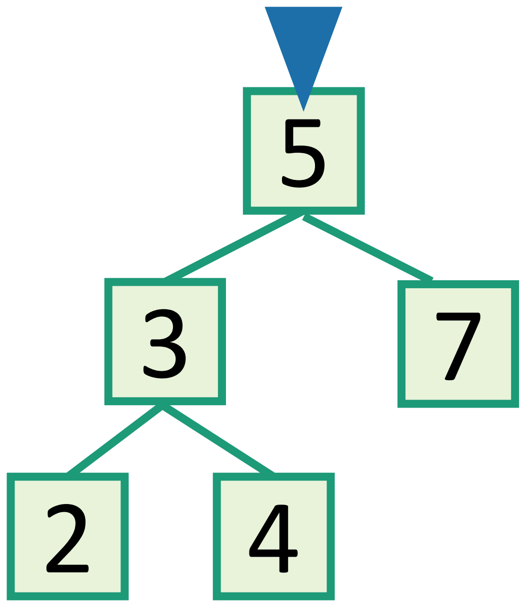

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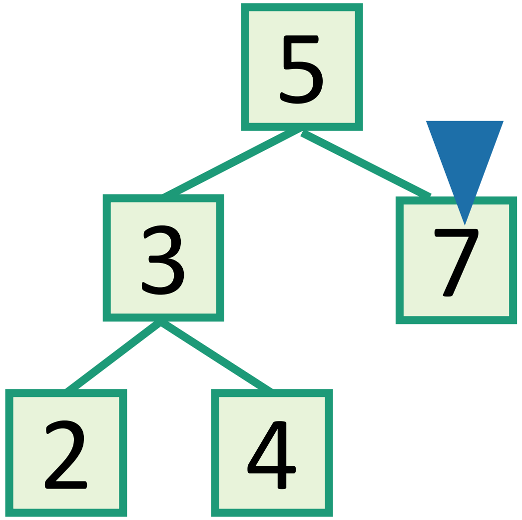

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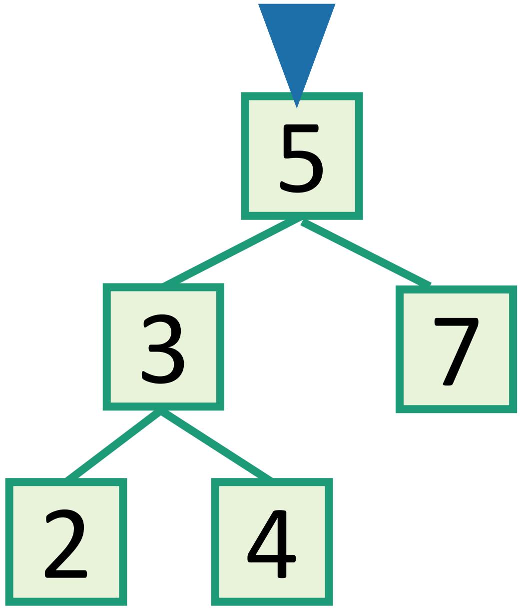


2 3 4 5 7 Sorted!

- Runs in time O(n).

