Indian Institute of Information Technology Allahabad
B. Tech. 2nd Semester
Probability and Statistics (PNS)
Quiz-II (April 23, 2024 (9:30 AM - 9:45 AM))

Enrollment No.: ..... **Total Marks: 5**

Important Instructions:

1. Attempt the question on this sheet only. No extra sheet will be provided.

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1. Suppose, we estimate the sum of n random real numbers, by rounding each to the nearest integer and adding the resulting integers. Let X_i be the error committed by rounding the i^{th} number, i.e., $X_i \sim U(-1/2, 1/2)$. Use the central limit theorem (CLT) to find the probability that the total error committed is at most $\pm\sqrt{n}$.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976

Solution:

Given :- $X_i \sim U(-1/2, 1/2)$

$E(X_i) = \frac{\alpha+\beta}{2} = 0$ — (1/2) $V(X_i) = \frac{(\beta-\alpha)^2}{12} = \frac{1}{12}$ — (1/2)

Let $X = \sum_{i=1}^{n} X_i$: Total error.

By CLT $\rightarrow$ $X \sim N(0, n/12)$ — (1)

$P[|X| \le \sqrt{n}] \approx P[-\sqrt{n} \le X \le \sqrt{n}]$ — (1)

$= P[-\sqrt{12} \le z \le \sqrt{12}]$

$= P[-3.46 \le z \le 3.46]$ — (1)

$= 2P[z \le 3.46] - 1$

$= 2 \times 0.99973 - 1$ — (1)

$= 0.99946$