Distributed Mutual Exclusion

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(*) Large parts of this lecture borrowed from Sukumar Ghosh's book.

Distributed Mutual Exclusion

Mostly from Sukumar Ghosh's book and handsout:

1 – Introduction

2 – Solutions Using Message Passing

3 – Token Passing Algorithms

4 – The Group Mutual Exclusion Problem

Also in Ghosh's book (not covered by this lecture):

- Solution on the shared memory model
  - Peterson algorithm
- Mutual exclusion using special instruction
  - Solution using Test-and-Set
  - Solution using DEC LL and SC instruction

Why Do We Need Distributed Mutual Exclusion (DME) ?

Atomicity exists only up to a certain level:

- Atomic instructions
- Defines the granularity of the computation
  - Types of possible interleaving
    - Assembly Language Instruction?
    - Remote Procedure Call?

Why Do We Need Distributed Mutual Exclusion (DME) ?

Some applications are:

- Resource sharing
- Avoiding concurrent update on shared data
- Controlling the grain of atomicity
- Medium Access Control in Ethernet
- Collision avoidance in wireless broadcasts

Why Do We Need Distributed Mutual Exclusion (DME) ?

Example: Bank Account Operations

shared n : integer

Process P

Account receives amount nP

Computation: n = n +nP:

P1. Load Reg_P, n

P2. Add Reg_P, nP

P3. Store Reg_P, n

Process Q

Account receives amount nQ

Computation: n = n +nQ:

Q1. Load Reg_Q, n

Q2. Add Reg_Q, nQ

Q3. Store Reg_Q, n

Why Do We Need DME? (example cont'd)

Possible Interleaves of Executions of P and Q:

- 2 give the expected result n= n + nP + nQ
  - P1, P2, P3, Q1, Q2, Q3
  - Q1, Q2, Q3, P1, P2, P3
- 5 give erroneous result n = n+nQ
  - P1, Q1, P2, Q2, P3, Q3
  - P1, P2, Q1, Q2, P3, Q3
  - P1, Q1, Q2, P2, P3, Q3
  - Q1, P1, Q2, P2, P3, Q3
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- 5 give erroneous result n = n + nP
  - Q1, P1, Q2, P2, Q3, P3
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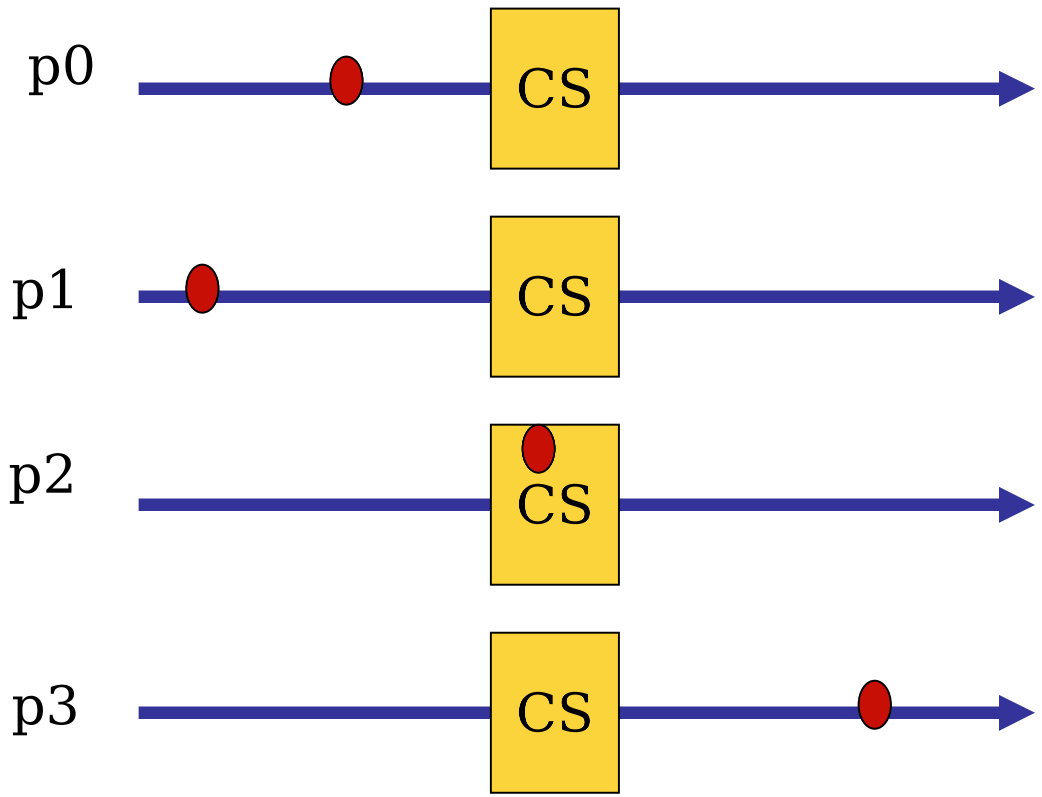
  - Q1, P1, P2, Q2, Q3, P3
  - P1, Q1, P2, Q2, Q3, P3
  - P1, P2, Q1, Q2, Q3, P3

Solutions to the Mutual Exclusion Problem



Solutions to the Mutual Exclusion Problem (2)

- 2 classes of solutions:
  - Ad hoc solutions
  - Solutions based on non-preemptible resource allocation
- Both classes require a special code around the critical section

Ad-hoc case	Non-preempt. resource case
Enter protocol	**Request resource**
<critical section>	<critical section>
Exit protocol	**Release resource**

1 – Introduction

2 – Solutions Using Message Passing

3 – Token Passing Algorithms

4 – The Group Mutual Exclusion Problem

Introduction

Problem formulation

- Assumptions
  - n processes (n>1), numbered 0 ... n-1, noted Pi
    - form a distributed system
  - topology: completely connected graph (Kn)
  - each Pi **periodically** wants:
    1. enter the Critical Section (CS)
    2. execute the CS code
    3. eventually exits the CS code
- Devise a protocol that satisfies:

  ME1 : Mutual Exclusion

  ME2 : Freedom from deadlock

  ME3 : Progress

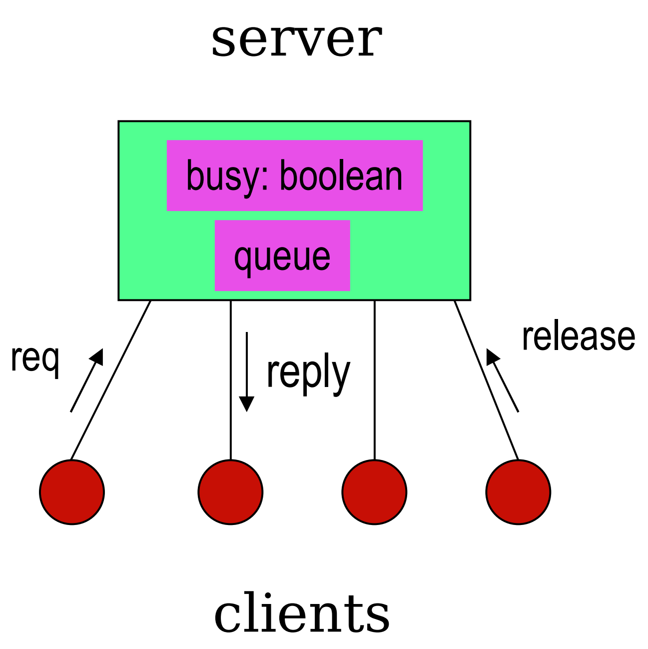
Introduction (2)

Zoom on Conditions...