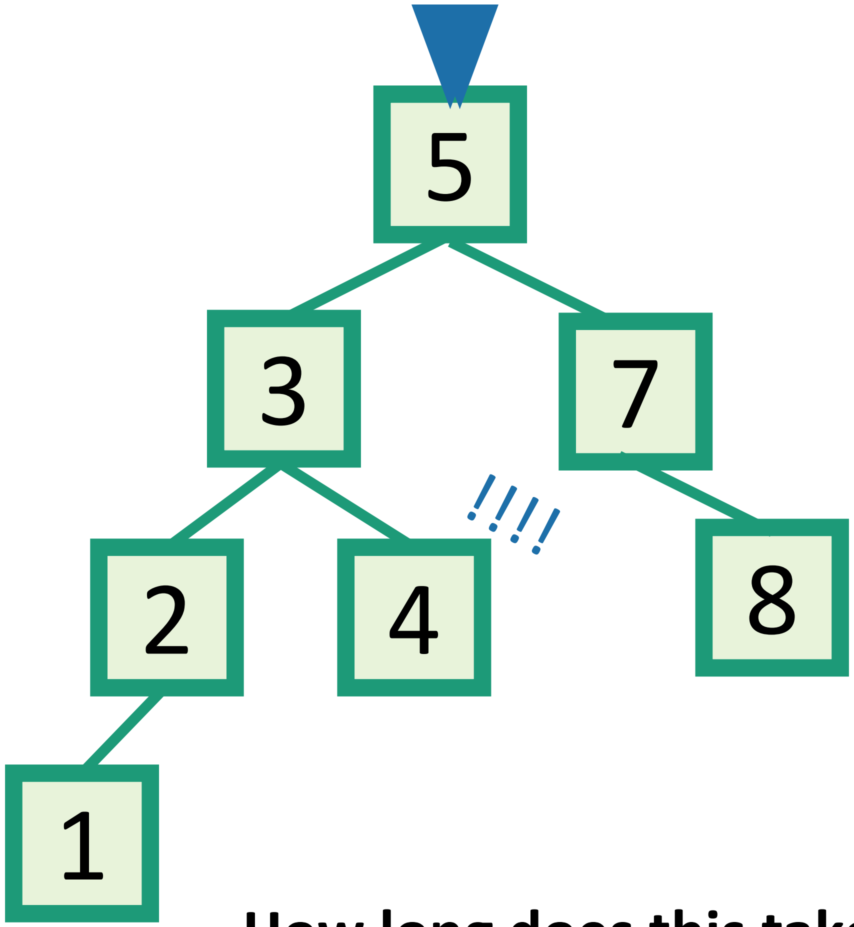
Back to the goal

Fast SEARCH/INSERT/DELETE

Can we do these?

SEARCH in a Binary Search Tree

definition by example



EXAMPLE: Search for 4.

EXAMPLE: Search for 4.5

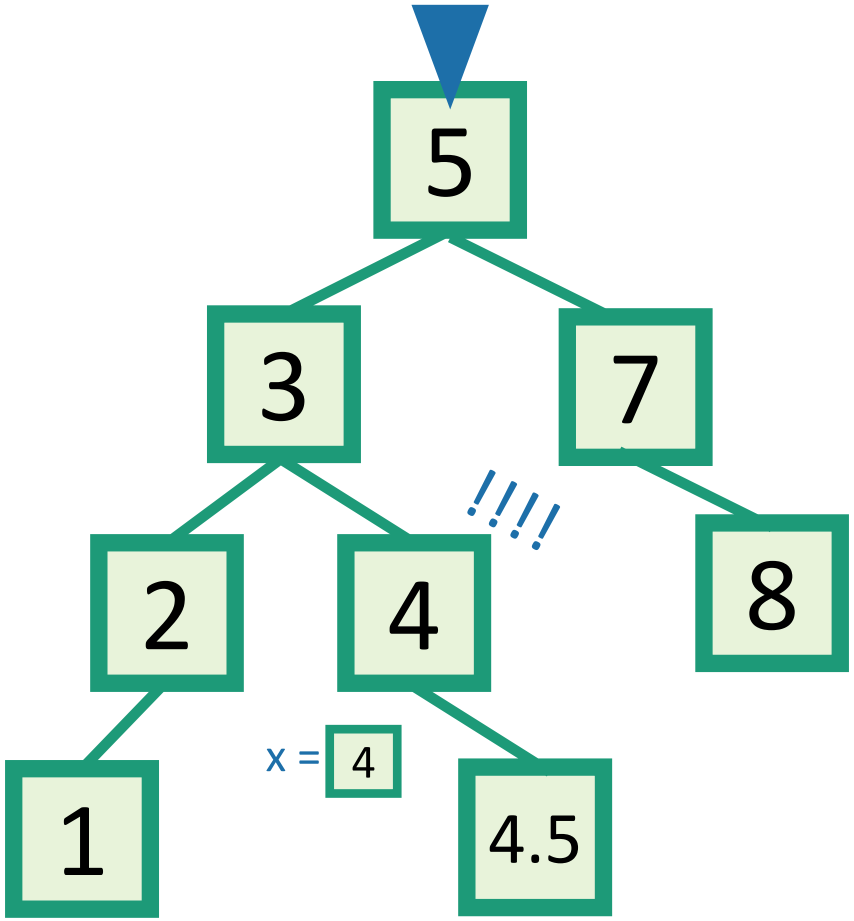
- It turns out it will be convenient to **return 4** in this case
- (that is, **return** the last node before we went off the tree)

Write pseudocode (or actual code) to implement this!

How long does this take?

O(length of longest path) = O(height)

