1(a) Min $W = 10y_1 + 8y_2$

subject to, $y_1 + 2y_2 \geq 5$

$2y_1 - y_2 \geq 12$

$y_1 + 3y_2 \geq 4$ — 2

$y_1 \geq 0$, y_2 unrestricted.

1(b) Min $W = 5y_1 + 3y_2 + 8y_3$

subject to, $y_1 - y_2 + 4y_3 = 5$

$2y_1 + 5y_2 + 7y_3 \geq 6$ — 2

y_1 unrestricted , $y_2 \leq 0$, $y_3 \geq 0$

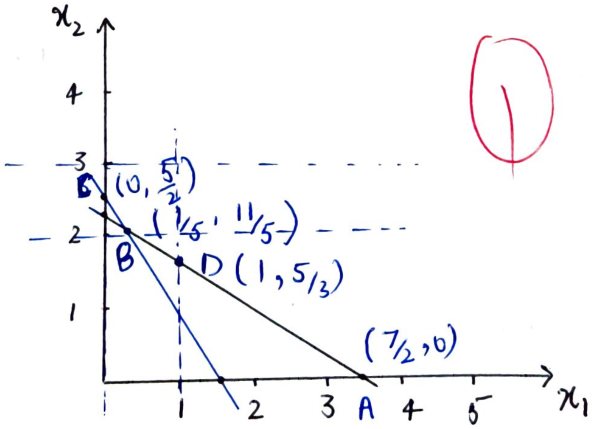
2. Min $Z = 5x_1 + 4x_2$

Subject to, $3x_1 + 2x_2 \geq 5$

$2x_1 + 3x_2 \geq 7$

x_1 and x_2 are non-negative integers.

Relaxing the integer conditions, the optimal non-integer solution is given by -



1

A	$(7/2, 0)$	$35/2$
B	$(1/5, 11/5)$	$49/5$
C	$(0, 5/2)$	10

$x_1 = \frac{1}{5}$, $x_2 = \frac{11}{5}$, $Z = \frac{49}{5}$

The value of Z represent initial lower bound. choosing x_1 as branching variable, two new constraints $x_1 \leq 0$, $x_1 \geq 1$ are created.

by adding in the given set of constraints as shown below.

LPP

LPP Subproblem 1

$$\text{Min } Z = 5x_1 + 4x_2$$

s.t., $3x_1 + 2x_2 \geq 5$

$2x_1 + 3x_2 \geq 7$

$x_1 \leq 0$ i.e. $x_1 = 0$

$(0, 5/2)$ $Z = 10$

LPP subproblem 2

$$\text{Min } Z = 5x_1 + 4x_2$$

s.t., $3x_1 + 2x_2 \geq 5$

$2x_1 + 3x_2 \geq 7$

$x_1 \geq 1$

$D(1, 5/3)$, $Z = \frac{35}{3} = 11.7$

If branching is done here we get $Z \geq 12$, which is already found

LPP subproblem 3

$$\text{Min } Z = 5x_1 + 4x_2$$

s.t., $3x_1 + 2x_2 \geq 5$

$2x_1 + 3x_2 \geq 7$

$x_1 = 0$

$x_2 \leq 2$

Infeasible as $x_1 = 0$

LPP subproblem 4

$$\text{Min } Z = 5x_1 + 4x_2$$

s.t., $3x_1 + 2x_2 \geq 5$

$2x_1 + 3x_2 \geq 7$

$x_1 = 0$

$x_2 \geq 3$

$x_1 = 0$, $x_2 = 3$

$Z = 12$

Integer solution

(Fathomed node)

(3) $\text{Max } Z = x_1 + 2x_2$

s.t., $x_1 + x_2 \leq 7$

$2x_1 \leq 11$

$2x_2 \leq 7$

x_1, x_2 are non-negative integers.

