

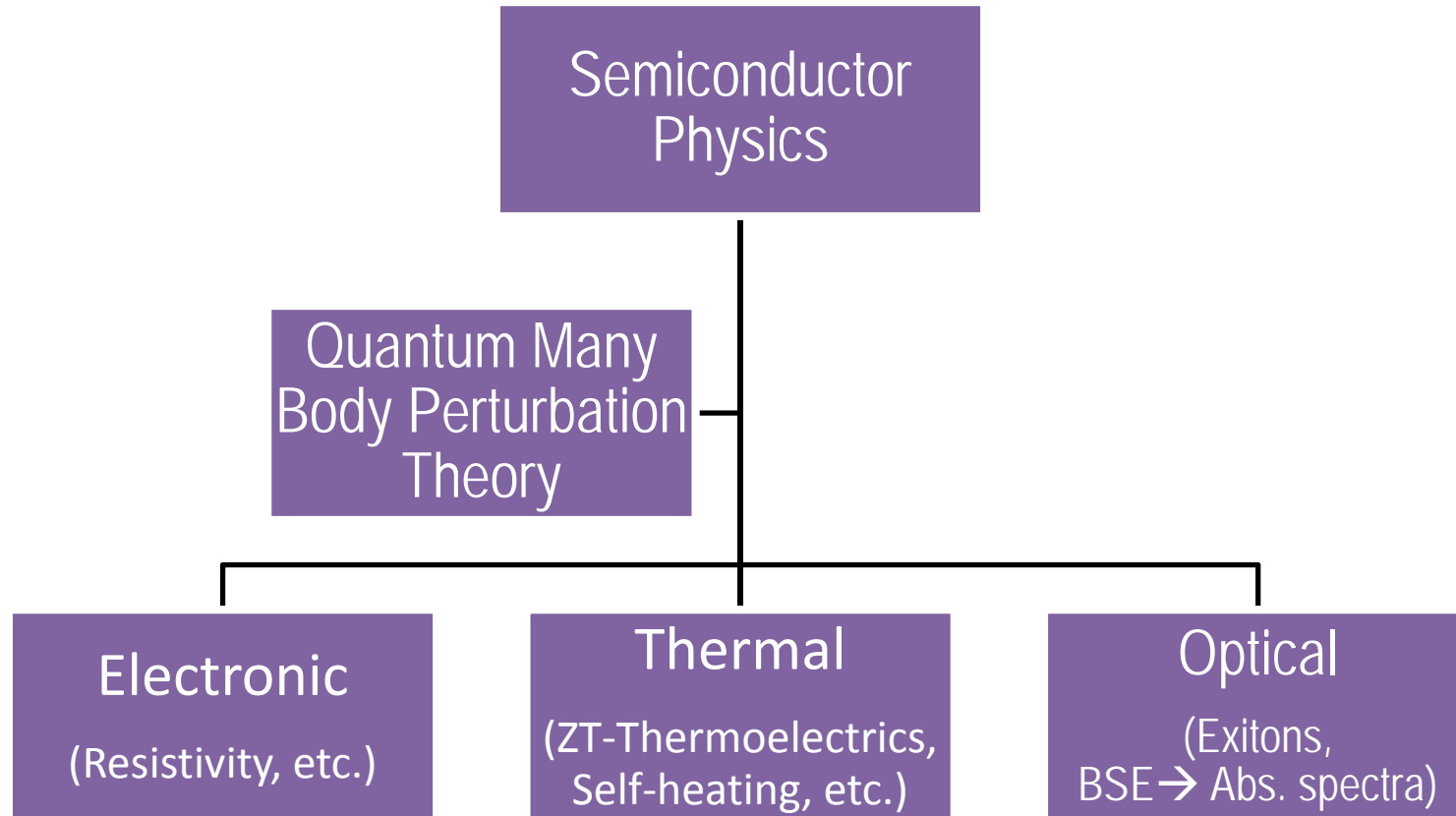
**Role of Feynman diagrams in energy band
structure of materials :
A post density functional theory approach**

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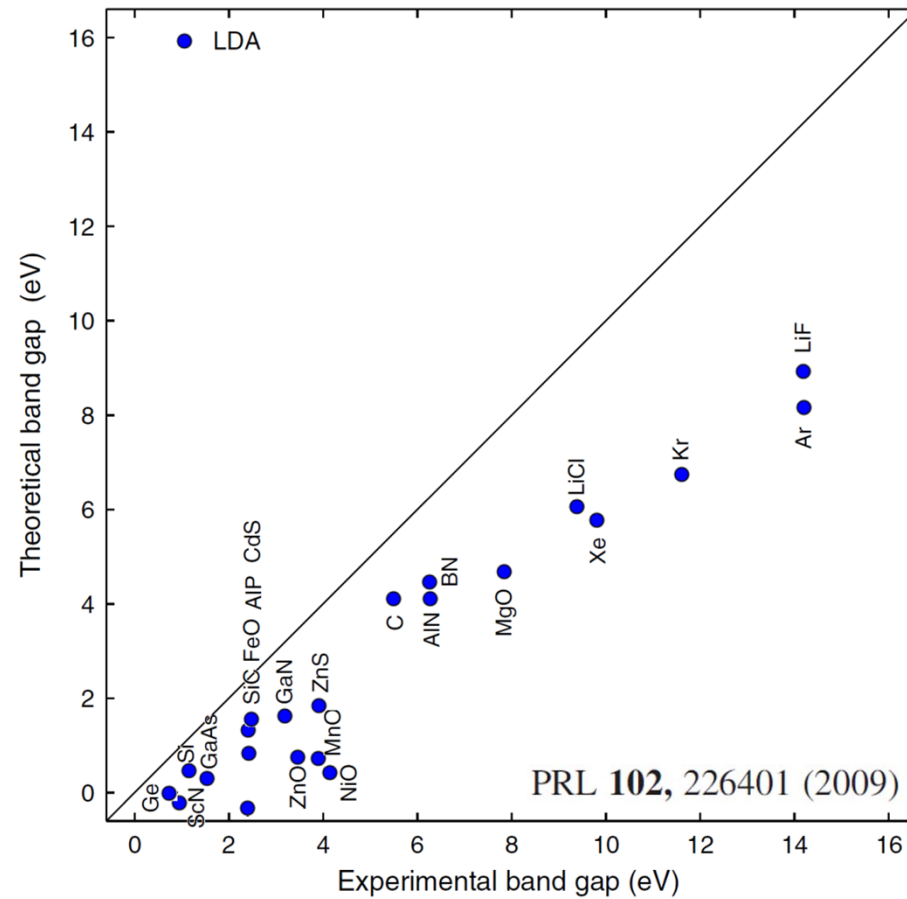
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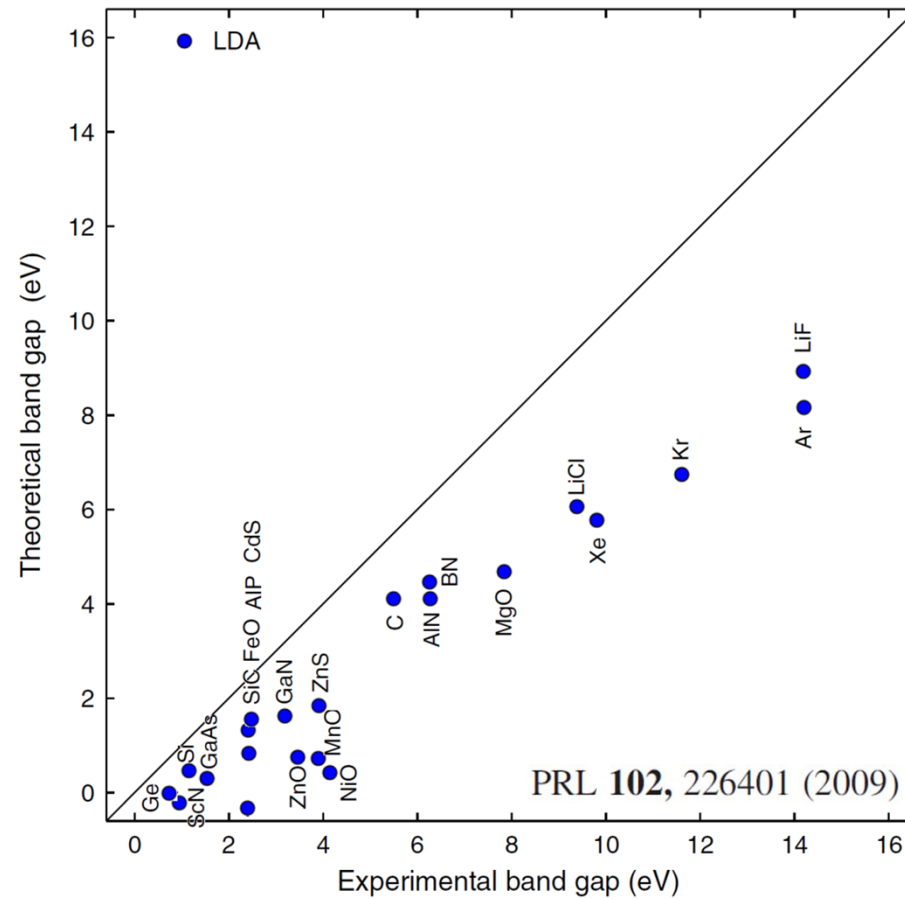
Research Activities : Semiconductor transport phenomena (computational)