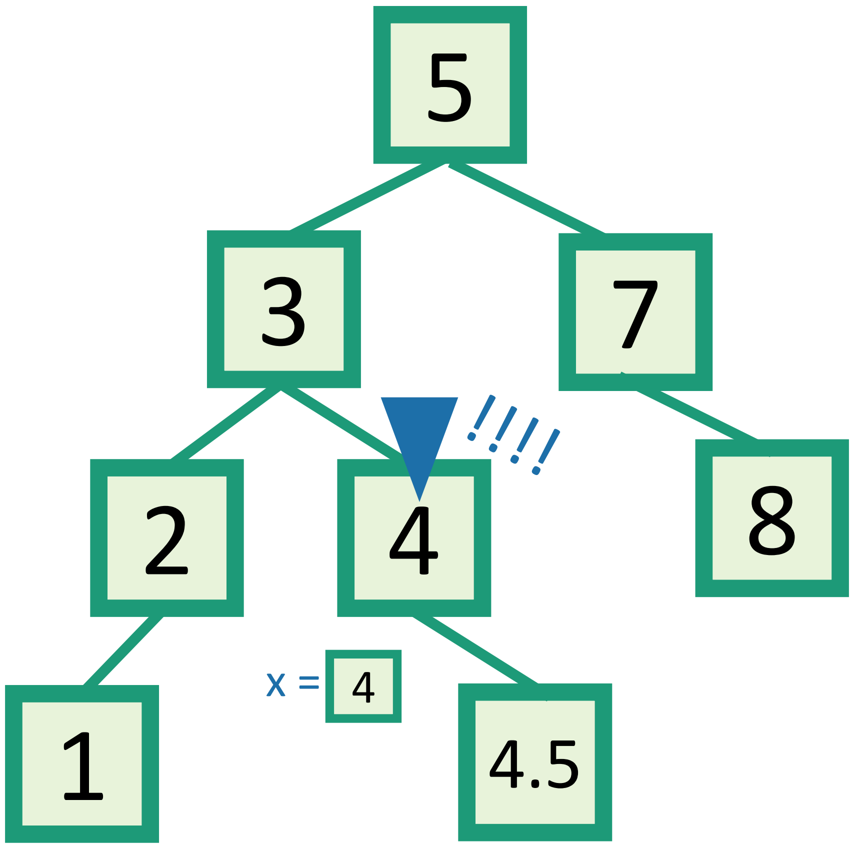
INSERT in a Binary Search Tree



EXAMPLE: Insert 4.5

- INSERT(key):
  - x = SEARCH(key)
  - **Insert** a new node with desired key at x…

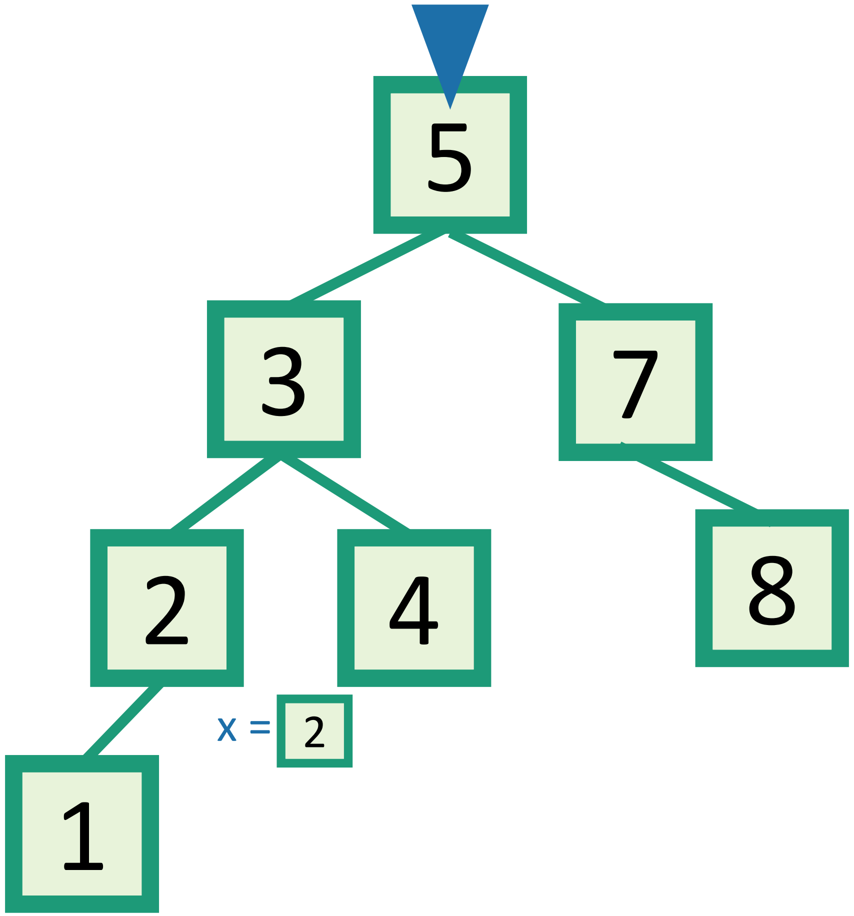
INSERT in a Binary Search Tree



EXAMPLE: Insert 4.5

- INSERT(key):
  - x = SEARCH(key)
  - **if** key > x.key:
    - Make a new node with the correct key, and put it as the right child of x.
  - **if** key < x.key:
    - Make a new node with the correct key, and put it as the left child of x.
  - **if** x.key == key:
    - **return**

DELETE in a Binary Search Tree



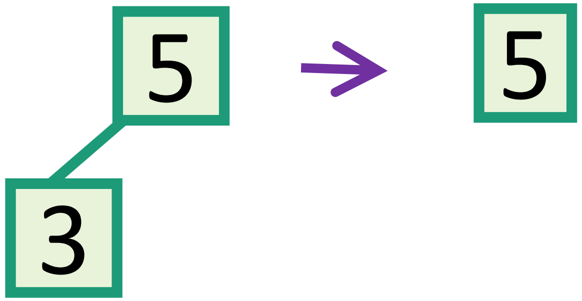
EXAMPLE: Delete 2

- DELETE(key):
  - x = SEARCH(key)
  - **if** x.key == key:
    - ....delete x....

