Hashing

Hashing …

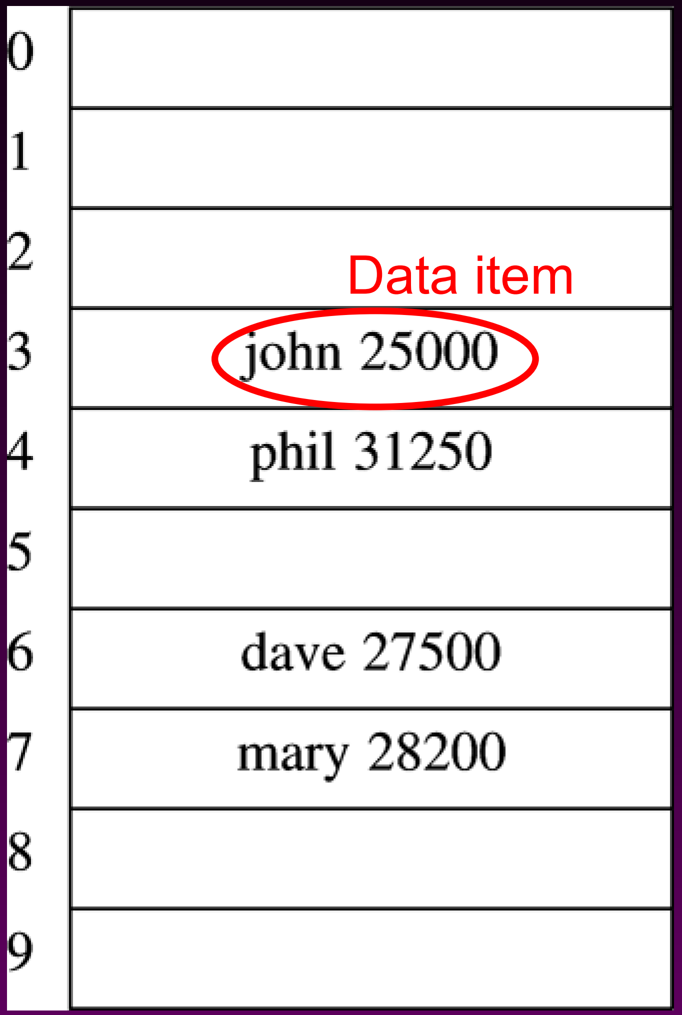
- Again, a (dynamic) set of elements in which we do ‘search’, ‘insert’, and ‘delete’
  - Linear ones: lists, stacks, queues, …
  - Nonlinear ones: trees, graphs (relations between elements are explicit)
  - Now for the case ‘relation is not important’, but want to be ‘efficient’ for searching (like in a dictionary)!
- Generalizing an ordinary array,
  - direct addressing!
  - An array is a direct-address table
- A set of N keys, compute the index, then use an array of size N
  - Key k at k -> direct address, now key k at h(k) -> hashing
- Basic operation is in O(1)!
- To ‘hash’ (is to ‘chop into pieces’ or to ‘mince’), is to make a ‘map’ or a ‘transform’ …

Hash Table

- Hash table is a data structure that support
  - `Finds, insertions, deletions` (deletions may be unnecessary in some applications)
- The implementation of hash tables is called hashing
  - A technique which allows the executions of above operations in constant average time
- Tree operations that requires any ordering information among elements are not supported
  - **`findMin`** and **`findMax`**
  - Successor and predecessor
  - Report data within a given range
  - List out the data in order