- ME1 : Mutual Exclusion
  - At most one process can remain in CS at any time
  - Safety property
- ME2 : Freedom from deadlock
  - At least one process is eligible to enter CS
  - Safety property
- ME3 : Progress
  - Every process trying to enter must eventually succeed
  - Liveness property
  - Violation called *livelock* or *starvation*
- A measure of fairness: bounded waiting
  - Specifies an upper bound on the number of times a process waits for its turn to enter SC

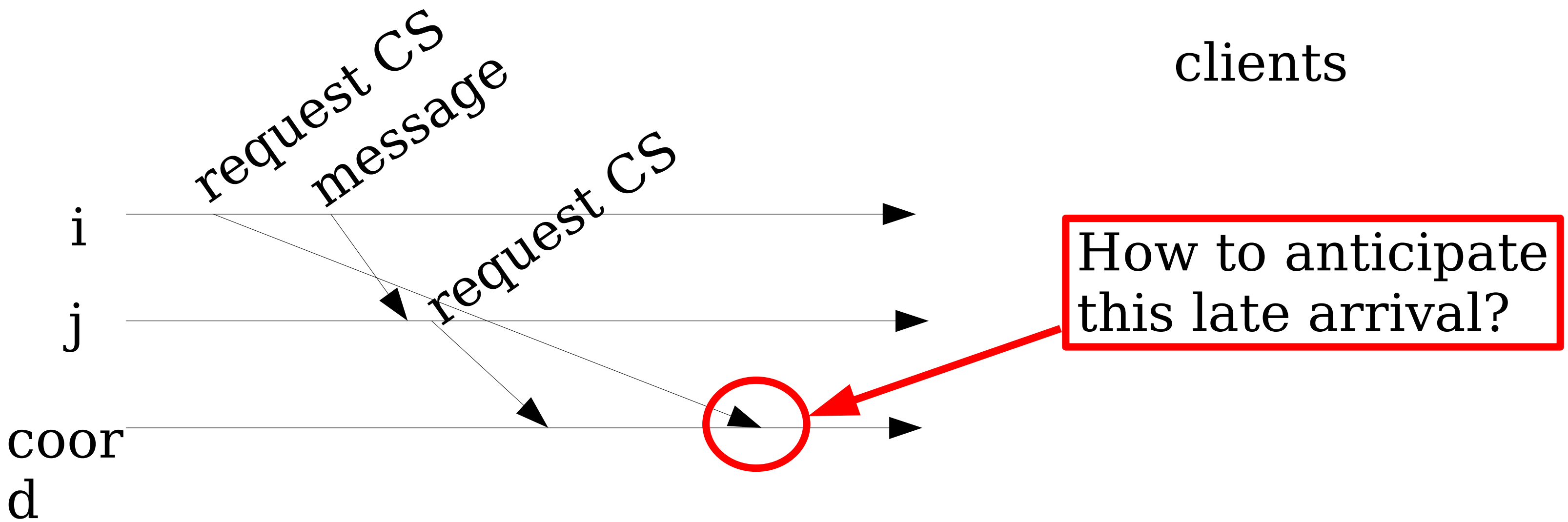
Introduction (3)

Centralized Solutions?



- Use a coordinator process
  - External process
  - One of the Pi-s
- Problems:
  - Single point of failure
  - Unable to achieve FIFO fairness

Example:



Lamport's Solution

- Assumptions:
  - Each communicaion channel is FIFO
  - Each process maintains a request queue Q
- Algorithm described by 5 rules

LA1. To request entry, send a time-stamped message to **every** other process and **enqueue to local Q**

LA2. Upon reception place request in Q and send time-stamped ACK but **once out of CS**

(possibly immediately if already out)

LA3. Enter CS when:

1. request first in Q (chronological order)
2. all ACK received from others

LA4. To exit CS, a process must:

1. delete request from Q
2. send time-stamped release message to others

LA5. When receiving a release msg, remove request from Q

Analysis of Lamport's Solution

Can you show that it satisfies all the properties (i.e. ME1, ME2, ME3) of a correct solution?

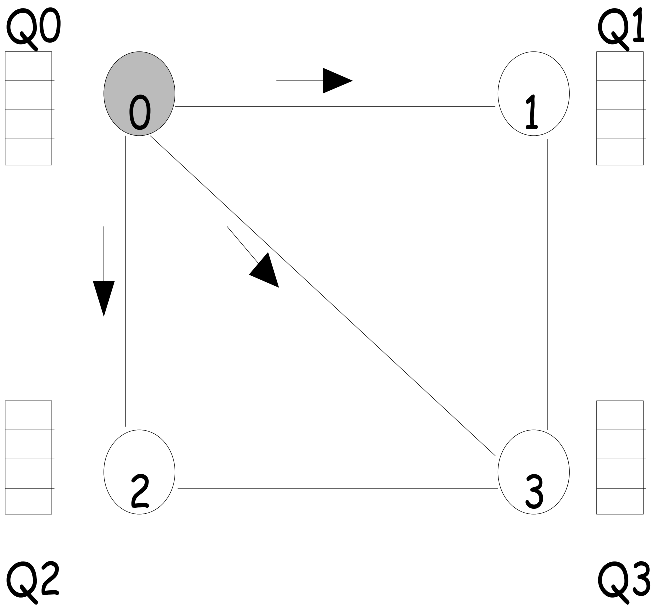
Observation. *Processes taking a decision to enter CS must have **identical views** of their local queues, when all ACKs have been received.*

Proof of ME1. *At most one process can be in its CS at any time.*

Suppose not, and both j,k enter their CS. This implies

- j in CS ⇒ Qj.ts.j < Qk.ts.k
- k in CS ⇒ Qk.ts.k < Qj.ts.j

Impossible.



Analysis of Lamport's Solution (2)

Proof of ME2. *(No deadlock)*

The waiting chain is acyclic.

i waits for j

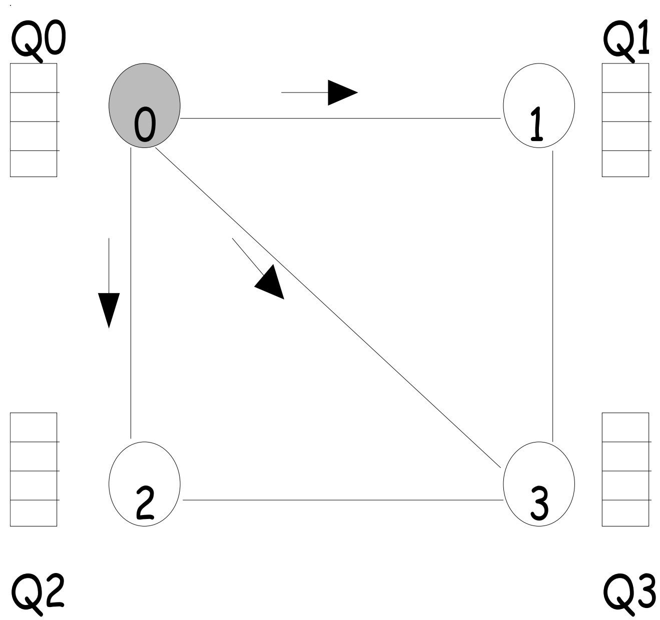
⇒ i is behind j in all queues

(or j is in its CS)

⇒ j does not wait for i

Proof of ME3. *(progress)*

New requests join the end of the queues, so new requests do not pass the old ones

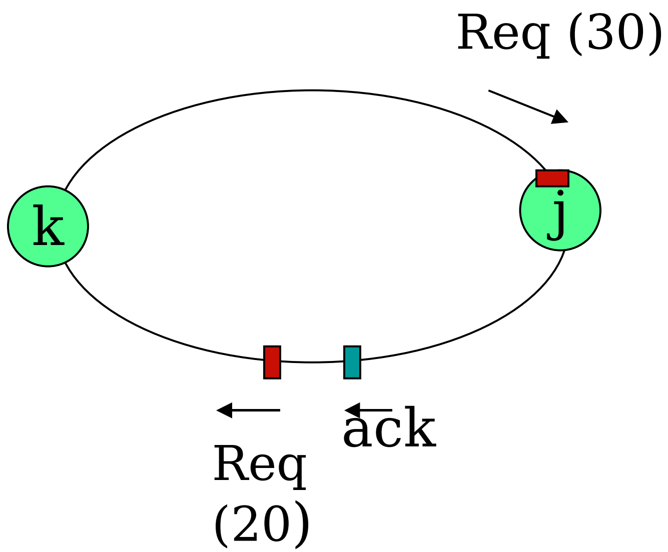


Analysis of Lamport's Solution (3)

Proof of FIFO fairness.

timestamp (j) < timestamp (k)

⇒ j enters its CS before k does so



Suppose not. So, k enters its CS before j. So k did not receive j's request. But k received the ack from j for its own req.

