

Indian Institute of Information Technology, Allahabad

BTECH IT/ECE/RGIT THIRD SEMESTER, 2016Campus: City

IT

Operating System

(Time allowed: TWO hours)

NOTE: There are six questions in this question paper. Answer **All** questions. The subquestions of Question 1 (multiple choice questions) are worth 0.5 mark each with a penalty of 0.25 marks for incorrect answer: the marks for each sub question in Question 2 (multiple choice questions) are worth 2 mark each. However, for each sub question in Question 2 you need to justify your answer, failing which your answer will not be considered. The marks of rest of the questions have been provided alongside each question.

SECTION A

MULTIPLE CHOICE QUESTIONS

Circle the preferred choice on the coloured answer sheet provided.

1. Let A and B and C be subsets of a universal set U . Which of the following sets is equal to the shaded region in the following Venn diagram?

(a) $(A \cup (B \cap C)) \setminus (A \cap (B \cup C)).$

(b) $A \cup (B \cap C).$

(c) $A \cap (B \cup C).$

(d) $(A \setminus B) \cap (A \setminus C).$

(e) $(A \setminus (B \cup C)) \cup (B \cap C).$

2. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Which of the following statements is NOT true?
- (a) if $g \circ f$ is 1–1 then f and g are both 1–1.
 - (b) if $g \circ f$ is 1–1 then f is 1–1.
 - (c) if $g \circ f$ is onto then g is onto.
 - (d) if $g \circ f$ is a bijection then f is 1–1 and g is onto.
 - (e) if f and g are both 1–1 then $g \circ f$ is 1–1.
3. A multiple-choice exam has 20 question, and each question has 5 possible answers. The same 5 answers for each question will appear on each exam paper, but they can appear in any order. The questions will appear in the same order on every exam paper. How many different exam papers can be printed?
- (a) $(5!)^{20}$.
 - (b) $\binom{20}{5}$.
 - (c) $\binom{20+5-1}{5}$.
 - (d) $P(20, 5)$.
 - (e) $\frac{20!}{5!}$.

SECTION B

4. (a) For each of the following propositions, determine whether it is a tautology, a contradiction or a contingent proposition. You may find it helpful to refer to the list of standard tautologies in the ATTACHMENT on page 4.
- (i) $p \rightarrow (q \wedge p)$.
 - (ii) $p \rightarrow (q \vee p)$.
 - (iii) $p \rightarrow \neg(q \wedge p)$.
 - (iv) $\neg(q \vee (q \rightarrow p))$.
- (b) Let A_1, A_2, \dots, A_n, B be propositions. What does it mean to say that the argument $A_1, A_2, \dots, A_n, \therefore B$ is *valid*? Identify the simple propositions in the following argument and determine whether or not it is valid:
- “If interest rates are not high, then house prices will rise. If interest rates are high, then the exchange rate will be low. Therefore if the exchange rate is not low then house prices will rise.”

(5 marks)

5. (a) Let $p(x, y)$ denote the predicate “ $x < y$ ”, with domain of definition $\mathbb{Z} \times \mathbb{Z}$. Translate the following statements into predicate logic.
- (i) “For every integer x there is an integer y with $x < y$.”
 - (ii) “For all integers x, y and z , if $x < y < z$ then $x < z$.”
 - (iii) “For each integer x there is an integer y such that $x < y$ but there is no integer z with $x < z < y$.”
- (b) (i) What does it mean to say that a proposition in predicate logic is a *tautology*?
- (ii) Show that $\forall x p(x) \rightarrow \exists x p(x)$ is a tautology.
- (iii) Show that $\exists x p(x) \rightarrow \forall x p(x)$ is not a tautology. (7 marks)
6. (a) Let A and B be sets, and let $f : A \rightarrow B$ be a function. Define a relation \approx on A by

$$x \approx y \quad \Leftrightarrow \quad f(x) = f(y).$$

Show that \approx is an equivalence relation.

- (b) Define the relation \approx on $\mathbb{R} \times \mathbb{R}$ by

$$(x, y) \approx (u, v) \quad \Leftrightarrow \quad x^2 + y^2 = u^2 + v^2.$$

By part (a) we know that this is an equivalence relation. For $x, y \in \mathbb{R}$, we denote the equivalence class of (x, y) by $[x, y]$. Suppose we define the sum of two equivalence classes by

$$[x, y] \oplus [u, v] = [\sqrt{x^2 + u^2}, \sqrt{y^2 + v^2}].$$

- (i) What does it mean to say that \oplus is *well-defined*?
- (ii) Show that \oplus is well-defined. (10 marks)

$p \Leftrightarrow \neg\neg p$	Double Negation
$p \wedge q \Leftrightarrow q \wedge p$	Commutative
$\forall q \Leftrightarrow q \vee p$	Laws
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$	De Morgan's
$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$	Laws

Standard tautologies

Candidate's Name: _____ ID No: _____

SECTION A

Circle the preferred answer.

If you make a mistake, mark a cross through your wrong choice and circle your next alternative.

- | | | | | | |
|----|---|---|---|---|---|
| 1. | a | b | c | d | e |
| 2. | a | b | c | d | e |
| 3. | a | b | c | d | e |