	x_1	x_2 ↓	S_1	S_2	S_3	RHS	
Z	-1	-2	0	0	0	0	
S_1	1	1	1	0	0	7	7/1
S_2	2	0	0	1	0	11	-
← S_3	0	2	0	0	1	7	7/2

	x_1 ↓	x_2	S_1	S_2	S_3		
Z	-1	0	0	0	1	7	
S_1	1	0	1	0	$-1/2$	$7/2$	$7/2$
S_2	2	0	0	1	0	11	$11/2$
x_2	0	1	0	0	$1/2$	$7/2$	$-$

	x_1	x_2	S_1	S_2	S_3	
Z	0	0	1	0	$1/2$	$21/2$
x_1	1	0	1	0	$-1/2$	$7/2$
S_2	0	0	-2	1	1	4
x_2	0	1	0	0	$1/2$	$7/2$

$$x_1 = \frac{7}{2} = 3 + \frac{1}{2}$$

$$x_2 = \frac{7}{2} = 3 + \frac{1}{2}$$

Constraints corresponding x_2

$$0 \cdot x_1 + 1 \cdot x_2 + 0 \cdot S_1 + 0 \cdot S_2 + \frac{1}{2} S_3 = \frac{7}{2}$$

Gromory's constraints,

$$\frac{1}{2} - \frac{1}{2} S_3 \leq 0$$

$$\Rightarrow -\frac{1}{2} S_3 \leq -\frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} S_3 + x_{G_1} = -\frac{1}{2} \quad -(1)$$

	x_1	x_2	S_1	S_2	S_3 ↓	x_{G_1}	
Z	0	0	1	0	$1/2$	0	$21/2$
x_1	1	0	1	0	$-1/2$	0	$7/2$
S_2	0	0	-2	1	1	0	4
x_2	0	1	0	0	$1/2$	0	$7/2$
x_{G_1}	0	0	0	0	$-1/2$	1	$-1/2$ → leaving

Applying dual-simplex

	x_1	x_2	S_1	S_2	S_3	x_{G_1}	
Z	0	0	1	0	0	1	10
x_1	1	0	1	0	0	-1	4
S_2	0	0	-2	1	0	2	3
x_2	0	1	0	0	0	1	3
S_3	0	0	0	0	1	-2	1

Optimal solution is $x_1 = 4$, $x_2 = 3$ & $Z = 10$

(4) The transportation cost matrix will include the given transportation cost plus variable cost. then, the initial basic feasible sol^n is given by – (5)

	I	II	III	IV	V	Supply
A	20	18 (80)	18 (20)	21	19	100
B	21	22	23 (65)	20	24 (60)	125
C	18 (60)	19	21	18 (105)	19 (10)	175
Demand	60	80	85	105	70	400

Total cost $= 18 \times 60 + 18 \times 80 + 18 \times 20 + 23 \times 65 + 18 \times 105$
$+ 24 \times 60 + 19 \times 10$
$= 7895$ — 2

	I	II	III	IV	V	Supply	
A	20	18 (80)	18 (20)	21	19	100	$u_1 = 0$
B	21	22	23 (65)	+20	−24 (60)	125	$u_2 = 5$
C	18 (60)	19	21	−18 (105)	+19 (10)	175	$u_3 = 0$
Demand	60	80	85	105	70		
	$V_1 = 18$	$V_2 = 18$	$V_3 = 18$	$V_4 = 18$	$V_5 = 19$		

— 2

Since the number of occupied cells (i.e. 7) is equal to $m + n - 1 = 7$ in feasible sol^n, there the sol^n is non-degenerate.

Now, calculating the opportunity cost.

$d_{11} = 2$, $d_{14} = 3$, $d_{15} = 0$, $d_{21} = -2$, $d_{22} = -1$

$d_{24} = -3$, $d_{32} = 1$ $d_{33} = 3$.

The value $d_{24} = -3$ indicates that the total transportation cost can be reduced.