This is impossible **if the channels are FIFO**

.

Message complexity = 3(N-1) (per trip to CS)

(N-1 requests + N-1 ack + N-1 release)

Ricart & Agrawala's Solution

What is new?

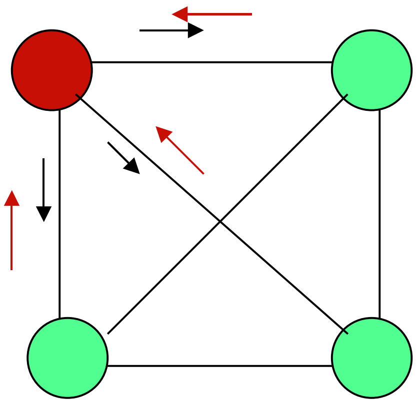
1. Broadcast a timestamped ***request*** to all.

2. Upon receiving a request, send ***ack*** if

-You do not want to enter your CS, or

-You are trying to enter your CS, but your timestamp is higher than that of the sender.

(If you are already in CS, then buffer the request)



3. **Enter CS**, when you receive ***ack*** *from all*.

4. Upon **exit from CS**, send ***ack*** to each pending request before making a new request.

(No release message is necessary)

Analysis of Ricart & Agrawala's Solution

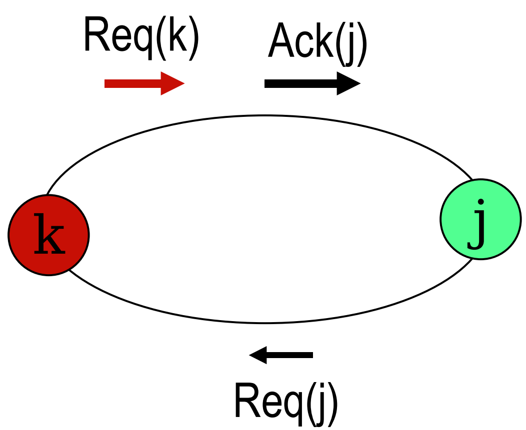
ME1. Prove that at most one process can be in CS.

ME2. Prove that deadlock is not possible.

ME3. Prove that FIFO fairness holds **even if channels are not FIFO**

Message complexity = 2(N-1)
(N-1 requests + N-1 acks - no release message)

TS(j) < TS(k)



Unbounded Time-stamps

Timestamps grow in an unbounded manner.

This makes real implementation impossible.

Can we somehow bound timestamps?

Think about it.

Maekawa's Solution

- First solution with a **sublinear** **O(sqrt N)** message complexity.
- "Close to" Ricart-Agrawala's solution, but each process is required to obtain permission from only a **subset** of peers

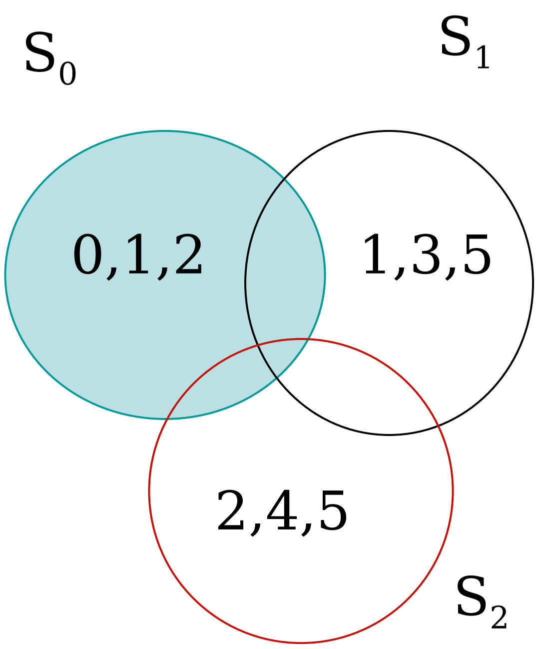
Maekawa's Algorithm

- With each process i, associate a subset S_i.Divide the set of processes into subsets that satisfy the following two conditions:

$i \in S_i$

$\forall\, i,j :\ 0 \le i,j \le n-1 ::\ S_i \cap S_j \ne \emptyset$

- **Main idea**. Each process i is required to receive permission from S_i only. Correctness requires that multiple processes will never receive permission from all members of their respective subsets.



Maekawa's Algorithm

Example. Let there be seven processes 0, 1, 2, 3, 4, 5, 6

$S_0 = \{0, 1, 2\}$

$S_1 = \{1, 3, 5\}$

$S_2 = \{2, 4, 5\}$

$S_3 = \{0, 3, 4\}$

$S_4 = \{1, 4, 6\}$

$S_5 = \{0, 5, 6\}$

$S_6 = \{2, 3, 6\}$

Maekawa's Algorithm (example cont'd)

Version 1 {Life of process I}

1. Send timestamped ***request*** to each process in $\mathbf{S_i}$.
2. Request received → send ***ack*** to process with the *lowest timestamp*. Thereafter, "**lock**" (i.e. **commit**) yourself to that process, and keep others waiting.
3. Enter CS if you receive an *ack* from **each member** in S_i.
4. To exit CS, send *release* to every process in S_i.
5. Release received → **unlock** yourself. Then send ack to the next process with the lowest timestamp.

$S_0 = \{0, 1, 2\}$

$S_1 = \{1, 3, 5\}$

$S_2 = \{2, 4, 5\}$

$S_3 = \{0, 3, 4\}$

$S_4 = \{1, 4, 6\}$

$S_5 = \{0, 5, 6\}$

$S_6 = \{2, 3, 6\}$

Analysis of Maekawa's Algorithm (version 1)

***ME1.* At most one process can enter its critical section at any time.**

Let **i** and **j** attempt to enter their Critical Sections

$\mathbf{S_i} \cap \mathbf{S_j} \neq \varphi$ there is a process $\mathbf{k} \in \mathbf{S_i} \cap \mathbf{S_j}$

Process **k** will **never** send ack to both**.**

So it will act as the arbitrator and establishes ME1

$S_0 = \{0, 1, 2\}$

$S_1 = \{1, 3, 5\}$

$S_2 = \{2, 4, 5\}$

$S_3 = \{0, 3, 4\}$

$S_4 = \{1, 4, 6\}$

$S_5 = \{0, 5, 6\}$

$S_6 = \{2, 3, 6\}$

Analysis of Maekawa's Algorithm (version 1)

ME2. No deadlock. Unfortunately deadlock is possible! Assume 0, 1, 2 want to enter their critical sections.

From $\mathbf{S_0}$**= {0,1,2}**, **0,2** send *ack* to **0**, but **1** sends *ack* to **1**;

From $\mathbf{S_1}$**= {1,3,5}**, **1,3** send *ack* to **1**, but **5** sends *ack* to **2**;

From $\mathbf{S_2}$**= {2,4,5}**, **4,5** send *ack* to **2**, but **2** sends *ack* to **0**;

Now, 0 waits for 1 **(to send a release),** 1 waits for 2 **(to send a release),** , and 2 waits for 0 **(to send a release),** . So deadlock **is** possible!

S_0 = {0, 1, 2}

S_1 = {1, 3, 5}

S_2 = {2, 4, 5}

S_3 = {0, 3, 4}

S_4 = {1, 4, 6}

S_5 = {0, 5, 6}

S_6 = {2, 3, 6}

Maekawa's Algorithm (version 2)

Avoiding deadlock

If processes always receive messages **in increasing order of timestamp**, then deadlock "could be" avoided. But this is too strong an assumption.