A visit to the energy band structure in semiconductors



A visit to the energy band structure in semiconductors



Why should this happen? And how to fix this problem ?

Understanding how a band gap is measured first

Electronic Band Gap

Optical Band Gap

Understanding how a band gap is measured first

Electronic Band Gap

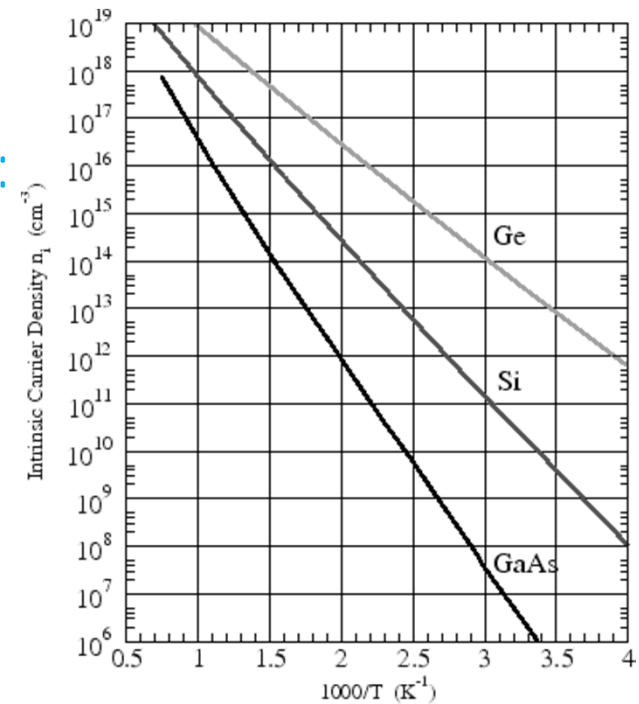
Hall Effect Measurement :

Understanding how a band gap is measured first

Electronic Band Gap

Hall Effect Measurement :

- ❑ Find out carrier densities at different temperatures.
- ❑ The slope in log scale will give you the band gap.



Understanding how a band gap is measured first

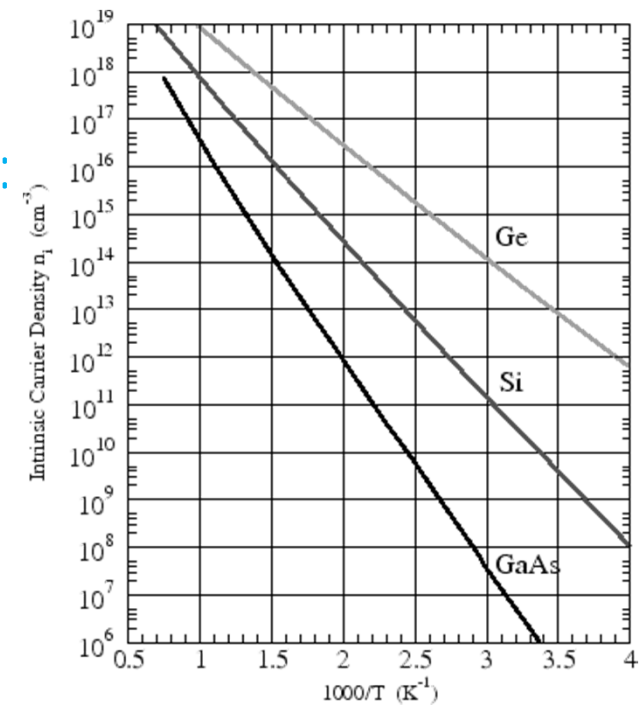
Electronic Band Gap

Hall Effect Measurement :

- ❑ Find out carrier densities at different temperatures.
- ❑ The slope in log scale will give you the band gap.