General Idea

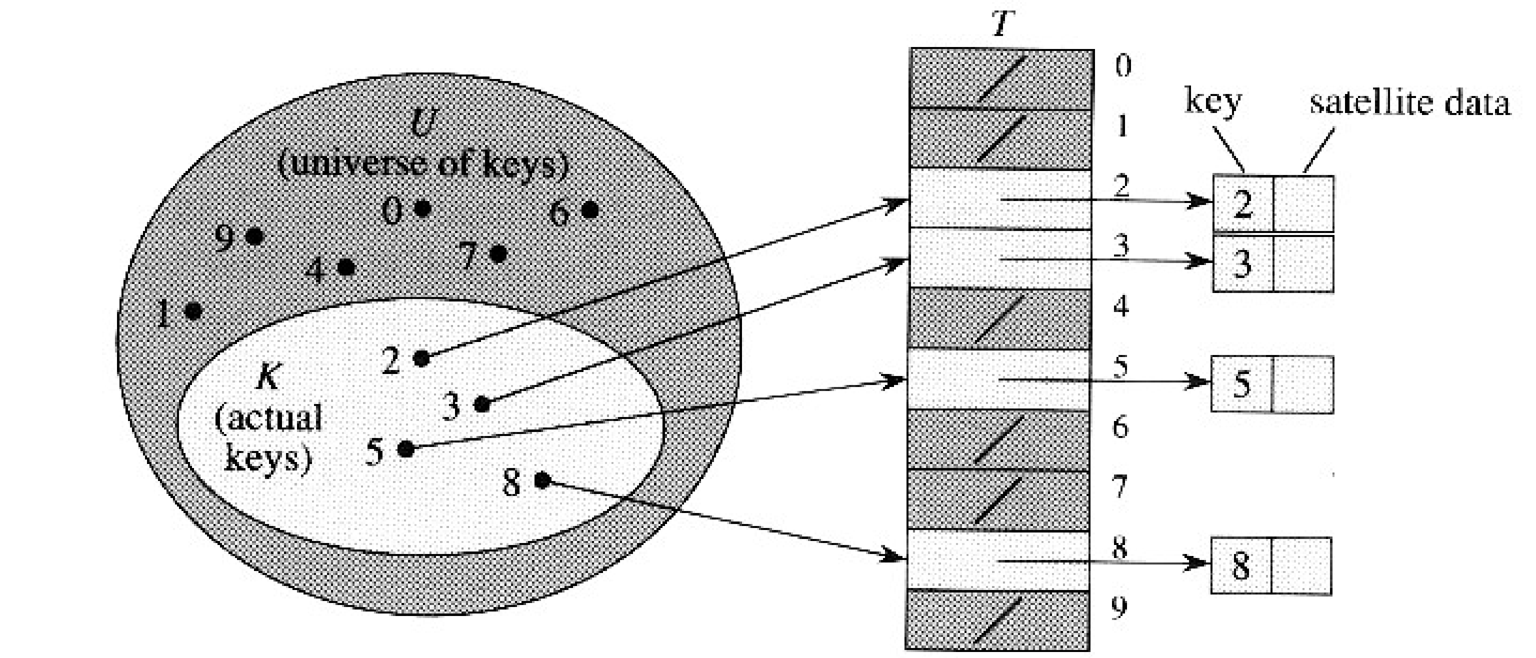
- The ideal hash table data structure is an array of some fixed size, containing the items
- A search is performed based on key
- Each key is mapped into some position in the range `0` to `TableSize-1`
- The mapping is called hash function