Version 2 uses three ***additional*** messages:

- ***failed***
- ***inquire***
- ***relinquish***

S_0 = {0, 1, 2}

S_1 = {1, 3, 5}

S_2 = {2, 4, 5}

S_3 = {0, 3, 4}

S_4 = {1, 4, 6}

S_5 = {0, 5, 6}

S_6 = {2, 3, 6}

Maekawa's Algorithm (version 2)

New features in version 2

- Send ***ack*** and set **lock** as usual.
- If **lock is set** and a request with a larger timestamp arrives, send ***failed*** *(you have no chance)*. If the incoming request has a lower timestamp, then send ***inquire*** *(are you in CS?) to the locked process*.
- Receive ***inquire*** and at least one ***failed*** message → send ***relinquish***. The recipient resets the lock.

$S_0 = \{0, 1, 2\}$

$S_1 = \{1, 3, 5\}$

$S_2 = \{2, 4, 5\}$

$S_3 = \{0, 3, 4\}$

$S_4 = \{1, 4, 6\}$

$S_5 = \{0, 5, 6\}$

$S_6 = \{2, 3, 6\}$

Comments on Maekawa's Algorithm (version 2)

- Let $K = |S_i|$. Let each process be a member of D subsets. When N = 7, K = D = 3. When K=D, N = K(K-1)+1. So $K = O(\sqrt{N})$
  (from theory of finite projective planes)

- The message complexity of Version 1 is $3\sqrt{N}$. Maekawa's analysis of Version 2 reveals a complexity of $7\sqrt{N}$

- *Sanders identified a bug in version 2 …*

Exercises

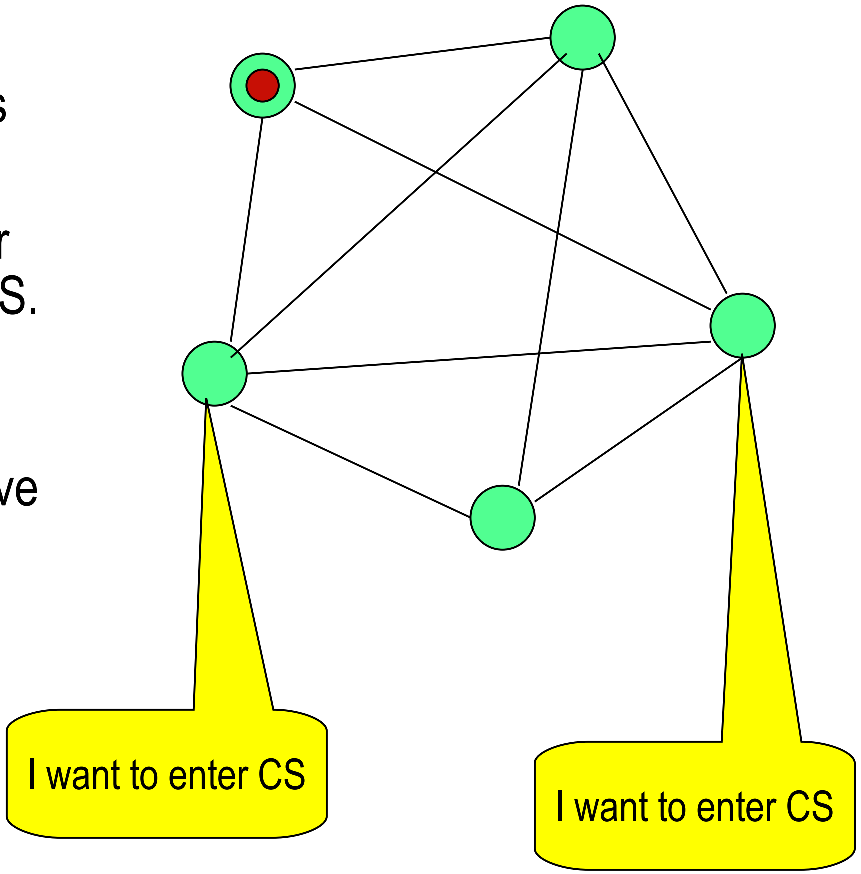
- In Ricart and Agrawala's distributed mutual exclusion algorithm, show that:
  a) Processes enter their critical sections in the order of their request timestamps
  b) Correctness is guaranteed even if the channels are not FIFO
- A Generalized version of the mutual exclusion problem in which up to L processes ($L \geq 1$) are allowed to be in their critical sections simultaneously is known as the **L-exclusion** problem. Precisely, if fewer than L processes are in the CS at any time and one more process wants to enter it, it must be allowed to do so. Modify R.-A. algorithm to solve the L-exclusion problem.

Suzuki-Kasami Solution

Completely connected network of processes

There is **one token** in the network. The holder of the token has the permission to enter CS.

Any other process trying to enter CS must acquire that token. Thus the token will move from one process to another based on demand.



Suzuki-Kasami Algorithm

Process i broadcasts **(i, num)**

Sequence number of the request

Each process maintains

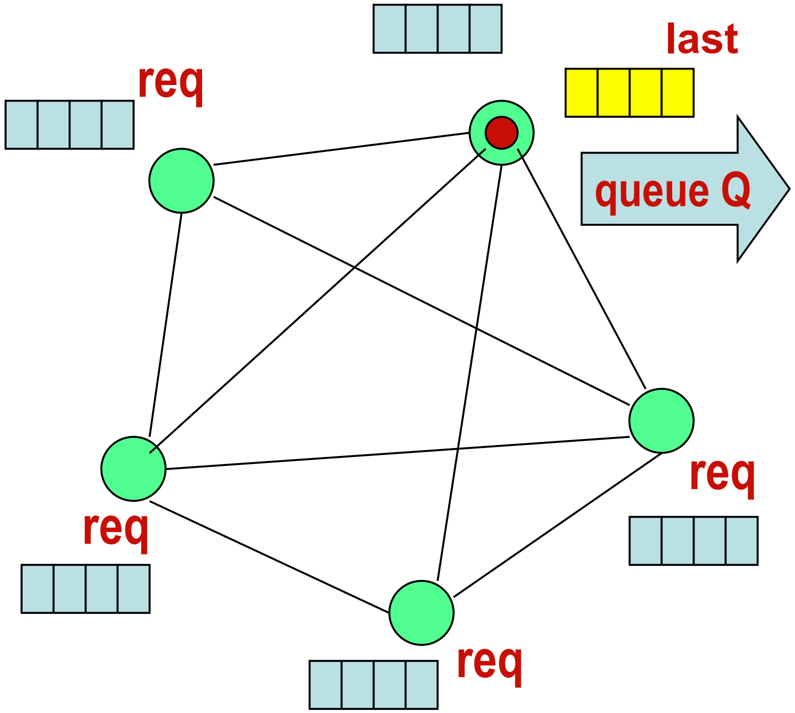
-an array **req**: **req[j]** denotes the sequence no of the ***latest request*** from process j

(Some requests will be stale soon)

Additionally, the holder of the token maintains

-an array **last**: **last[j]** denotes the sequence number of *the latest visit* to CS for process j.