However, beware about the Hall-factor coefficient r_H .

$$n_H = \frac{n_S}{r_H}$$



Understanding how a band gap is measured first

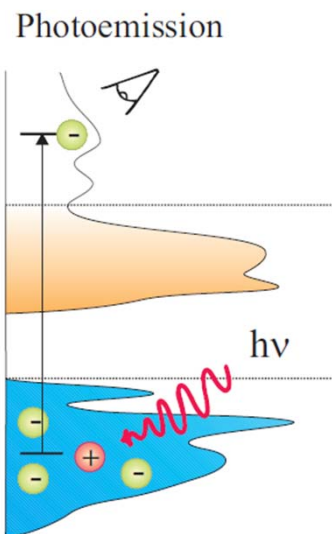
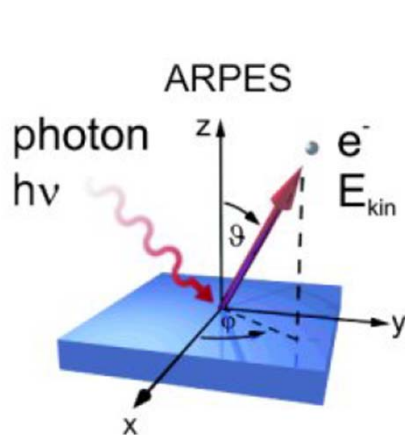
Optical Band Gap

Angle Resolved Photo-Emission Spectroscopy Measurement :

Understanding how a band gap is measured first

Optical Band Gap

Angle Resolved Photo-Emission Spectroscopy Measurement :



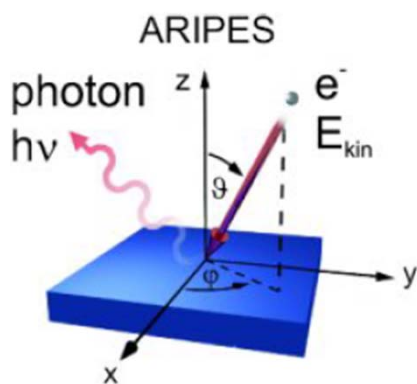
Removal of Electron:

$$\epsilon_r = E(N,s) - E(N-1,s)$$

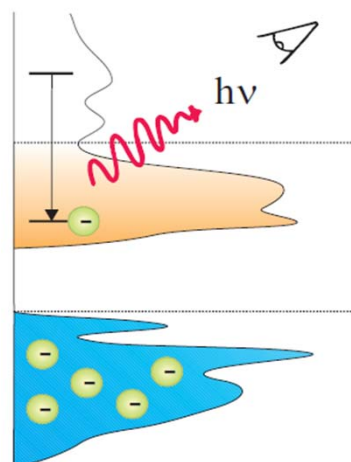
Understanding how a band gap is measured first

Optical Band Gap

Angle Resolved Photo-Emission Spectroscopy Measurement :



Inverse Photoemission



Addition of Electron:

$$\epsilon_a = E(N+1,s) - E(N,s)$$

Understanding how a band gap is measured first

Optical Band Gap

Angle Resolved Photo-Emission Spectroscopy Measurement :

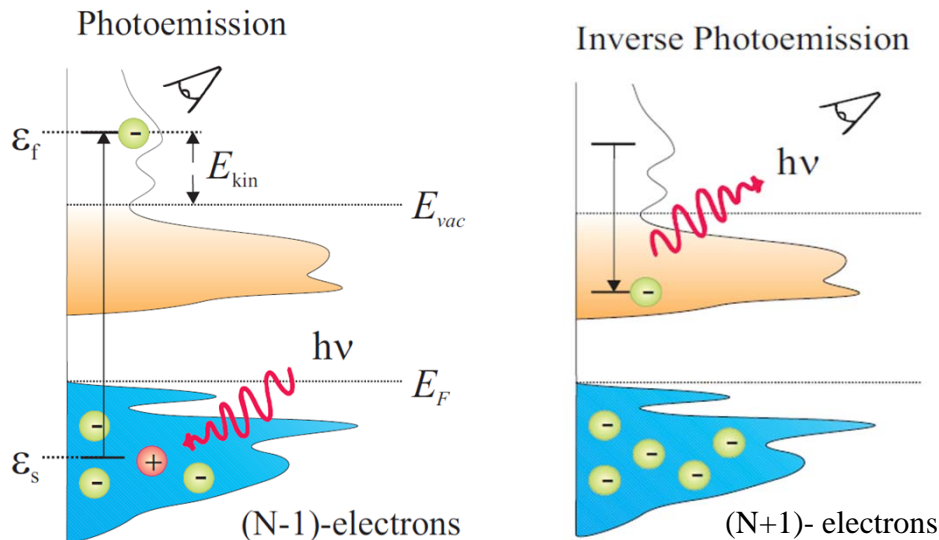
Fundamental Band Gap:

$$\mathcal{E}_G = \mathcal{E}_a - \mathcal{E}_r$$

Understanding how a band gap is measured first

Optical Band Gap

Angle Resolved Photo-Emission Spectroscopy Measurement :



The states $E(N+1,s)$ and $E(N-1,s)$ are called as the **excited states** and contains all the information that you need for the correction of the underestimated band gap.

Understanding how DFT calculate the band-gap


DFT underlying principles: Hohenberg-Kohn Theorem

- For any interacting particle system in an external potential, the density is uniquely determined.

Understanding how DFT calculate the band-gap

DFT underlying principles: Hohenberg-Kohn Theorem

- For any interacting particle system in an external potential, the density is uniquely determined.
- A universal *functional* for the energy $E[n]$ can be defined in terms of the density. The exact ground state is the global minimum value of this functional.