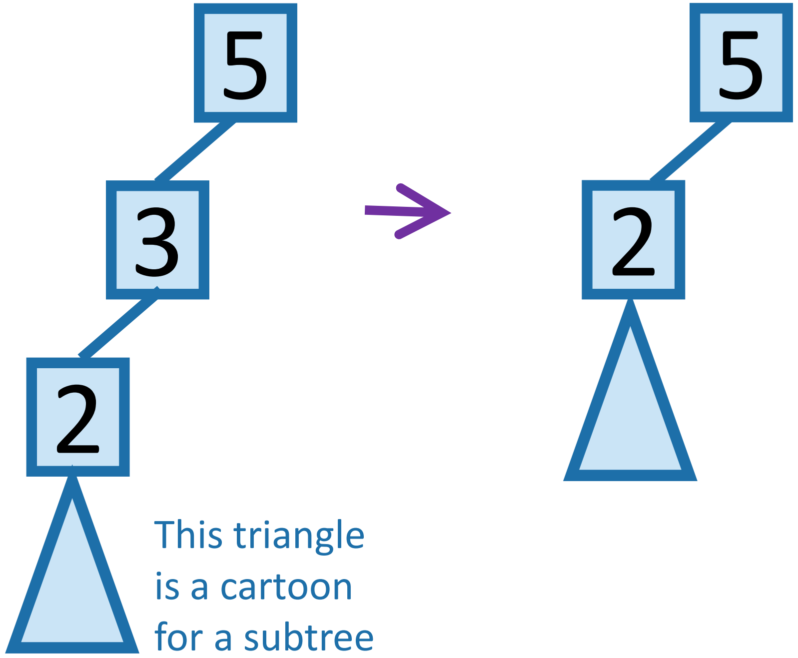
DELETE in a Binary Search Tree

several cases (by example)

say we want to delete 3



Case 1: if 3 is a leaf, just delete it.

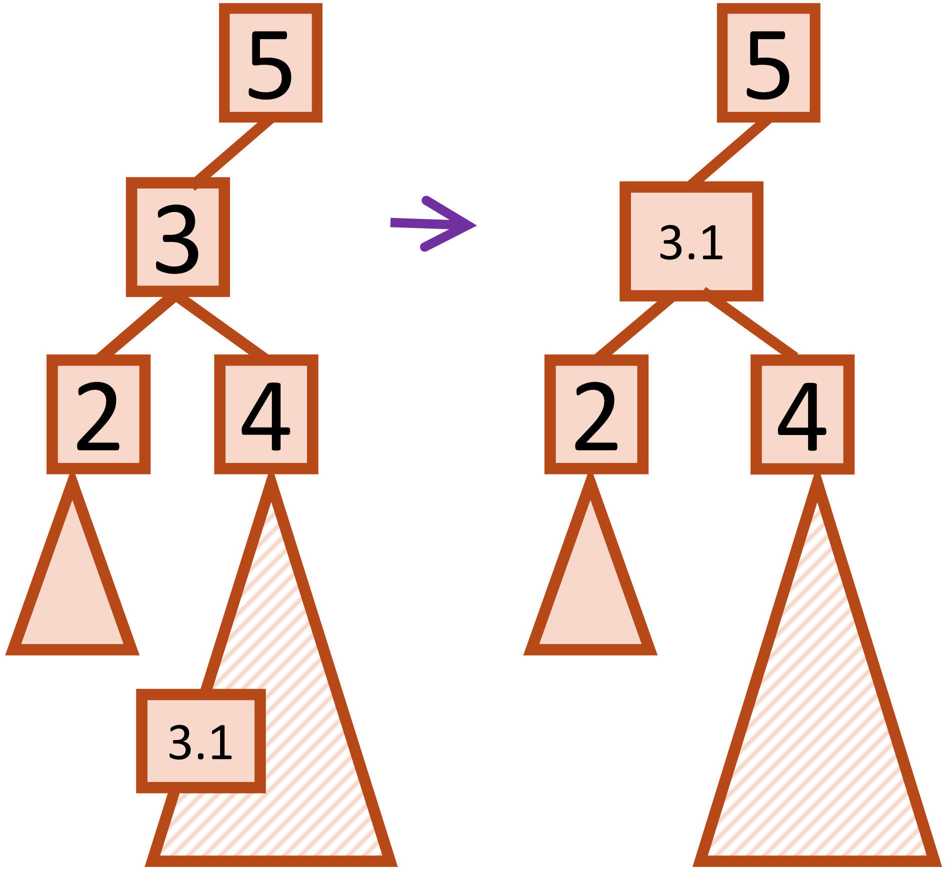


Case 2: if 3 has just one child, move that up.

Write pseudocode for all of these!