- **a queue Q** of waiting processes

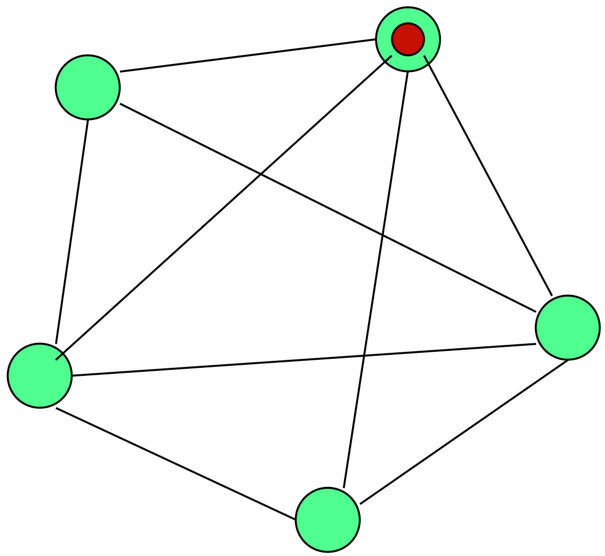


req: array[0..n-1] of integer

last: array [0..n-1] of integer

Suzuki-Kasami Algorithm (2)

When a process **i** receives a request **(k, num)** from process **k,** it sets **req[k] to max(req[k], num).**



The holder of the token

--Completes its CS

--Sets **last[i]:=** its own **num**

--Updates **Q** by retaining each process **k** only if **1+ last[k] = req[k]**

(*This guarantees the freshness of the request)*

--Sends the token to the ***head of Q***, along with the array **last** and the ***tail of Q***

Req: array[0..n-1] of integer

Last: Array [0..n-1] of integer

In fact, **token** $\equiv$ **(Q, last)**

Suzuki-Kasami Algorithm (3)

{Program of process j}

Initially, $\forall i$: req[i] = last[i] = 0

*** Entry protocol ***

req[j] := req[j] + 1

Send (j, req[j]) to all

Wait until token (Q, last) arrives

Critical Section

*** Exit protocol ***

last[j] := req[j]

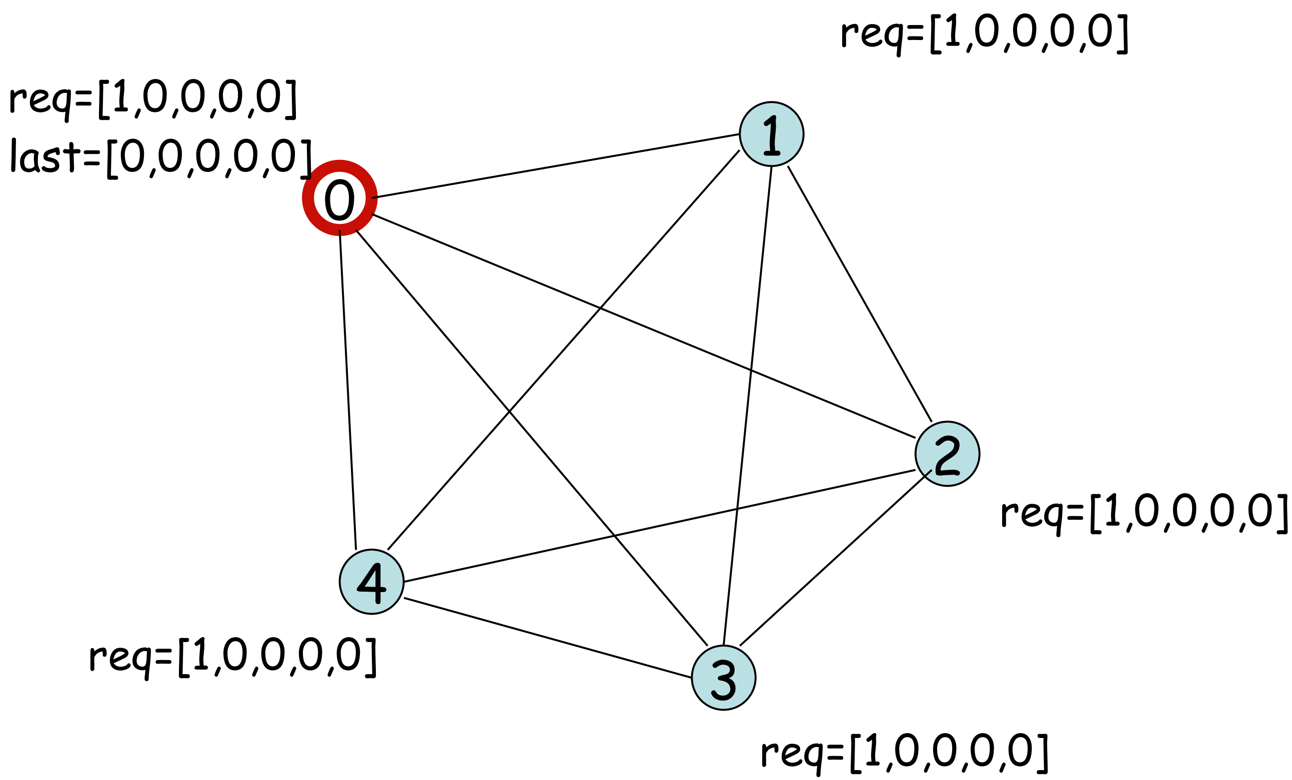
$\forall k \neq j$: $k \notin Q$ ^ req[k] = last[k] + 1 → append k to Q;

if Q is not empty → send (tail-of-Q, last) to head-of-Q **fi**

*** Upon receiving a request (k, num) ***

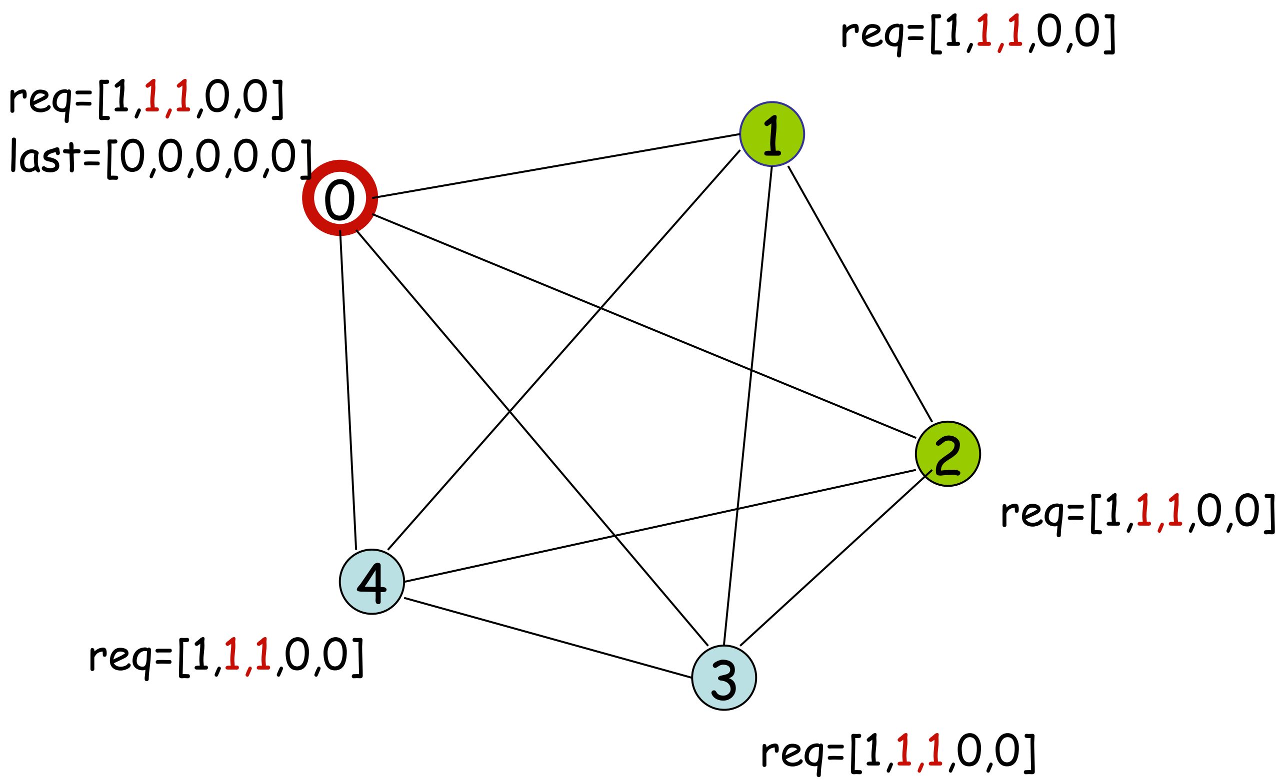
req[k] := max(req[k], num)

