Depth-First Search

Summary of BFS

- Graph and representations
- BFS, and BFS tree
- Complexity of BFS

Two representations: Adjacency List vs. Matrix

- **Two sizes: n = |V| and m=|E|,**
  - **m = O(n^2)**

- **Adjacency List**
  - More compact than adjacency matrices if graph has few edges

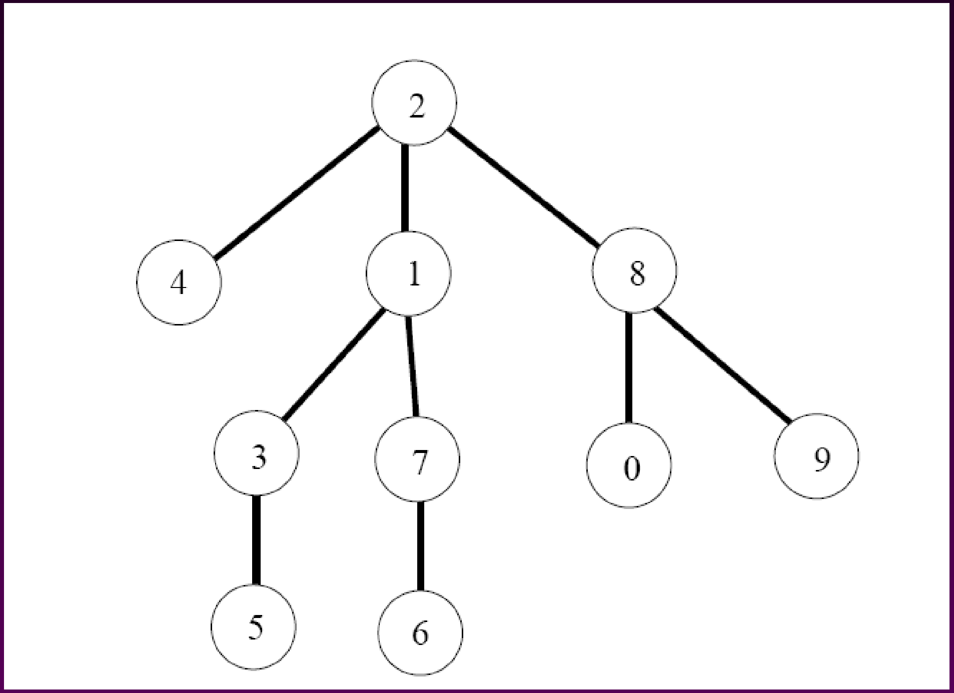
  - Requires a scan of adjacency list to check if an edge exists
  - Requires a scan to obtain all edges!

- **Adjacency Matrix**
  - Always require n^2 space
    - This can waste a lot of space if the number of edges are sparse

  - find if an edge exists in O(1)
  - Obtain all edges in O(n)

BFS Tree

BFS tree for vertex s=2.



Time Complexity of BFS
(Using Adjacency List)

Algorithm *BFS*(s)
Input: s is the source vertex
Output: Mark all vertices that can be visited from s.

```
for each vertex v
    do flag[v] := false;
Q = empty queue;
flag[s] := true;
enqueue(Q, s);
while Q is not empty
   do v := dequeue(Q);
      for each w adjacent to v
          do if flag[w] = false
               then flag[w] := true;
                    enqueue(Q, w)
```

O(n + m)

Each vertex will enter Q at most once.

Each iteration takes time proportional to deg(v) + 1 (the number 1 is to account for the case where deg(v) = 0 --- the work required is 1, not 0).