A hash table

Unrealistic Solution

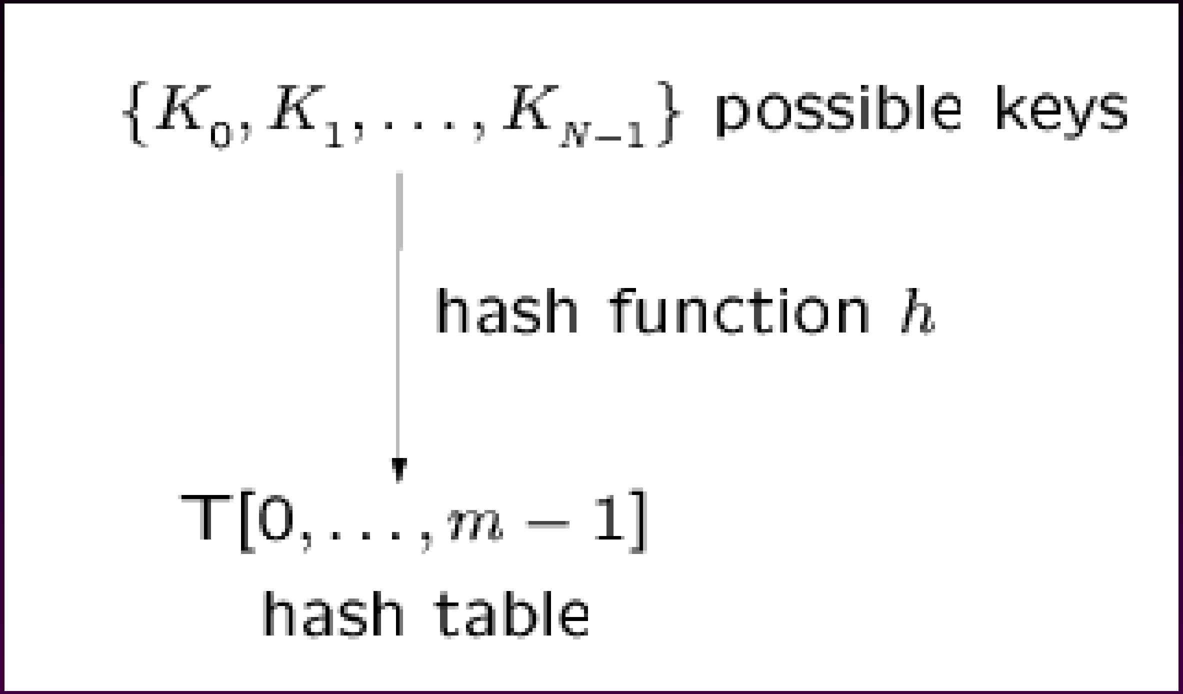
- Each position (**slot**) corresponds to a key in the universe of keys
  - T[k] corresponds to an element with key k
  - If the set contains no element with key k, then T[k]=NULL



Unrealistic Solution

- Insertion, deletion and finds all take O(1) (worst-case) time
- Problem: waste too much space if the universe is too large compared with the actual number of elements to be stored
  - E.g. student IDs are 8-digit integers, so the universe size is 10^8, but we only have about 7000 students

Hashing



Usually, m << N

$h(K_i)$ = an integer in [0, …, m-1] called the hash value of K_i

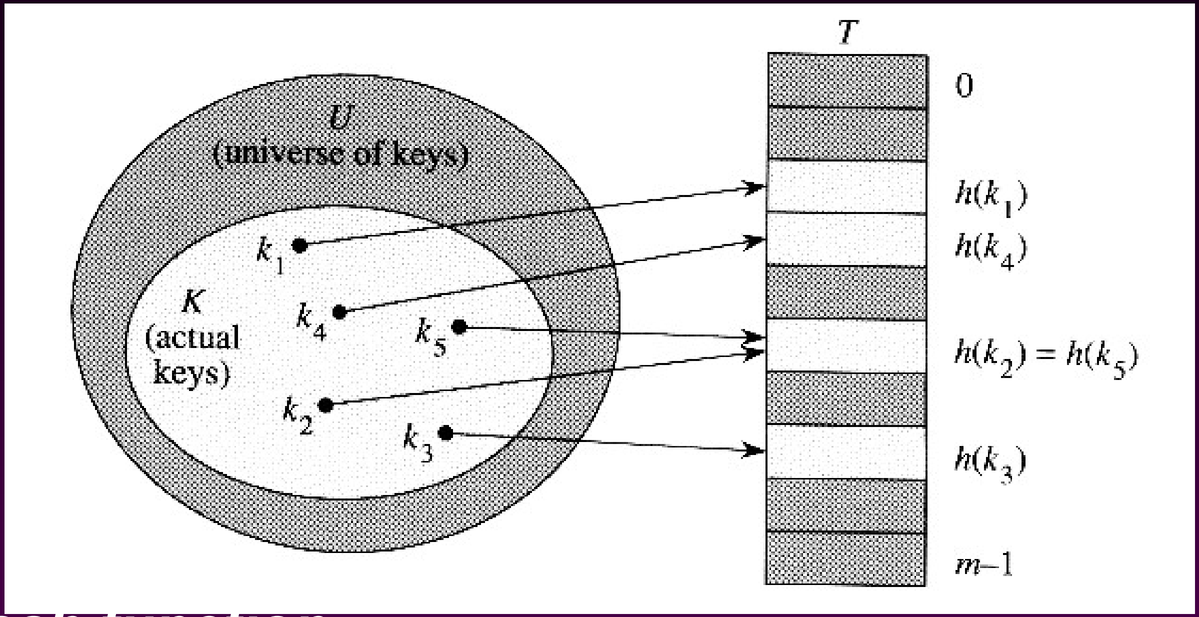
The keys are assumed to be natural numbers, if they are not, they can always be converted or interpreted in natural numbers.