DELETE in a Binary Search Tree

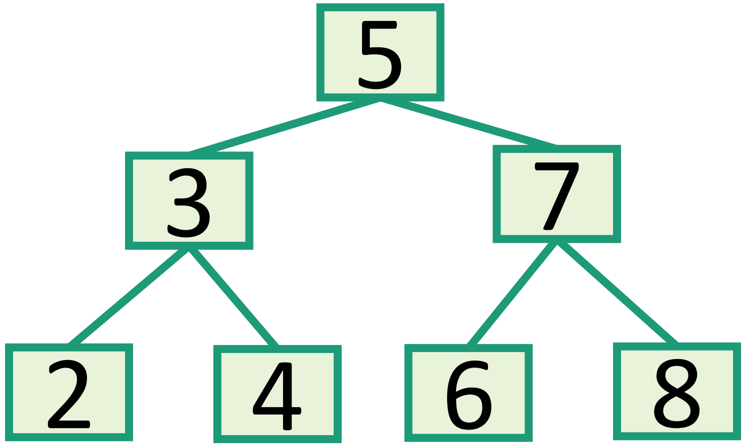
Case 3: if 3 has two children,
replace 3 with it's immediate successor.
(aka, next biggest thing after 3)



- Does this maintain the BST property?
  - Yes.
- How do we find the immediate successor?
  - SEARCH for 3 in the subtree under 3.right
- How do we remove it when we find it?
  - If [3.1] has 0 or 1 children, do one of the previous cases.
- What if [3.1] has two children?
  - It doesn't. (can not have two children)

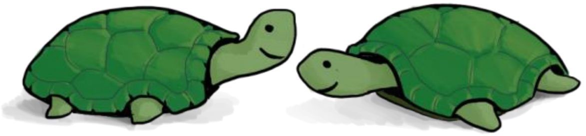
How long do these operations take?

- SEARCH is the big one.
  - Everything else just calls SEARCH and then does some small O(1)-time operation.



Time = O(height of tree)

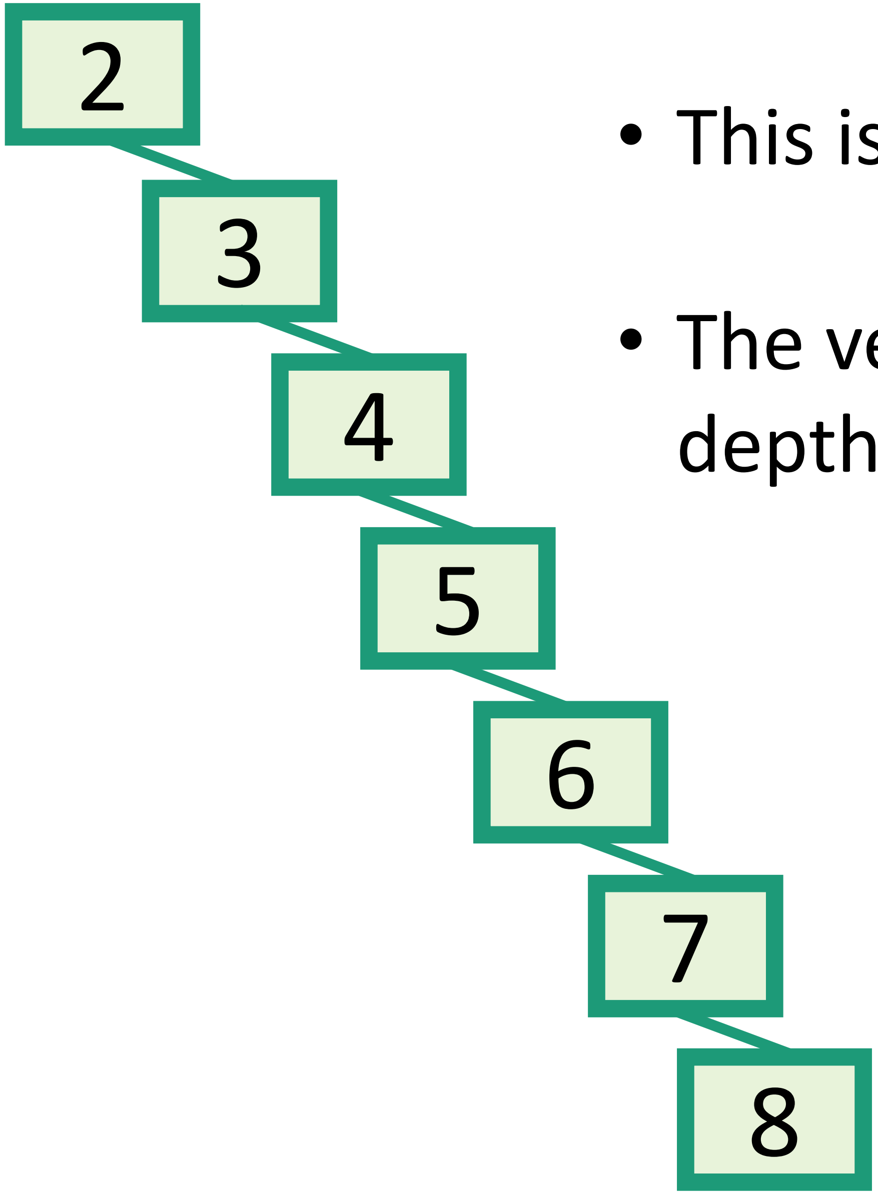
How long does search take?