Example of Suzuki-Kasami Algorithm Execution



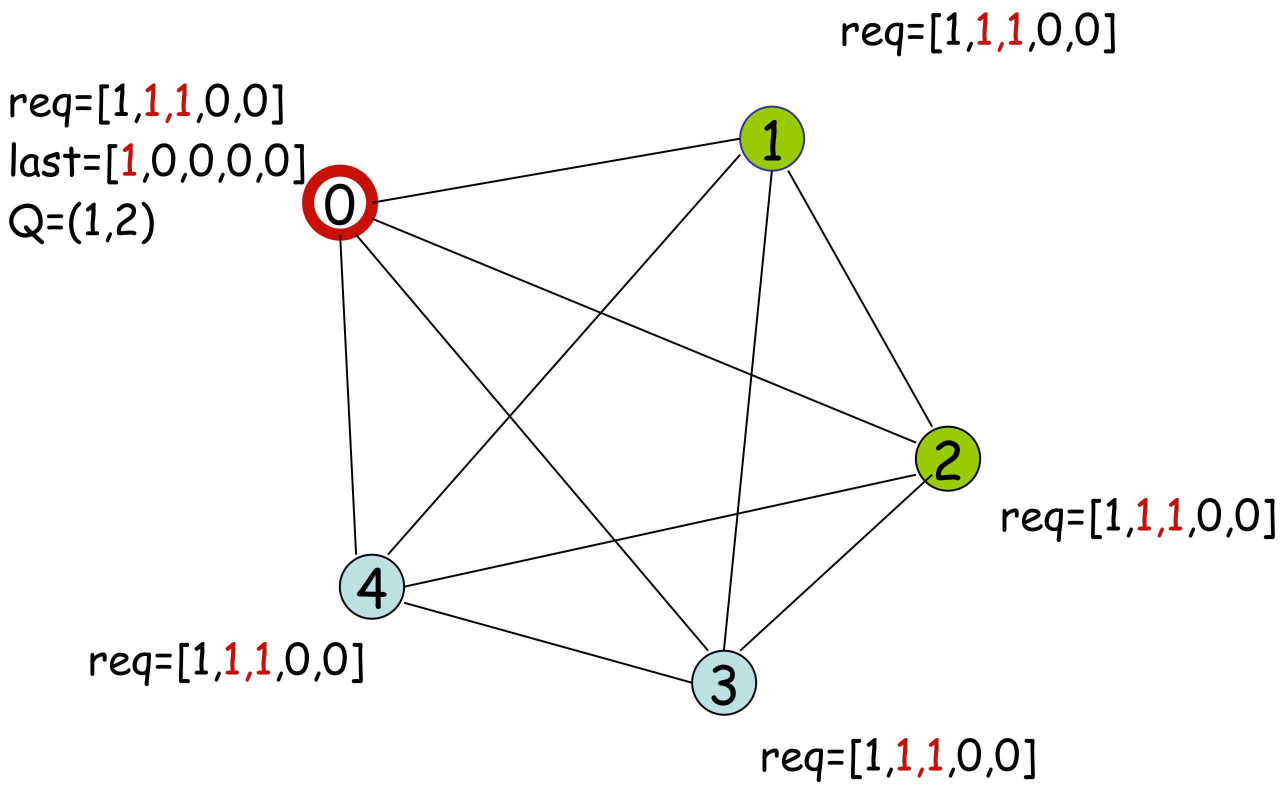
initial state: process 0 has sent a request to all, and grabbed the token