Time Complexity of BFS
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    do flag[v] := false;
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enqueue(Q, s);
while Q is not empty
    do v := dequeue(Q);
        for each w adjacent to v
            do if flag[w] = false
                then flag[w] := true;
                    enqueue(Q, w)
```

$O(n^2)$

Finding the adjacent vertices of v requires checking all elements in the row. This takes linear time O(n).

Summing over all the n iterations, the total running time is $O(n^2)$.

So, with adjacency matrix, BFS is $O(n^2)$ independent of the number of edges m. With adjacent lists, BFS is O(n+m); if $m=O(n^2)$ like in a dense graph, $O(n+m)=O(n^2)$.

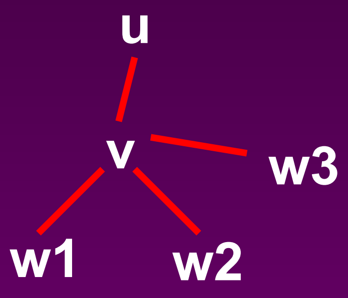
Depth-First Search (DFS)

- DFS is another popular graph search strategy
  - Idea is similar to pre-order traversal (visit node, then visit children recursively)

- DFS can provide certain information about the graph that BFS cannot
  - It can tell whether we have encountered a cycle or not

DFS Algorithm

- DFS will continue to visit neighbors in a recursive pattern
  - Whenever we visit v from u, we recursively visit all unvisited neighbors of v. Then we backtrack (return) to u.
