Practice Set - I

1. Items coming off a production line are marked defective (D) or non-defective (N). Items are observed and their condition noted. This is continued until two consecutive defectives are produced or four items have been checked, which ever occurs first. Describe the sample space for this experiment.

2. A particle starts at the origin and moves to and from on a straight line. At any move it jumps either 1 unit to the right or 1 unit to the left each with probability. All successive moves are/ independent. Given that the particle is at the origin at the 1/2. All successive moves are independent. Given that the particle is at the origin at the completion of the 6th move, find the probability that it never occupied a position to the left of the origin during previous moves.

3. A biased coin with probability p; $0 < p < 1$, of success (head) is tossed until for the first time the same result occurs three times in succession (that is, three heads or three tails in succession). Find the probability that the game will end at the seventh throw.

4. Show that the probability that exactly one of the events A or B occurs is equal to $P(A) + P(B) - 2P(A \cap B)$.

5. The coefficients a, b and c of the quadratic equation $ax^2 + bx + c = 0$ are determined by rolling a fair die three times in a row. What is the probability that both roots of the equation are real?. What is the probability that both roots of the equation are complex?

6. Suppose that n independent trials, each of which results in any of the outcomes 0, 1 and 2, with respective probabilities 0.3, 0.5 and 0.2, are performed. Find the probability that both outcome 1 and outcome 2 occur at least once.

7. An urn contains balls numbered from 1 to N. A ball is randomly drawn.
   (a) What is the probability that the number on the ball is divisible by 3 or 4?
   (b) What happens to the probability from the previous question when $n \to \infty$?

8. An Integrated MTech student has to take 5 courses a semester for 10 semesters. In each course he/she has a probability 0.5 of getting an 'A' grade. Assuming the grades to be independent in each course, what is the probability that he/she will have all 'A' grades in at least one semester.

9. In an electronics repair shop there are 10 TVs to be repaired 3 of which are from brand A, 3 from brand B and 4 from brand C. The order in which the TVs are repaired is random.
   (a) Find the probability that a TV from the brand A will be the first one to be repaired?
   (b) Find the probability that all three TVs from the brand B will be first repaired?
   (c) Find the probability that the TVs will be repaired in the order CBACBACABC?

10. Pick a number x at random out of the integers 1 through 30. Let A be the event that x is even, B that x is divisible by 3 and C that x is divisible by 5. Are the events A, B and C independent?

11. An omnibus company always requires its drivers to wait for 10 minutes at a particular bus stop. The bus you hope to get arrives at this stop anywhere between noon and 1 PM. Assume that you arrive at the stop randomly between 12:30 PM and 1:30 PM and plan to spend at most 10 minutes waiting for the bus. What is the probability that you catch your bus on any day?

12. The base and altitude of a right triangle are obtained by picking points randomly from $[0, a]$ and $[0, b]$, respectively. Find the probability that the area of the triangle so formed will be less than $ab = 4$.

13. Three players A,B and C play a series of games, none of which can be drawn and their probability of winning any game are equal. The winner of each game scores 1 point and the series is won by the player who first scores 4 points. Out of the first three games A won 2 games and B won 1 game. Find the probability that C will win the series.

14. A point P is randomly placed in a square with side of 1 cm. Find the probability that the distance from P to the nearest side does not exceed x cm.

15. Let there be n people in a room and p denote the probability that there are no common birthdays. Find an approximate value of p for $n = 10$.

16. Suppose a lift has 3 occupants A, B and C and there are three possible floors (1,2 and 3) on which they can get out. Assuming that each person acts independently of the others and that each person has an equally likely chance of getting off at each floor, calculate the probability that exactly one person will get out on each floor.

17. If n men, among whom are A and B , stand in a row, what is the probability that there will be exactly r men between A and B ?

18. In a town of $n+1$ inhabitants, a person tells a rumor to a second person, who in turn tells it to a third person, and so on. At each step the recipient of the rumor is chosen at random from the n people available. Find the probability that the rumor will be told r times without

    (a) returning to the originator,

    (b) being repeated to any person. Do the same problem when at each step the rumor is told to a gathering of N randomly chosen people.

    (c) 2 points are taken at random and independently of each other on a line segment of length m. Find the probability that the distance between the 2 points is less than $m/3$.

    (d) n points are taken at random and independently of one another inside a sphere of radius R. What is the probability that the distance from the centre of the sphere to the nearest point is not less than r?

(e) A car is parked among N cars in a row, not at either end. On his return, the owner finds that exactly r of the N places are still occupied. What is the probability that both neighboring places are empty?

19. Box I contains 3 red and 2 blue marbles while Box II contains 2 red and 8 blue marbles. A fair coin is tossed. If the coin turns up heads, a marble is chosen from Box I; if it turns up tails, a marble is chosen from Box II. Find the probability that a red marble is chosen.

20. 70% of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose that a light aircraft has disappeared. If it has an emergency locator, what is the probability that it will be discovered?