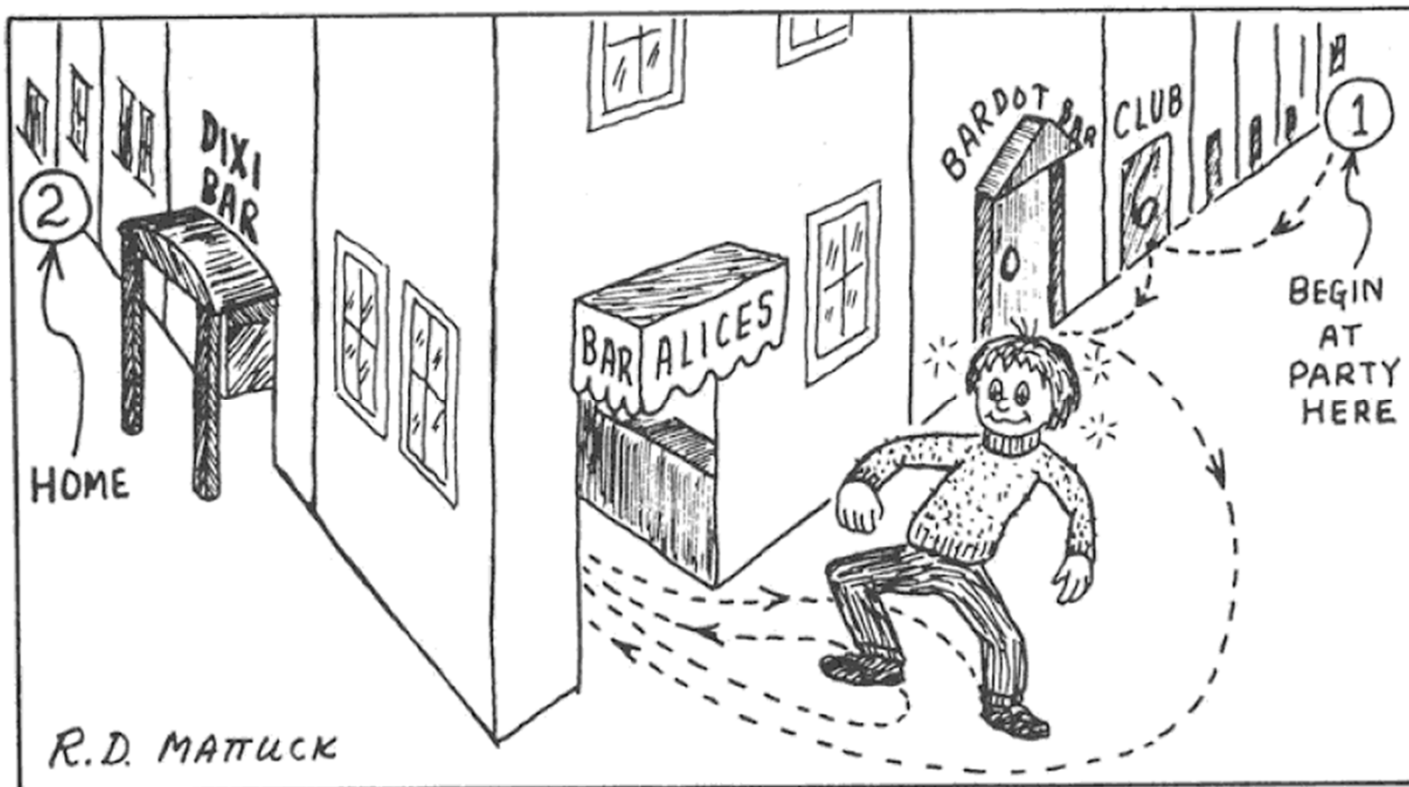


DFT is a ground state story

Include dynamic screening!
Electrons are screened.
This is known as GW formalism.

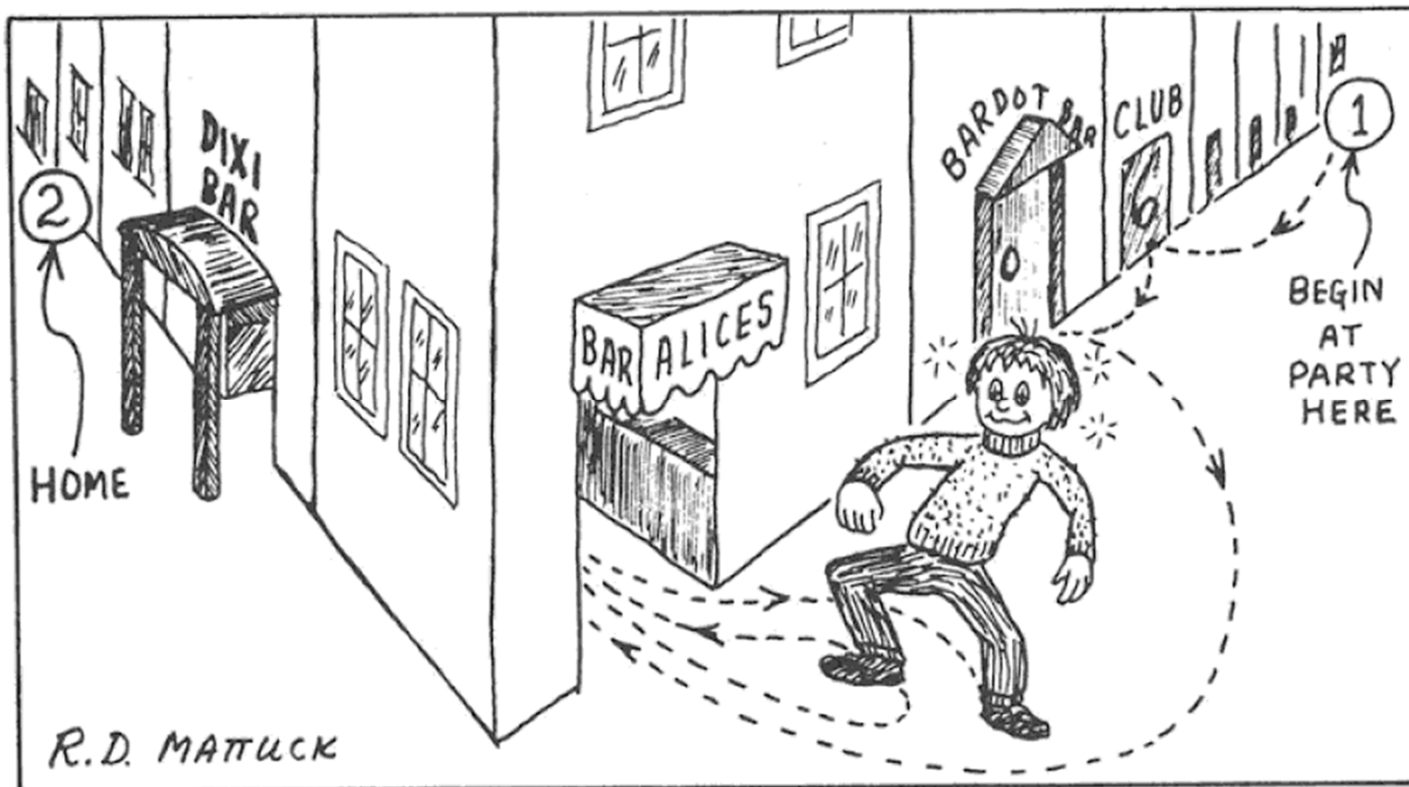
Lets understand this...

A strict drunker case: Random Walk Problem



What is the probability the drunker can end up in his home?