Example Applications

- Compilers use hash tables (symbol table) to keep track of declared variables.
- On-line spell checkers. After prehashing the entire dictionary, one can check each word in constant time and print out the misspelled word in order of their appearance in the document.
- Useful in applications when the input keys come in sorted order. This is a bad case for binary search tree. AVL tree and B+-tree are harder to implement and they are not necessarily more efficient.

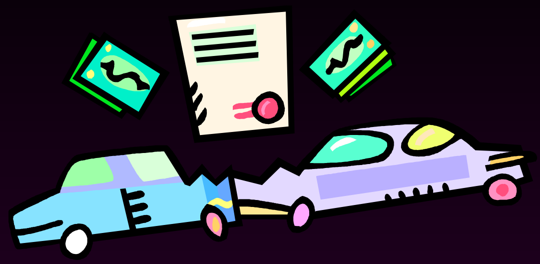
Hash Function

- With hashing, an element of key k is stored in T[h(k)]



- h: ***hash function***
  - maps the universe U of keys into the slots of a ***hash table*** T[0,1,...,m-1]
  - an element of key k ***hashes*** to slot h(k)
  - h(k) is the ***hash value*** of key k

Collision



- Problem: ***collision***
  - two keys may hash to the same slot
  - can we ensure that any two distinct keys get different cells?
    - No, if N>m, where m is the size of the hash table
- Task 1: Design a good hash function
  - that is fast to compute and
  - can minimize the number of collisions
- Task 2: Design a method to resolve the collisions when they occur

Design Hash Function

- A simple and reasonable strategy: h(k) = k mod m
  - e.g. m=12, k=100, h(k)=4
  - Requires only a single division operation (quite fast)

- Certain values of m should be avoided
  - e.g. if $m=2^p$, then h(k) is just the p lowest-order bits of k; the hash function does not depend on all the bits
  - Similarly, if the keys are decimal numbers, should not set m to be a power of 10

- It's a good practice to set the table size m to be a prime number

- Good values for m: primes not too close to exact powers of 2
  - e.g. the hash table is to hold 2000 numbers, and we don't mind an average of 3 numbers being hashed to the same entry
    - choose m=701

Deal with String-type Keys

- Can the keys be strings?
- Most hash functions assume that the keys are natural numbers
  - if keys are not natural numbers, a way must be found to interpret them as natural numbers
- Method 1: Add up the ASCII values of the characters in the string
  - Problems:
    - Different permutations of the same set of characters would have the same hash value
    - If the table size is large, the keys are not distribute well. e.g. Suppose m=10007 and all the keys are eight or fewer characters long. Since ASCII value <= 127, the hash function can only assume values between 0 and 127*8=1016

- Method 2

a,…,z and space

27^2

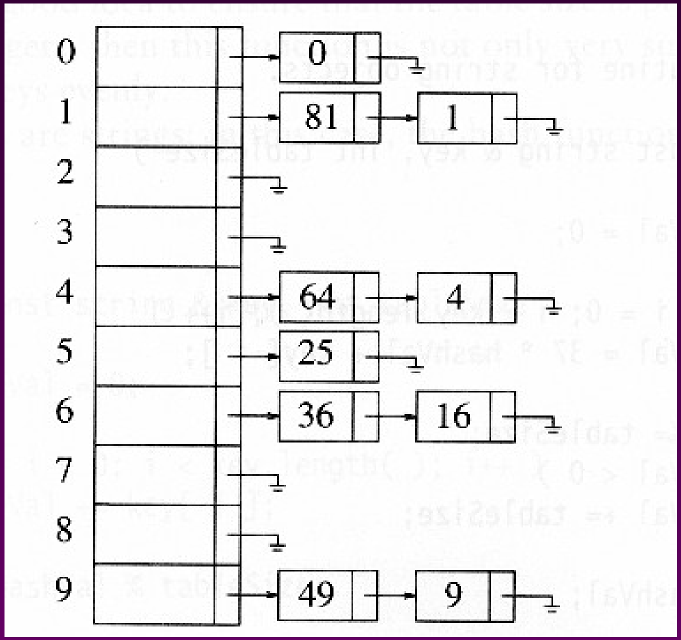
```
int hash( const string & key, int tableSize )
{
    return ( key[ 0 ] + 27 * key[ 1 ] + 729 * key[ 2 ] ) % tableSize;
}
```