Trees have depth O(log(n)). **Done!**

Wait a second…

Search might take time O(n)



- This is a valid binary search tree.
- The version with n nodes has depth n, **not** O(log(n)).

What to do?

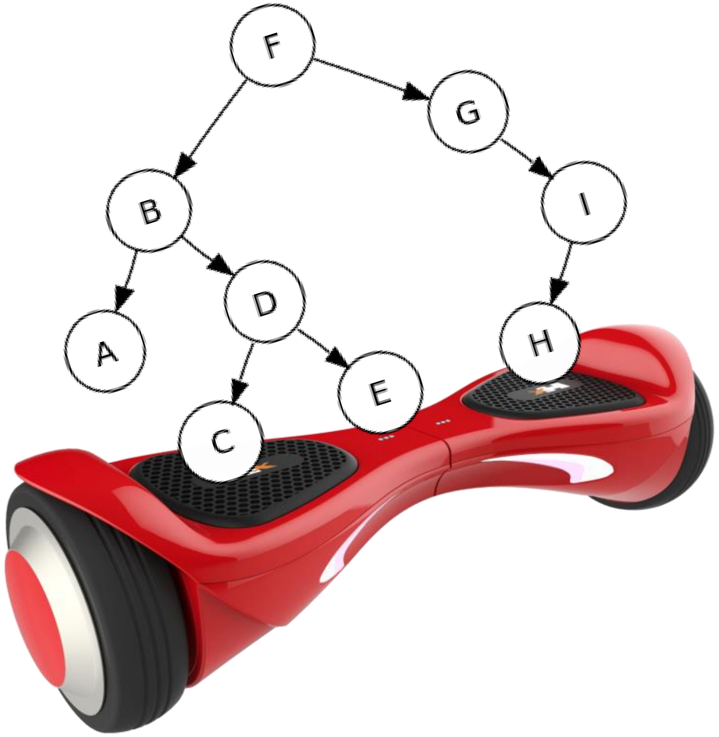
How often is "every so often" in the worst case? It's actually pretty often!

- Goal: Fast SEARCH/INSERT/DELETE
- All these things take time O(height)
- And the height might be big!!! ☹

- Idea 0:
  - Keep track of how deep the tree is getting.
  - If it gets too tall, re-do everything from scratch.
    - At least $\Omega(n)$ every so often....