The new transportation schedule so obtained is,

	I	II	III	IV	V	Supply	
A	20	$-$18(80)	+18(20)	21	19	100	$u_1 = -3$
B	21	22	$-$23(65)	+20(60)	24	125	$u_2 = 2$
C	18(60)	19+	21	$-$18(45)	19(70)	175	$u_3 = 0$
Demand	60	80	85	105	70		
	$v_1 = 18$	$v_2 = 21$	$v_3 = 21$	$v_4 = 18$	$v_5 = 19$		

— 2

$d_{11} = 5,\ d_{14} = 6,\ d_{15} = 3,\ d_{21} = 1,\ d_{22} = -1$

$d_{25} = 3 \quad d_{32} = -2 \quad d_{33} = 0$

	I	II	III	IV	V	Supply	
A	20	$-$18(35)	+18(65)	21	19	100	$u_1 = -1$
B	+21	22	$-$23(20)	20(105)	24	125	$u_2 = 4$
C	$-$18(60)	+19(45)	21	18	19	175	$u_3 = 0$
Demand	60	80	85	105	70		
	$v_1 = 18$	$v_2 = 19$	$v_3 = 19$	$v_4 = 16$	$v_5 = 19$		

— 2

~~$d_{11} = 20 = 1$.~~

$d_{11} = 3,\ d_{14} = 6,\ d_{15} = 1,\ d_{21} = -1,\ d_{22} = -1,\ d_{25} = 1$

$d_{33} = 2,\ d_{34} = 2$

	I	II	III	IV	V	Supply	
A	20	18(15)	18(85)	21	19	100	$u_1 = -1$
B	21(20)	22	23	20(105)	24	125	$u_2 = 3$
C	18(40)	19(65)	21	18	19(70)	175	$u_3 = 0$
Demand	60	80	85	105	70		
	$v_1 = 18$	$v_2 = 19$	$v_3 = 19$	$v_4 = 17$	$v_5 = 19$		

$d_{11} = 20 - (-1 + 18) = 3,\ d_{14} = 5,\ d_{15} = 1,\ d_{22} = 0,\ d_{23} = 1$

$d_{25} = 2,\ d_{33} = 2,\ d_{34} = 1$

Since, none of the unoccupied cells have a negative opportunity cost value, therefore the total transportation cost cannot be reduced.

Total cost $= 18 \times 15 + 18 \times 85 + 22 \times 20 + 20 \times 105 + 18 \times 60$
$+ 19 \times 45 + 19 \times 70$
$= 7605.$ — 2

(5) (a) Since cost matrix is not balanced, therefore add one dummy column (road, R_5) with a zero cost elements. The revised cost matrix is given as–

	R_1	R_2	R_3	R_4	R_5
C_1	9	14	19	15	0
C_2	7	17	20	19	0
C_3	9	18	21	18	0
C_4	10	12	18	19	0
C_5	10	15	21	16	0

	R_1	R_2	R_3	R_4	R_5
C_1	2	2	1	$\boxed{0}$	0
C_2	$\boxed{0}$	5	2	4	0
C_3	2	6	3	3	$\boxed{0}$
C_4	3	$\boxed{0}$	0	4	0
C_5	3	3	3	1	0

— 2

$$\begin{matrix} 2 & 2 & 1 & \boxed{0} & 1 \\ \boxed{0} & 5 & 2 & 4 & 1 \\ 1 & 5 & 2 & 2 & \boxed{0} \\ 3 & \boxed{0} & 0 & 4 & 1 \\ 2 & 2 & 2 & 0 & 0 \end{matrix}$$

$$\begin{matrix} 2 & 1 & \boxed{0} & 0 & 1 \\ \boxed{0} & 4 & 1 & 4 & 1 \\ 1 & 4 & 1 & 2 & \boxed{0} \\ 4 & \boxed{0} & 0 & 5 & 2 \\ 2 & 1 & 1 & \boxed{0} & 0 \end{matrix}$$ — 2