  - If the first 3 characters are random and the table size is 10,0007 => a reasonably equitable distribution
  - Problem
    - English is not random
    - Only 28 percent of the table can actually be hashed to (assuming a table size of 10,007)
- Method 3
  - computes $\sum_{i=0}^{KeySize-1} Key[KeySize-i-1]*37^i$
  - involves all characters in the key and be expected to distribute well

Collision Handling: (1) Separate Chaining

- Lilke ‘equivalent classes’ or clock numbers in math

- Instead of a hash table, we use a table of linked list
- keep a linked list of keys that hash to the same value



Keys:
Set of squares

Hash function:
h(K) = K mod 10

Separate Chaining Operations

- To insert a key K
  - Compute h(K) to determine which list to traverse
  - If T[h(K)] contains a null pointer, initiatize this entry to point to a linked list that contains K alone.
  - If T[h(K)] is a non-empty list, we add K at the beginning of this list.

- To delete a key K
  - compute h(K), then search for K within the list at T[h(K)]. Delete K if it is found.

Separate Chaining Features

- Assume that we will be storing *n* keys. Then we should make *m* the next larger prime number. If the hash function works well, the number of keys in each linked list will be a small constant.

- Therefore, we expect that each search, insertion, and deletion can be done in constant time.

- Disadvantage: Memory allocation in linked list manipulation will slow down the program.

- Advantage: deletion is easy.

Collision Handling: (2) Open Addressing

- Instead of following pointers, compute the sequence of slots to be examined

- Open addressing: relocate the key K to be inserted if it collides with an existing key.
  - We store K at an entry different from T[h(K)].

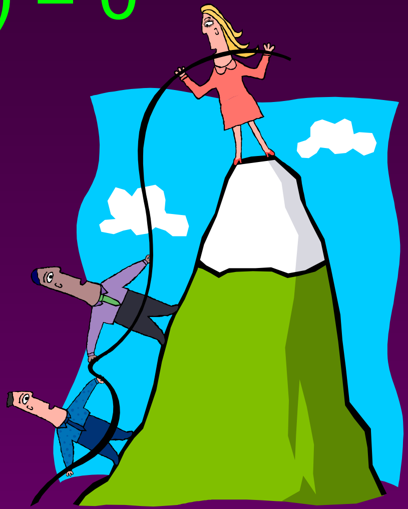
- Two issues arise
  - what is the relocation scheme?
  - how to search for K later?

- Three common methods for resolving a collision in open addressing
  - Linear probing
  - Quadratic probing
  - Double hashing

Open Addressing Strategy

- To insert a key K, compute $h_0(K)$. If $T[h_0(K)]$ is empty, insert it there. If collision occurs, probe alternative cell $h_1(K)$, $h_2(K)$, ... until an empty cell is found.

- $h_i(K) = (hash(K) + f(i)) \bmod m$, with $f(0) = 0$
  - f: ***collision resolution strategy***



Linear Probing

- **f(i) =i**
  - cells are probed sequentially (with wrap-around)
  - **$h_i(K) = (hash(K) + i) \bmod m$**

- Insertion:
  - Let K be the new key to be inserted, compute hash(K)
  - For i = 0 to m-1
    - compute L = (hash(K) + I) mod m
    - T[L] is empty, then we put K there and stop.
  - If we cannot find an empty entry to put K, it means that the table is full and we should report an error.