- Turns out that's not a great idea. Instead we turn to…

Self-Balancing Binary Search Trees

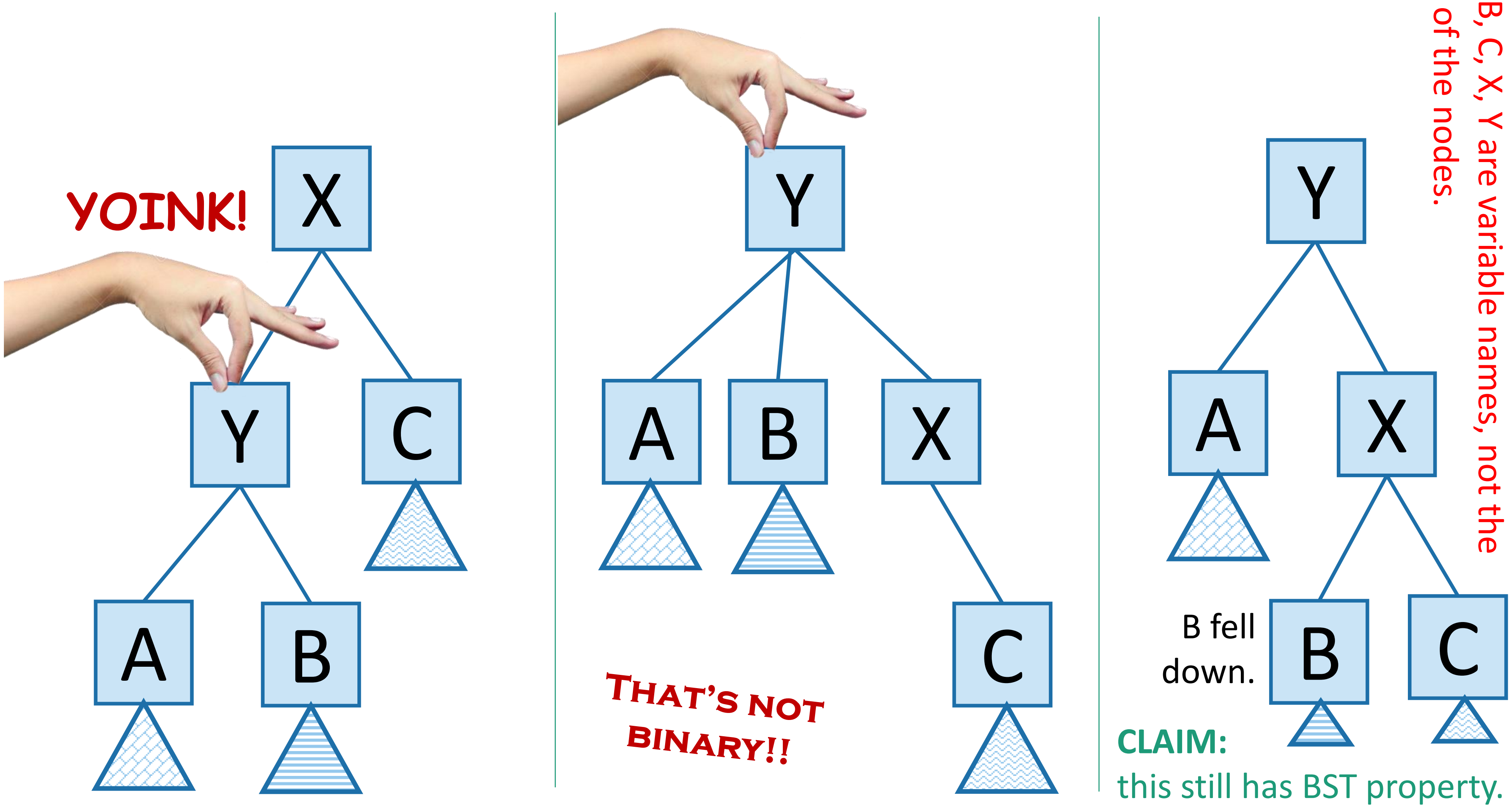


Idea 1: Rotations

No matter what lives underneath A,B,C, **this takes time O(1).** (Why?)

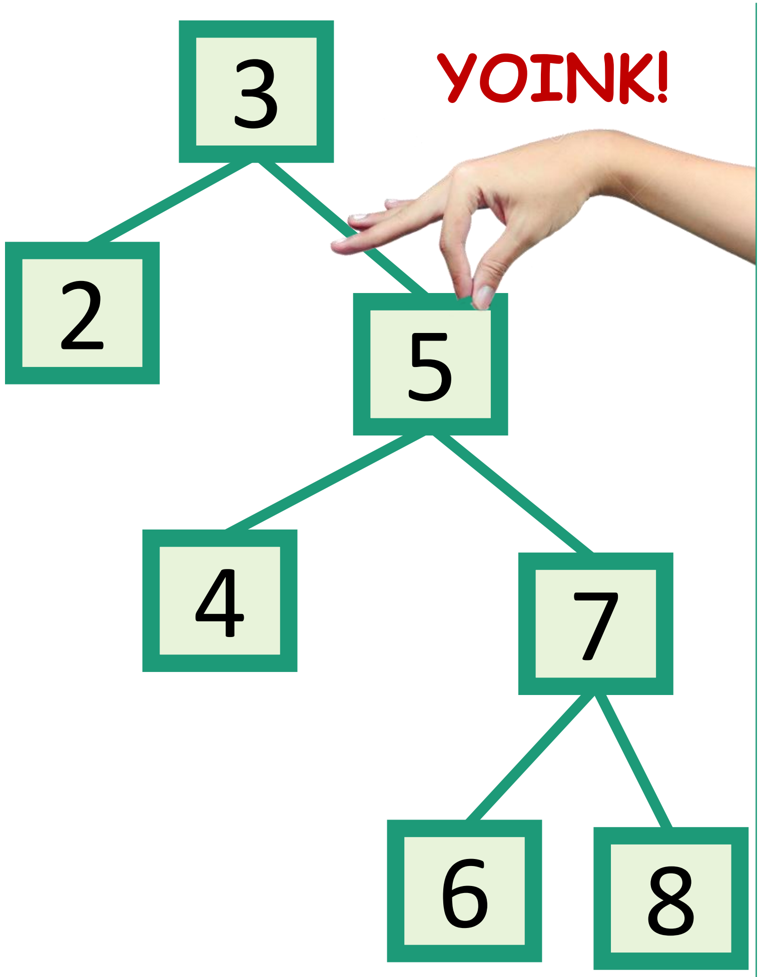
- Maintain Binary Search Tree (BST) property, while moving stuff around.