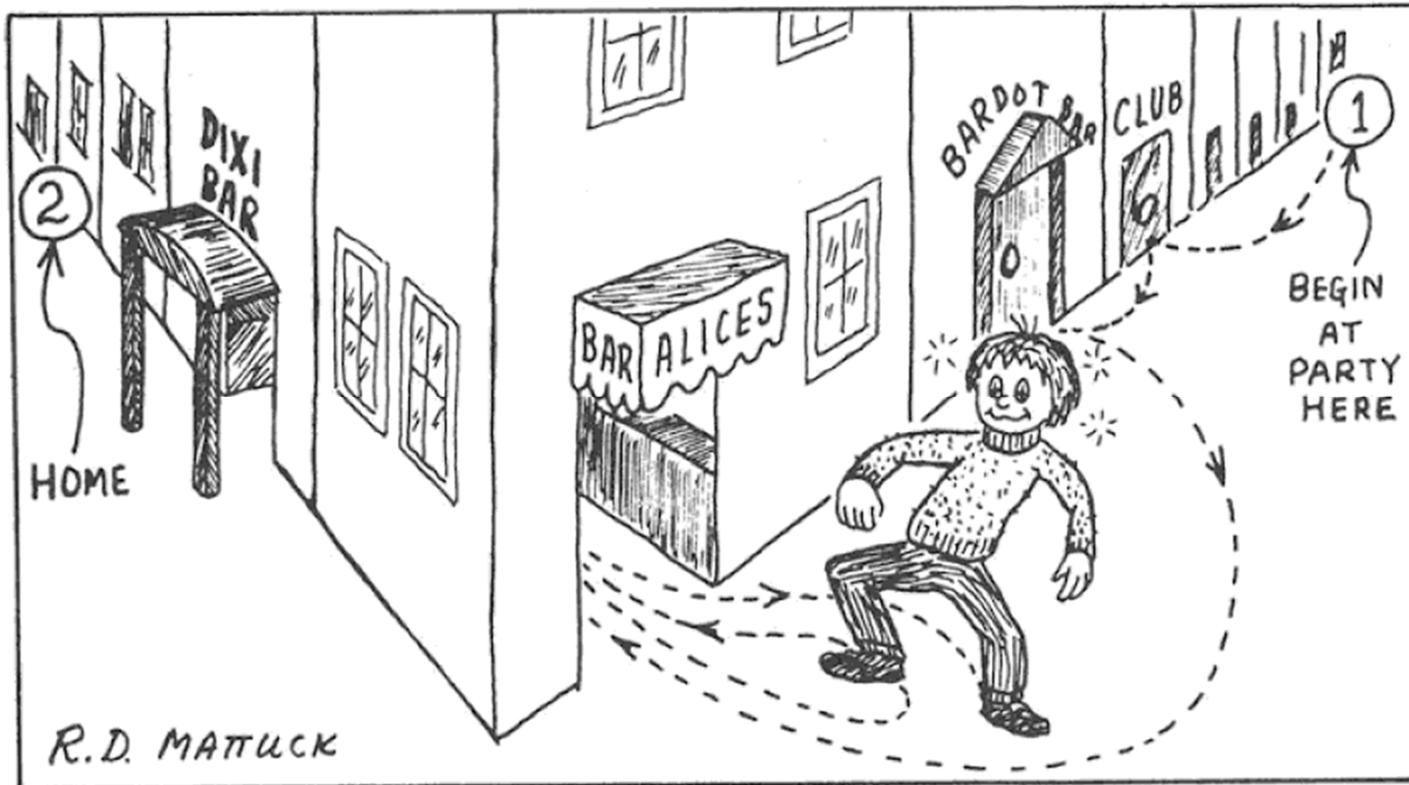
A strict drunker case: Random Walk Problem



1st way:

Go free from 1 to 2 without stopping = $P_0(2,1)$

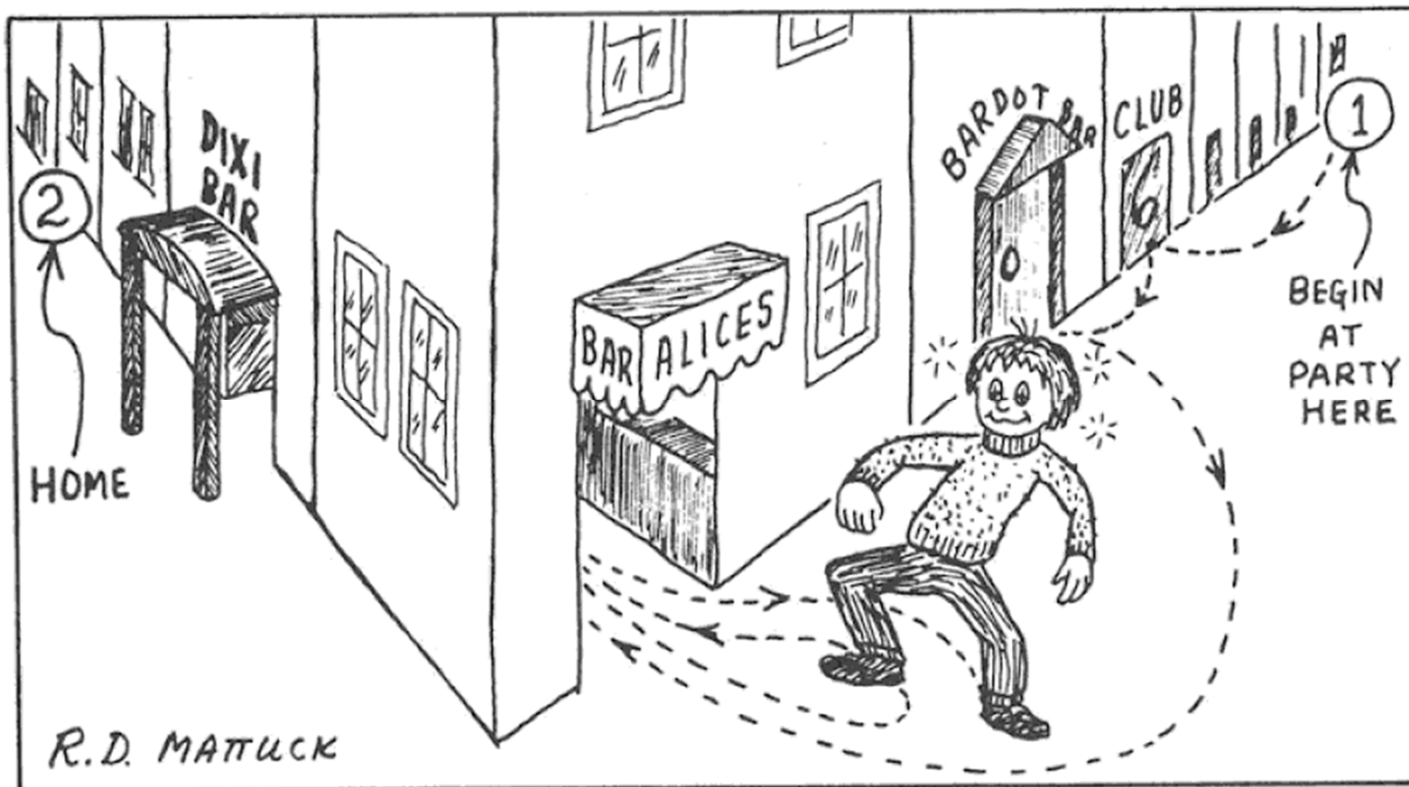
A strict drunker case: Random Walk Problem



2nd way:

Go freely from 1 to bar A ($P_0(A,1)$) and then stop for a drink ($P(A)$) and then go freely from A to 2 ($P_0(2,A)$).