  - Note: it is possible that w2 was unvisited when we recursively visit w1, but became visited by the time we return from the recursive call.



DFS Algorithm

Algorithm $DFS(s)$
1. **for** each vertex v
2. **do** $\boxed{\mathit{flag}[v] := \text{false;}}$
3. $RDFS(s)$;

Flag all vertices as not visited

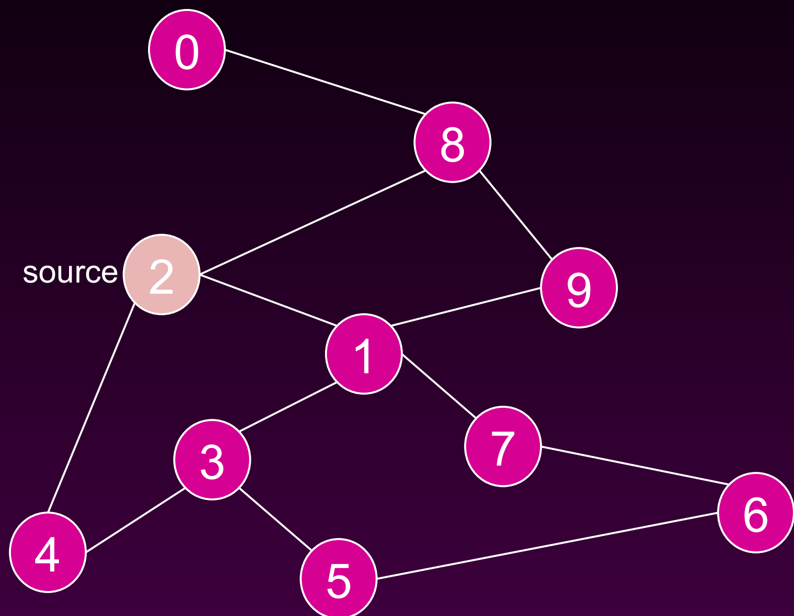
Algorithm $RDFS(v)$
1. $\boxed{\mathit{flag}[v] := \text{true;}}$
2. **for** each neighbor w of v
3. **do if** $\mathit{flag}[w] = \text{false}$
4. **then** $RDFS(w)$;

Flag yourself as visited

For unvisited neighbors, call RDFS(w) recursively

We can also record the paths using pred[ ].

Example



Adjacency List

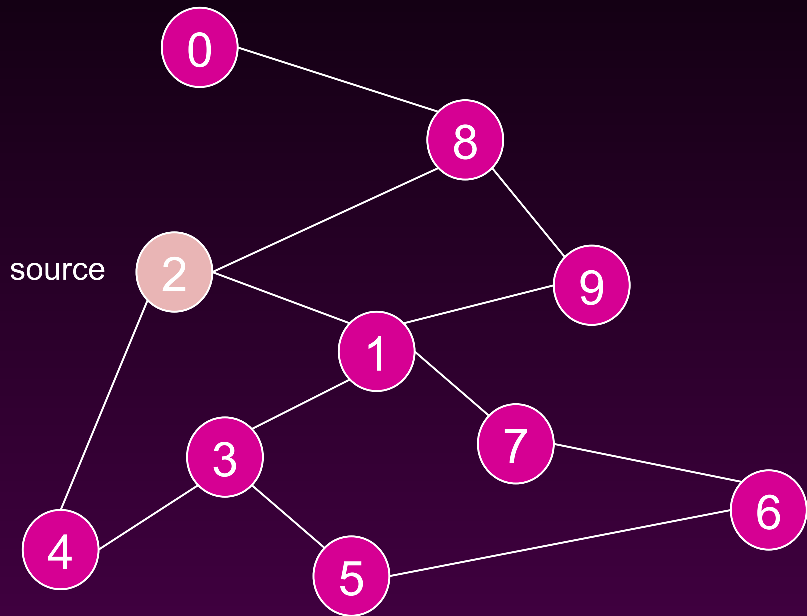
0	8
1	3 7 9 2
2	8 1 4
3	4 5 1
4	2 3
5	3 6
6	7 5
7	1 6
8	2 0 9
9	1 8

Visited Table (T/F)

		Pred
0	F	-
1	F	-
2	F	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	F	-
9	F	-

Initialize visited table (all False)

Initialize Pred to -1



Adjacency List

0	8			
1	3	7	9	2
2	8	1	4	
3	4	5	1	
4	2	3		
5	3	6		
6	7	5		
7	1	6		
8	2	0	9	
9	1	8		

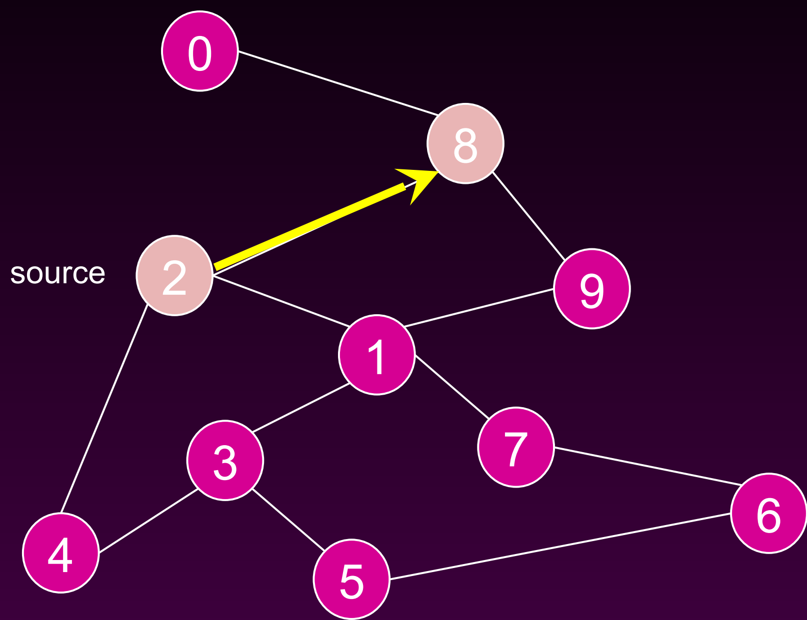
Visited Table (T/F)

		Pred
0	F	-
1	F	-