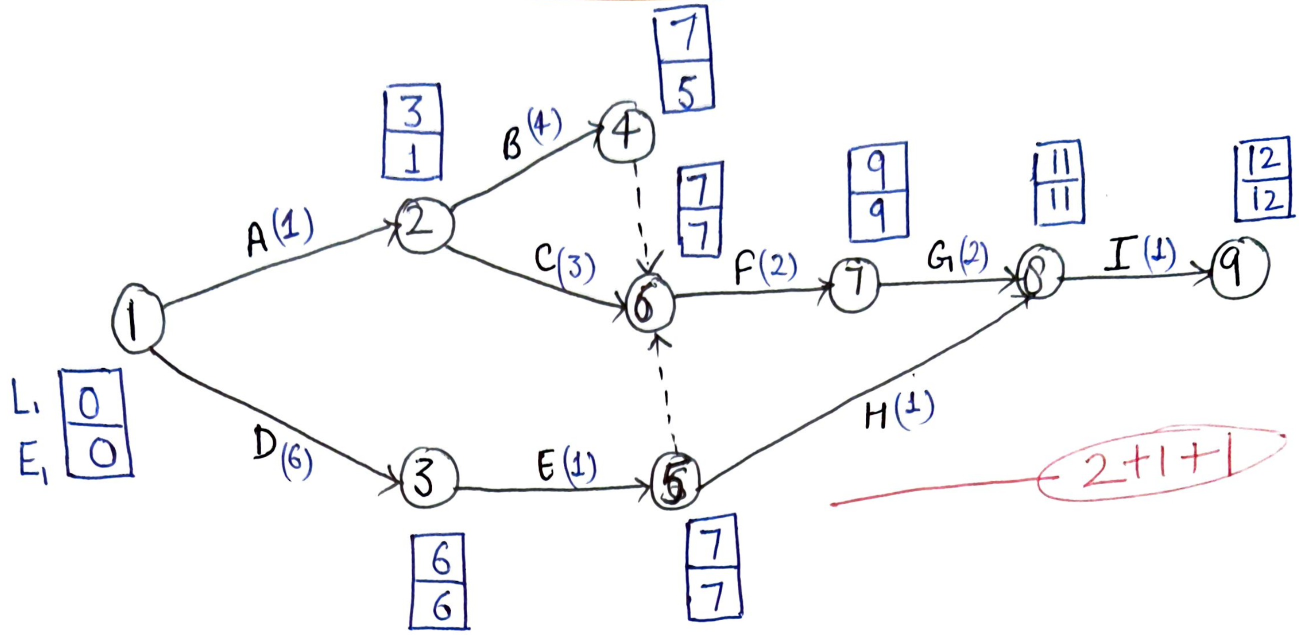
The total min cost & optimal assignment are as follows -

Road	Contractor	Cost (Rs in lakh)
R_1	C_1	7
R_2	C_2	12
R_3	C_3	19
R_4	C_4	16
R_5	C_5	0
		54

— 1

(b) Since the total cost exceeds 50 lakh, the excess amount of Rs 4 lakh (= 54 - 50) is to be sought as supplementary grant. — 1

(c) Contracter C_3 who has been assigned to dummy row R_5 loses out in bid. — 1



2+1+1

tivity ,j)	Duration (t_{ij})	Earliest time		Latest time		Float	
		Start (E_i)	Finish ($E_i + t_{ij}$)	Start ($L_j - t_{ij}$)	Finish (L_j)	Total $(L_j - t_{ij}) - E_i$	Free $(E_j - E_i) - t_{ij}$
2 A	1	0	1	2	3	2	0
4 B	4	1	5	3	7	2	0
6 C	3	1	4	4	7	3	3
-8 H	1	7	8	10	11	3	~~3~~ 2

2

The critical path of the project is -

1 - 3 - 5 - 6 - 7 - 8 - 9

and the critical activities are D, E, F, G, I

Total project completion time is 12 weeks.

2