Example of Suzuki-Kasami Algorithm Execution



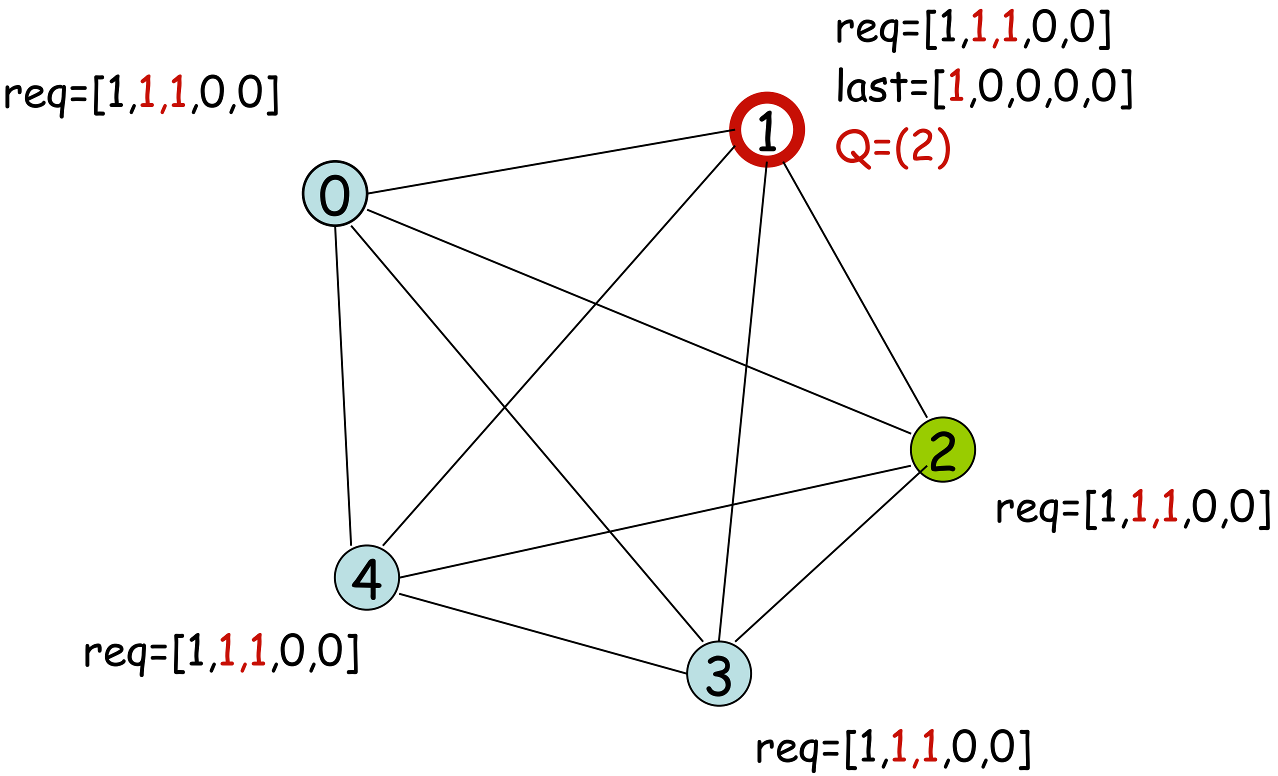
1 & 2 send requests to enter CS

Example of Suzuki-Kasami Algorithm Execution



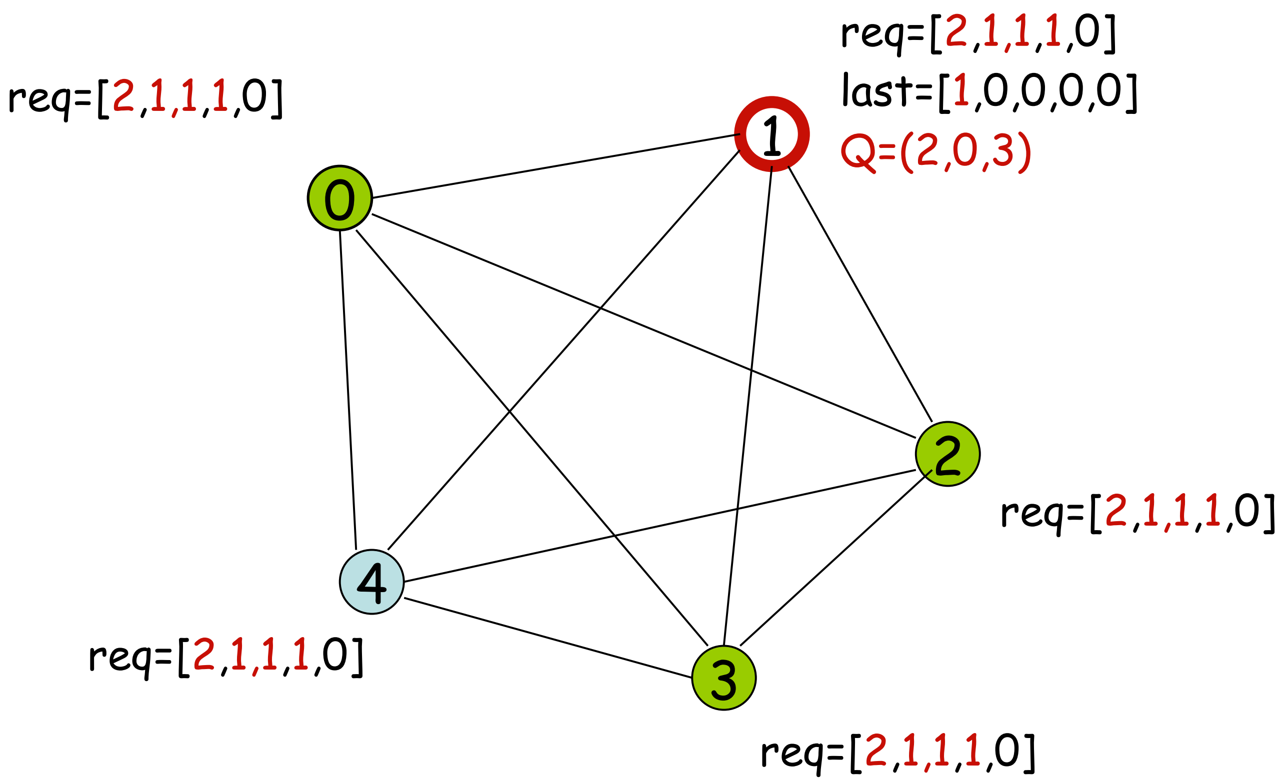
0 prepares to exit CS

Example of Suzuki-Kasami Algorithm Execution



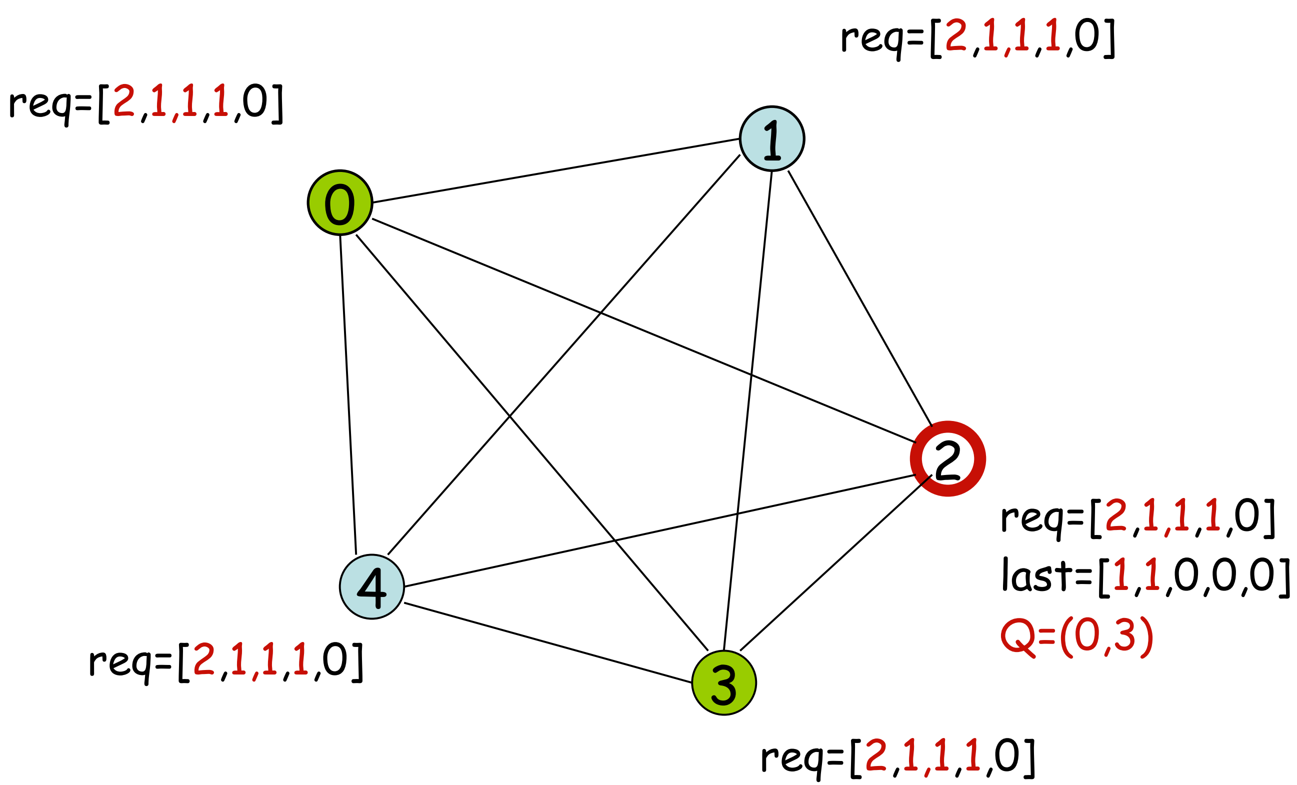
0 passes token (Q and last) to 1

Example of Suzuki-Kasami Algorithm Execution



0 and 3 send requests

Example of Suzuki-Kasami Algorithm Execution



1 sends token to 2

Raymond's Solution

- Improved version of token-based solution
  - Uses a tree-topology
- Idea:
  - At any time, one node holds the token
    - The holder is the root of the tree
  - Every edge is assigned a direction
    - Route reqests towards the root
    - If edge from Pi to Pj, Pj called holder of Pi
  - When the token moves, some edges change direction

Raymond's Algorithm

Outline

Each node has a **holder** variable and a local **Q.** Only first request forwarded to holder.

R1. A node **enters CS when it has token**. Otherwise (no token), registers request in local Q

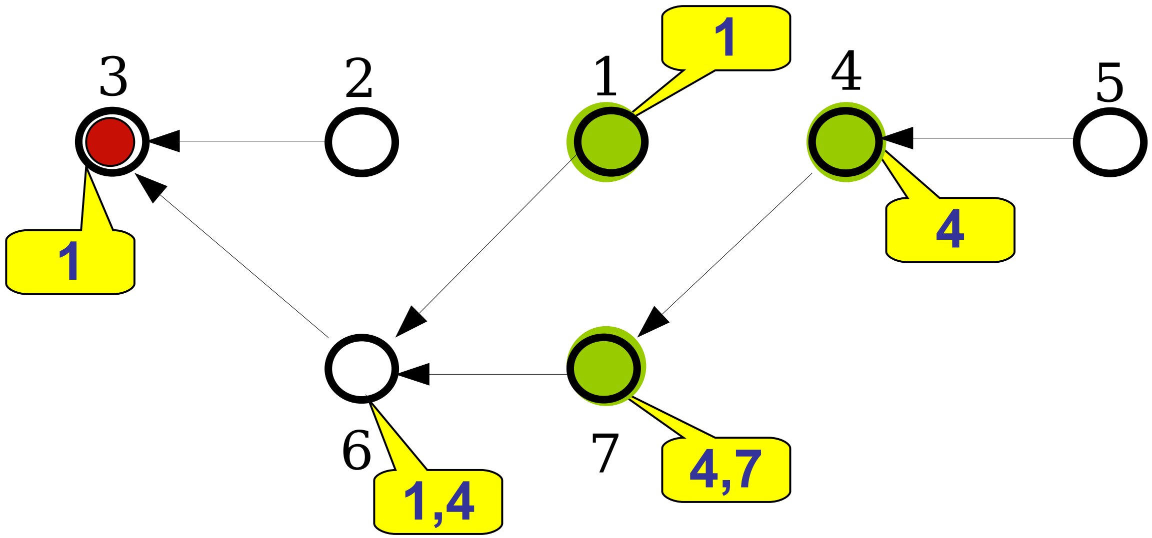
R2. A node Pj with non empty Q sends 1st request to its holder, **unless already sent** and awating for token.