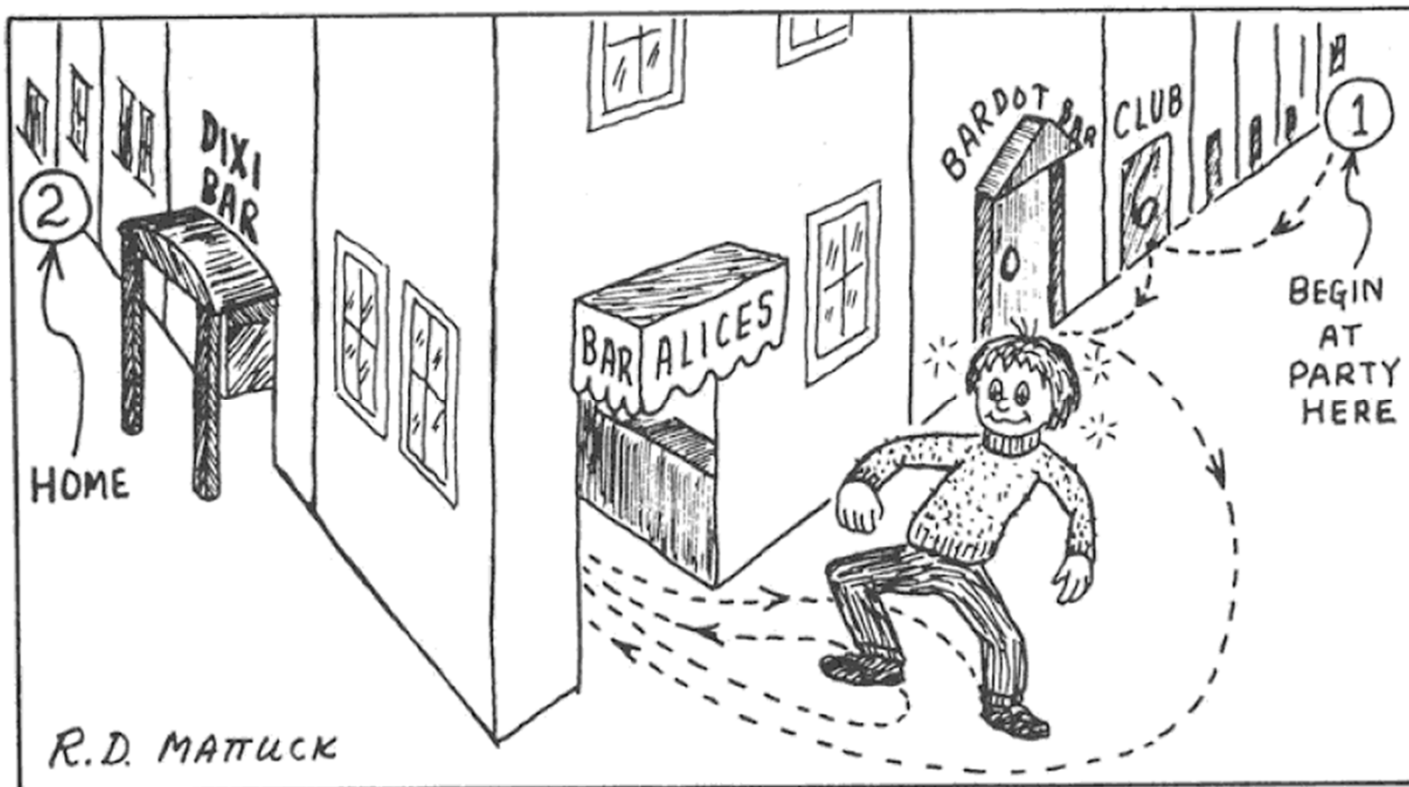
A strict drunker case: Random Walk Problem



2nd way:

$$P_0(A,1) \times P(A) \times P_0(2,A)$$

A strict drunker case: Random Walk Problem

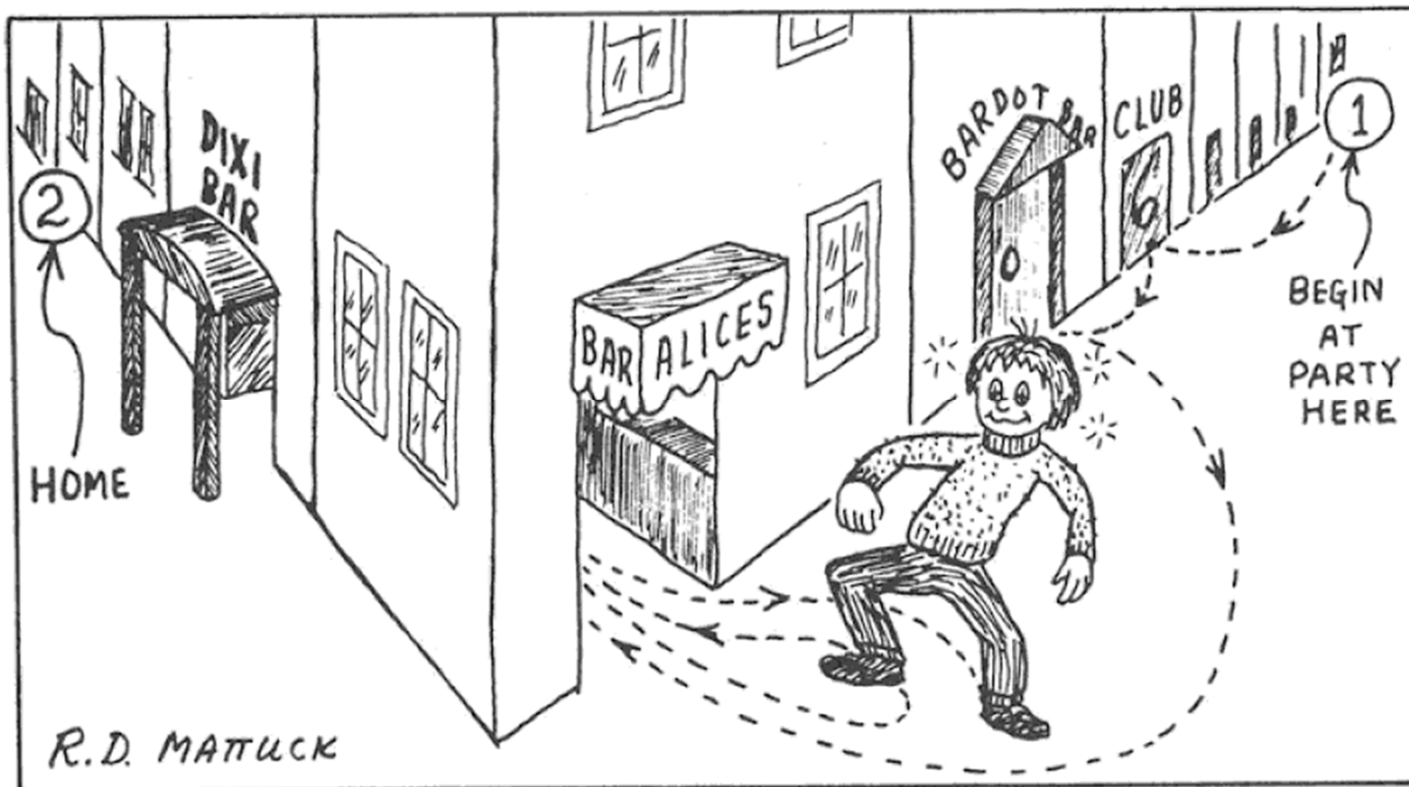


3rd way:

$$P_0(B,1) \times P(B) \times P_0(2,B)$$

... and so on

A strict drunker case: Random Walk Problem



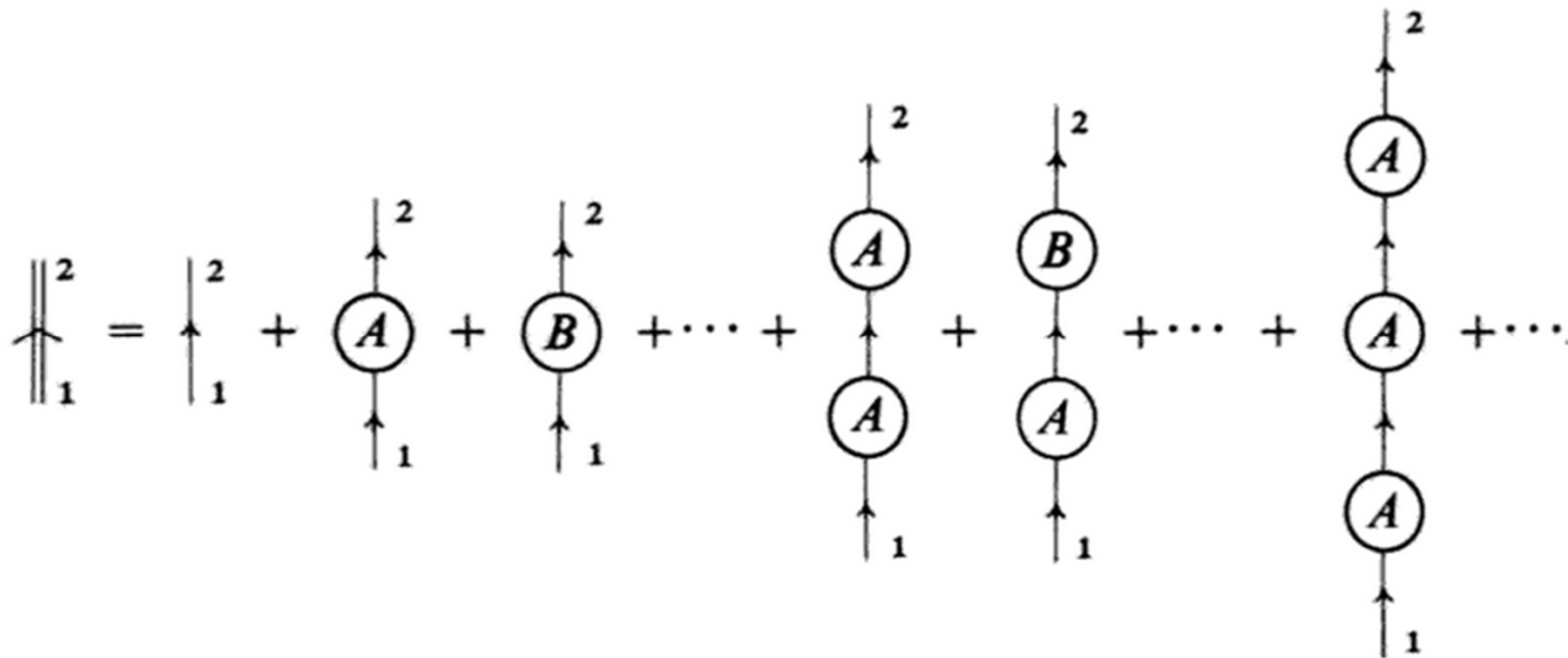
Total probability:

$$P(2,1) = P_0(2,1) + P_0(A,1) P(A) P_0(2,A) + P_0(B,1) P(B) P_0(2,B) + P_0(A,1) P(A) P_0(B,A) P(B) P_0(2,B) + \dots$$

Feynman's view

Total probability:

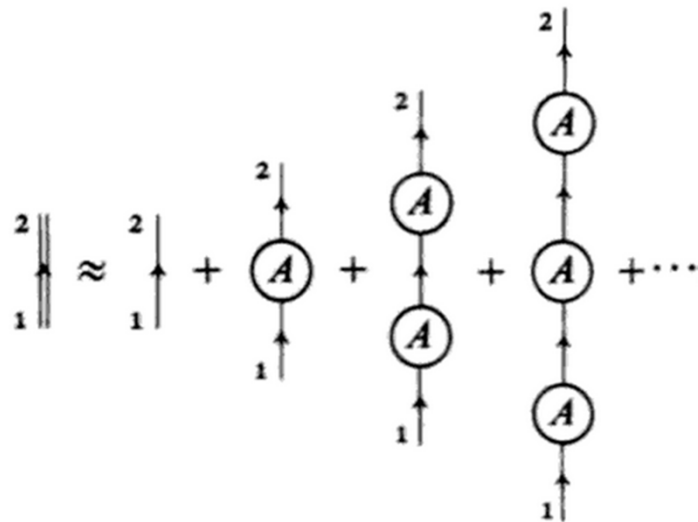
$$P(2,1) = P_0(2,1) + P_0(A,1) P(A) P_0(2,A) + P_0(B,1) P(B) P_0(2,B) + P_0(A,1) P(A) P_0(B,A) P(B) P_0(2,B) + \dots$$



Feynman's view

Diagrammatically:

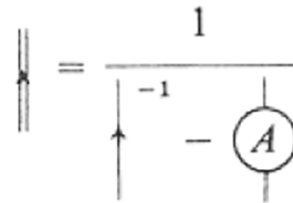
$$P(2,1) = \frac{1}{a^{-1} - P(A)}$$



$$\parallel = \uparrow \times \left\{ 1 + \begin{array}{c} \uparrow \\ \text{A} \end{array} + \left(\begin{array}{c} \uparrow \\ \text{A} \end{array} \right)^2 + \left(\begin{array}{c} \uparrow \\ \text{A} \end{array} \right)^3 + \dots \right\}$$

$$= \uparrow \times \left(\frac{1}{1 - \begin{array}{c} \uparrow \\ \text{A} \end{array}} \right) = \frac{1}{\uparrow^{-1} - \begin{array}{c} \uparrow \\ \text{A} \end{array}}$$

Quasi-particle GW Formalism



This is my Green's function propagator formalism.