Linear Probing Example

- $h_i(K) = (hash(K) + i) \bmod m$
- E.g, inserting keys 89, 18, 49, 58, 69 with hash(K)=K mod 10

	Empty Table	After 89	After 18	After 49	After 58	After 69
0				49	49	49
1					58	58
2						69
3						
4						
5						
6						
7						
8			18	18	18	18
9		89	89	89	89	89

To insert 58, probe T[8], T[9], T[0], T[1]

To insert 69, probe T[9], T[0], T[1], T[2]

Primary Clustering

- We call a block of contiguously occupied table entries a cluster
- On the average, when we insert a new key K, we may hit the middle of a cluster. Therefore, the time to insert K would be proportional to half the size of a cluster. That is, the larger the cluster, the slower the performance.
- Linear probing has the following disadvantages:
  - Once h(K) falls into a cluster, this cluster will definitely grow in size by one. Thus, this may worsen the performance of insertion in the future.
  - If two clusters are only separated by one entry, then inserting one key into a cluster can merge the two clusters together. Thus, the cluster size can increase drastically by a single insertion. This means that the performance of insertion can deteriorate drastically after a single insertion.
  - Large clusters are easy targets for collisions.

Quadratic Probing Example

- `f(i) = i²`
- $h_i(K) = (\, hash(K) + i^2 \,) \bmod m$
- E.g., inserting keys 89, 18, 49, 58, 69 with hash(K) = K mod 10

	Empty Table	After 89	After 18	After 49	After 58	After 69
0				49	49	49
1						
2					58	58
3						69
4						
5						
6						
7						
8			18	18	18	18
9		89	89	89	89	89

To insert 58, probe T[8], T[9], T[(8+4) mod 10]

To insert 69, probe T[9], T[(9+1) mod 10], T[(9+4) mod 10]

Quadratic Probing

- Two keys with different home positions will have different probe sequences
  - e.g. m=101, h(k1)=30, h(k2)=29
  - probe sequence for k1: 30,30+1, 30+4, 30+9
  - probe sequence for k2: 29, 29+1, 29+4, 29+9
- If the table size is prime, then a new key can always be inserted if the table is at least half empty (see proof in text book)
- ***Secondary clustering***
  - Keys that hash to the same home position will probe the same alternative cells
  - Simulation results suggest that it generally causes less than an extra half probe per search
  - To avoid secondary clustering, the probe sequence need to be a function of the original key value, not the home position

Double Hashing

- To alleviate the problem of clustering, the sequence of probes for a key should be independent of its primary position => use two hash functions: hash() and hash2()

- **`f(i) = i * hash2(K)`**
  - E.g. hash2(K) = R - (K mod R), with R is a prime smaller than m

Double Hashing Example

- $h_i(K)$ = (hash(K) + f(i)) mod m; hash(K) = K mod m
- f(i) = i * hash2(K); hash2(K) = R - (K mod R),
- Example: m=10, R = 7 and insert keys 89, 18, 49, 58, 69

	Empty Table	After 89	After 18	After 49	After 58	After 69
0						69
1						
2						
3					58	58
4						
5						
6				49	49	49
7						
8			18	18	18	18
9		89	89	89	89	89

To insert 49, hash2(49)=7, 2nd probe is T[(9+7) mod 10]

To insert 58, hash2(58)=5, 2nd probe is T[(8+5) mod 10]

To insert 69, hash2(69)=1, 2nd probe is T[(9+1) mod 10]

Choice of hash2()

- Hash2() must never evaluate to zero
- For any key K, hash2(K) must be relatively prime to the table size m. Otherwise, we will only be able to examine a fraction of the table entries.
  - E.g.,if hash(K) = 0 and hash2(K) = m/2, then we can only examine the entries T[0], T[m/2], and nothing else!
- One solution is to make m prime, and choose R to be a prime smaller than m, and set

  hash2(K) = R – (K mod R)
- Quadratic probing, however, does not require the use of a second hash function
  - likely to be simpler and faster in practice

Deletion in Open Addressing

- Actual deletion cannot be performed in open addressing hash tables
  - otherwise this will isolate records further down the probe sequence
- Solution: Add an extra bit to each table entry, and mark a deleted slot by storing a special value DELETED (*tombstone*)

Perfect hashing

- Two-level hashing scheme
- The first level is the same as with ‘chaining’
- Make a secondary hash table with an associated hash function h_j, instead of making a list of the keys hashing to the same slot