R3. When root receives request, **sends to** neighbor at the **head of its local Q** after exiting CS. And changes **holder** to that node.

R4. When receiving a token, node Pj does:

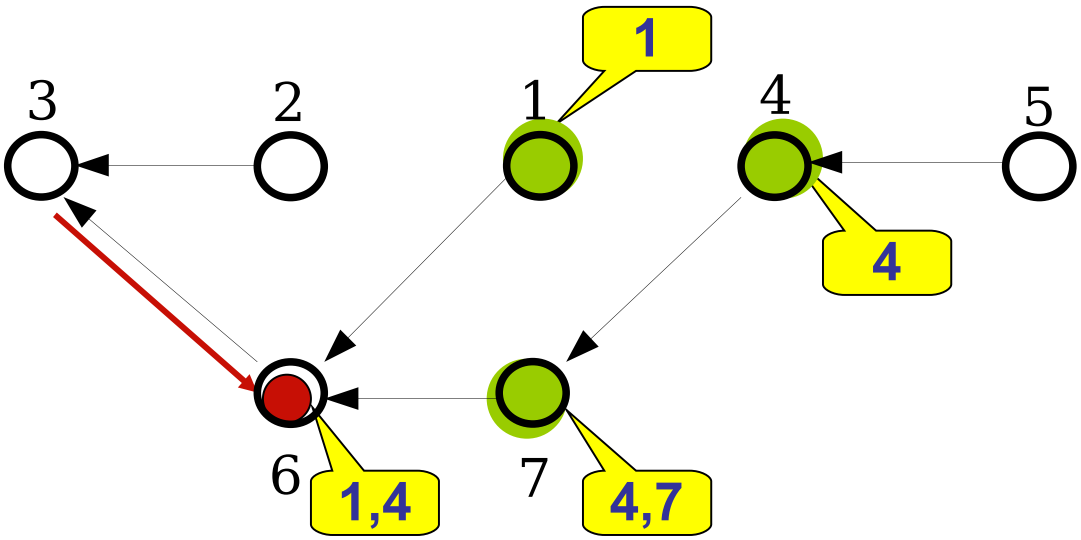
- forward to neighbor at head of its local Q
- delete request from Q
- set **holder** to that neighbor
- if there are pending requests in Q, send another request to **holder**

Example of Raymond's Algorithm Execution



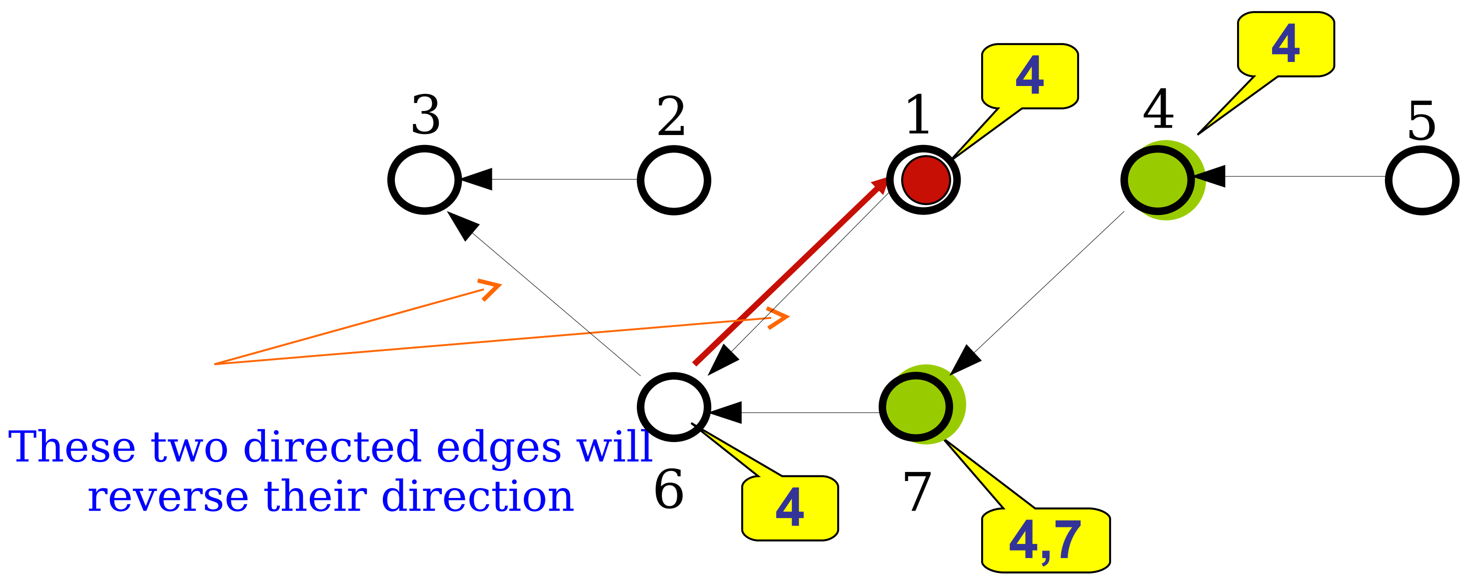
1,4,7 want to enter their CS

Example of Raymond's Algorithm Execution



3 sends the token to 6

Example of Raymond's Algorithm Execution



6 forwards the token to 1

The message complexity is O(diameter) of the tree. Extensive empirical measurements show that the average diameter of randomly chosen trees of size n is O(log n). Therefore, the authors claim that the average message complexity is O(log n)

Exercises

- In Suzuki-Kasami algorithm, prove the liveness property that any process requesting a token eventually receives the token. Also compute an upper bound on the number of messages exchanged in the system before the token is received.
- Repeat previous exercise with Raymond's algorithm.

Introduction

- Problem proposed and solved by Young in 1999
  - N processes, each belongs to one of M forums
  - Four conditions must hold:
    1. Mutual exclusion. At most one forum in session at a time.
    2. Freedom from deadlock. At any time, at least one process should be able to make a move
    3. Bounded waiting. Every forum chosen by a process must be in session in bounded time
    4. Concurrent entry. Once a forum is in session, concurrent entry in session is guaranteed for all willing processes.

Simplistic Centralized Solution

- Assume only 2 forums F and F'.
- Each process has a *flag* with values in {F, F', ⊥}
- Coordinator reads flags of each process in ascending order from 0 to N-1
  - Guarantees that first active Pi always served
  - followed by others requesting same forum

- Satisfies all requirement **except** bounded waiting
  - Possible starvation for one forum if processes keep entering always the same
  - Solved by electing a leader
    - first to enter forum
    - no more process allowed to join when leader leaves

Joung's Solution

- Each process cycles through 4 phases
  - request, in-cs, in-forum, passive
- Each process has flag={state,op}
  - state=phase, and op={F,F',⊥}

- First version (for Pi, forum F):

```
turn: F or F'
while ∃ Pj s.t. flag[j]=(in-cs,F')
do
      flag[i] = (request,F)
      while (turn ≠ F' and not all-passive(F')) do nop done
      flag[i] = (in-cs, F)
done
attend forum F
turn = F'
flag[i] = (passive, ⊥)
```

First Version Improved

- Fair with respect to forums
  - turn variable
  - note that a process has to wait for all other candidate to F' to be out of in-cs
- Not fair for processes
  - If several processes request F, at least one will succeed
  - A process sleeping in NOP may not notice a forum change from F' to F and then F' again
- Young's solution:
  - Introduce a leader for each session (as in centralized)
  - Each Pi has a variable successor[i] in (F, F', ⊥)
    - denote which is next forum
  - Only one leader can capture successors
  - A Pk with successor[k] = F enters in session F if leader of F in session