Mathematically (in FT-plane):

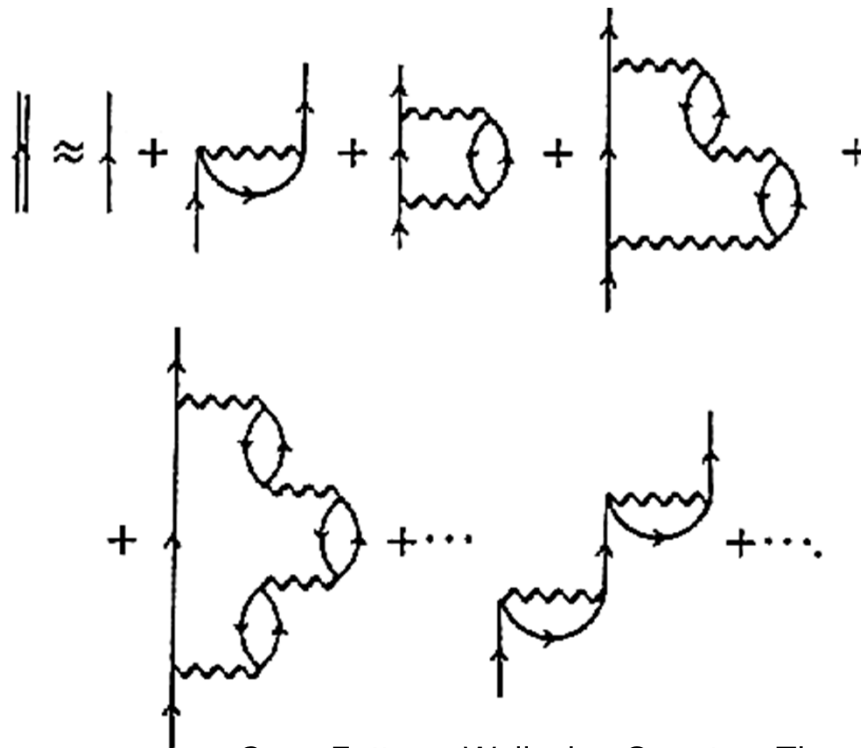
$$G^+(k, \omega) = \frac{1}{\left[G_0^+(k, \omega) \right]^{-1} - V_{kk}}$$

Quasi-particle GW Formalism

Picture example of a particle Green's function:

Ring Diagram

"Random-Phase Approximation in post- DFT"



See : Fetter & Wallecka: Quantum Theory of Many Particle Systems

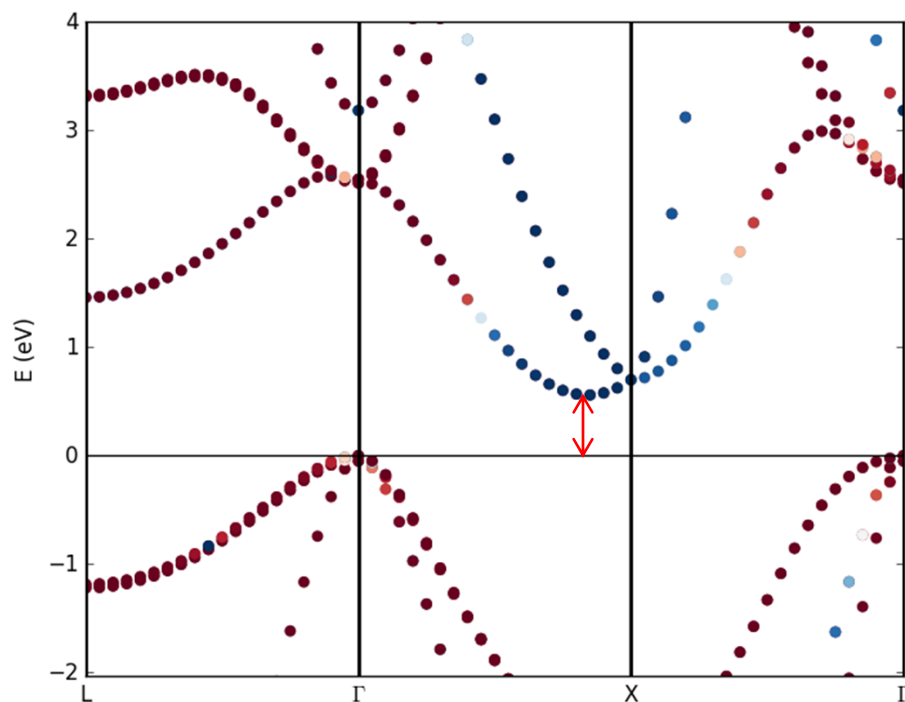
Quasi-particle GW Formalism

Hartree-Fock Term:

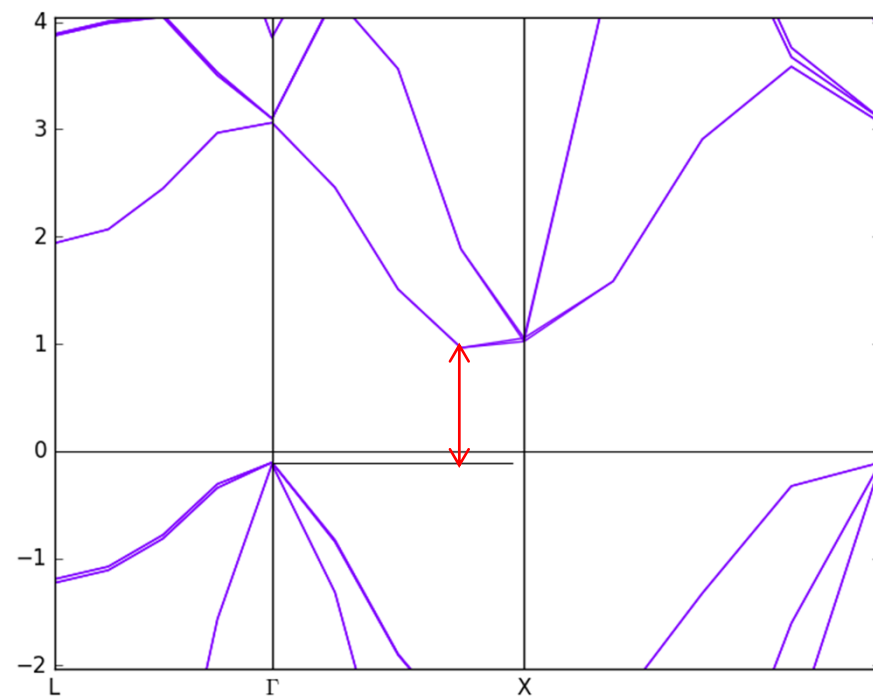
$$\begin{aligned}
 \text{Diagram 1} &\approx \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} + \text{Diagram 8} + \text{Diagram 9} + \dots \\
 &= \text{Diagram 2} \times \left[1 + \text{Diagram 3} + \text{Diagram 4} + \text{Diagram 5} + \text{Diagram 6} + \text{Diagram 7} + \text{Diagram 8} + \dots \right] \\
 &= \text{Diagram 2} \times \left[1 + \text{Diagram 3} + \text{Diagram 4} \right] + \text{Diagram 5} \times \left[\text{Diagram 3} + \text{Diagram 4} \right]^2 + \dots \\
 &= \frac{\text{Diagram 2}}{1 - \text{Diagram 3} - \text{Diagram 4}} = \frac{1}{\text{Diagram 2}^{-1} - (\text{Diagram 3} + \text{Diagram 4})}.
 \end{aligned}$$

See : Fetter & Wallecka: Quantum Theory of Many Particle Systems

Quasi-particle GW Formalism (Relativistic Correction)



Si band gap DFT $\sim 0.55\text{eV}$



Si band gap $\text{GW}_0 \sim 1.07\text{ eV}$
GW Approximation

Thank You all.