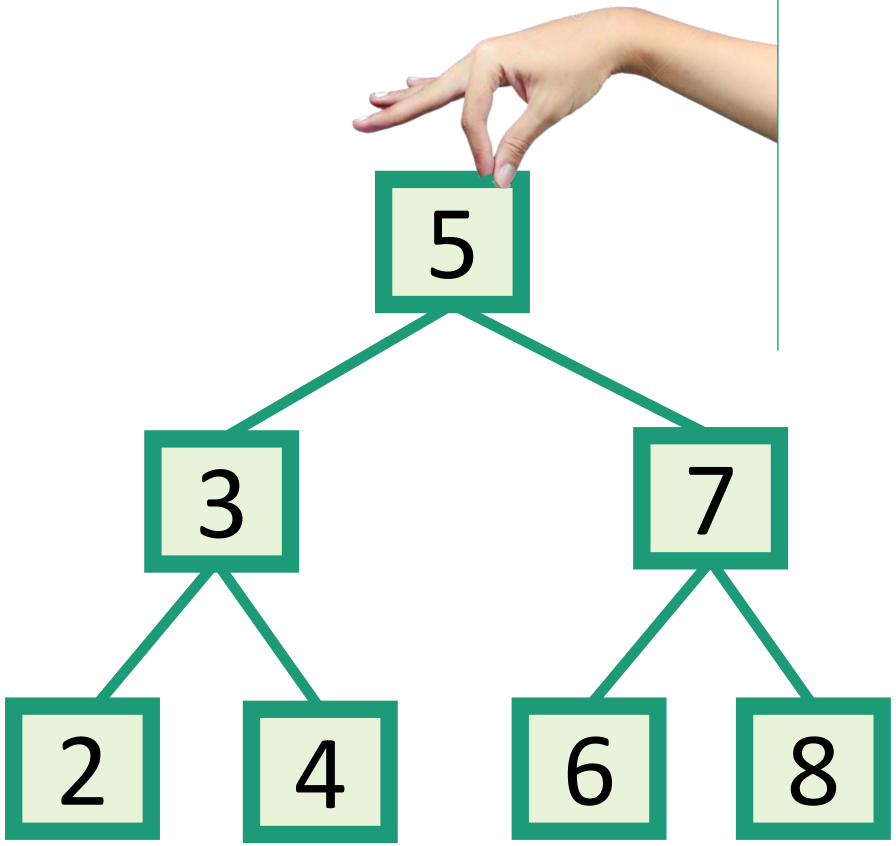
Note: A, B, C, X, Y are variable names, not the contents of the nodes.



CLAIM:
this still has BST property.

This seems helpful





Strategy?

- Whenever something seems unbalanced, do rotations until it's okay again.

This is pretty vague.

What do we mean by "seems unbalanced"?

What's "okay"?

Idea 2: have some proxy for balance

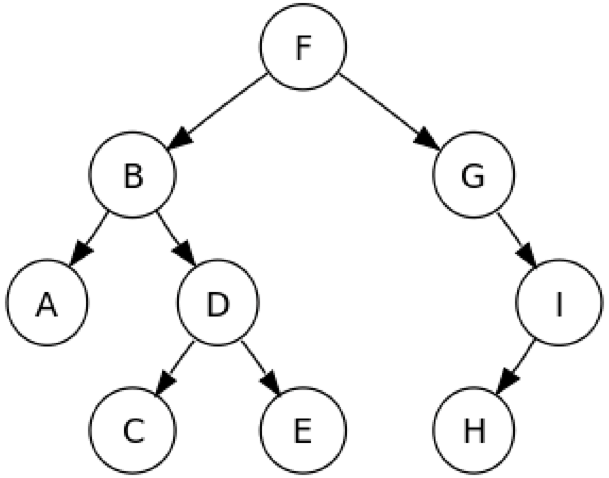
- Maintaining perfect balance is too hard.
- Instead, come up with some proxy for balance:
  - If the tree satisfies [SOME PROPERTY], then it's pretty balanced.
  - We can maintain [SOME PROPERTY] using rotations.

There are actually several ways to do this, but we'll see:

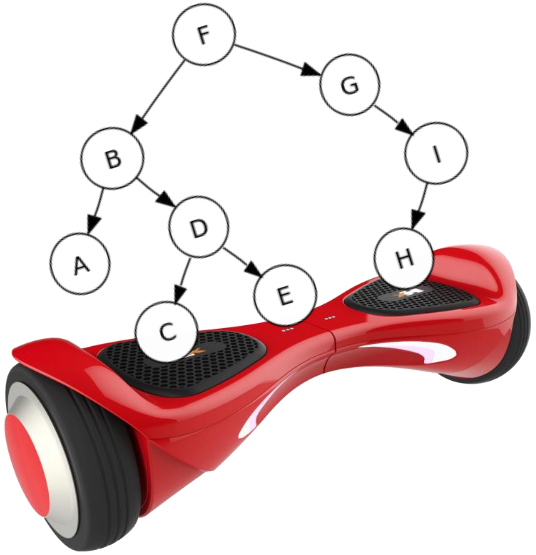
1. AVL Tree (In this course)
2. Multiway-Search Tree (2-4 Tree)
3. Red-Black Tree

Recap

- Begin a brief foray into data structures!
- Binary search trees
  - They are better when they're balanced.



this will lead us to…



- Self-Balancing Binary Search Trees
  - AVL Tree
  - Multiway-Search Tree
  - Red-Black Tree

Acknowledgement

- Stanford University

Thank You