2	T	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	F	-
9	F	-

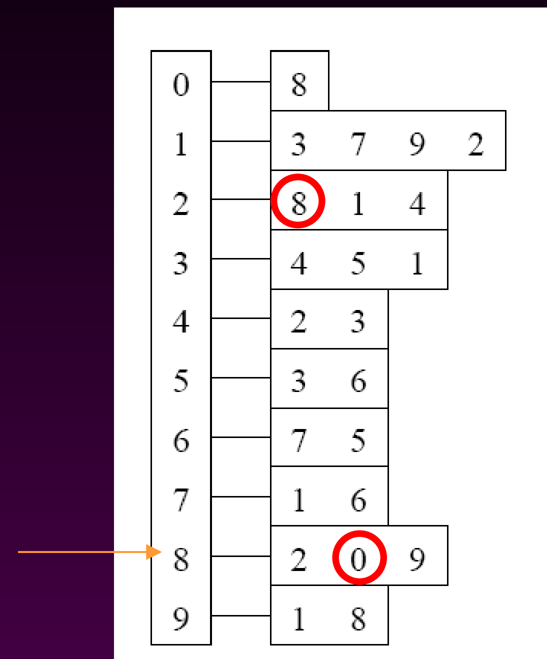
Mark 2 as visited

RDFS(2)

Now visit RDFS(8)



Adjacency List



Visited Table (T/F)

		Pred
0	F	-
1	F	-
2	T	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	T	2
9	F	-

Mark 8 as visited

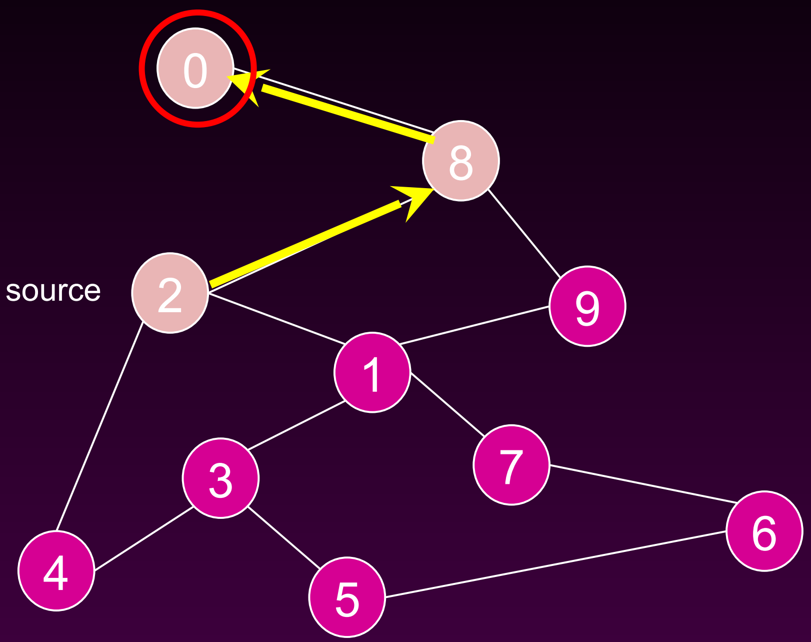
mark Pred[8]

Recursive calls

RDFS(2)

RDFS(8)

2 is already visited, so visit RDFS(0)



Adjacency List

0	8
1	3 7 9 2
2	8 1 4
3	4 5 1
4	2 3
5	3 6
6	7 5
7	1 6
8	2 0 9
9	1 8

Visited Table (T/F)

		Pred
0	T	8
1	F	-
2	T	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	T	2
9	F	-

Mark 0 as visited

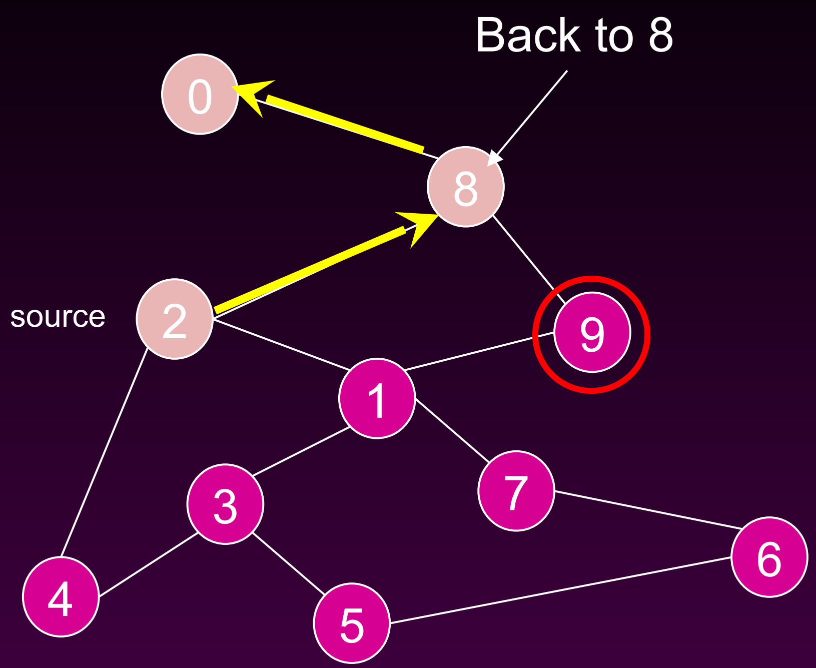
Mark Pred[0]

Recursive calls

RDFS(2)

RDFS(8)

RDFS(0) -> no unvisited neighbors, return to call RDFS(8)



Adjacency List

0	8			
1	3	7	9	2
2	8	1	4	
3	4	5	1	
4	2	3		
5	3	6		
6	7	5		
7	1	6		
8	2	0	9	
9	1	8		

Visited Table (T/F)

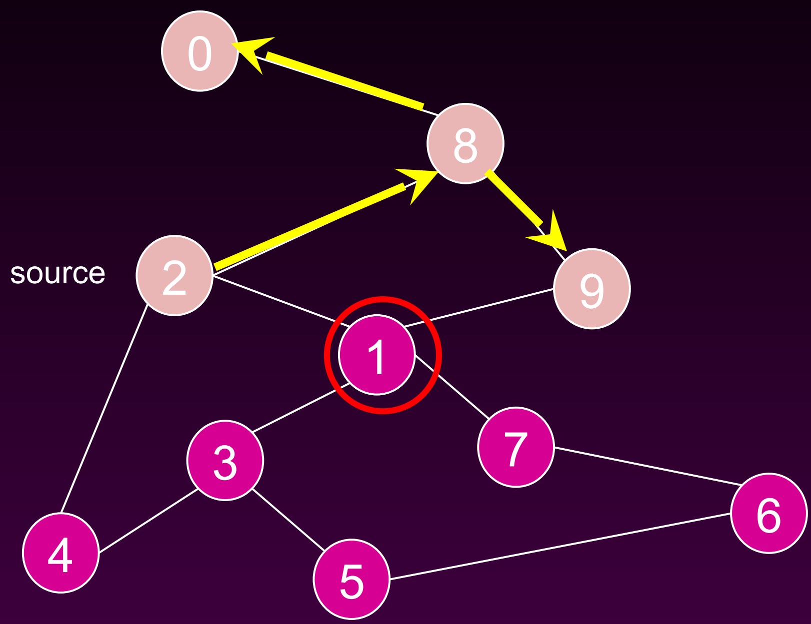
		Pred
0	T	8
1	F	-
2	T	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	T	2
9	F	-

Recursive calls

RDFS(2)

RDFS(8)

Now visit 9 -> RDFS(9)



Adjacency List

0	8			
1	3	7	9	2
2	8	1	4	
3	4	5	1	
4	2	3		
5	3	6		
6	7	5		
7	1	6		
8	2	0	9	
9	1	8		

Visited Table (T/F)

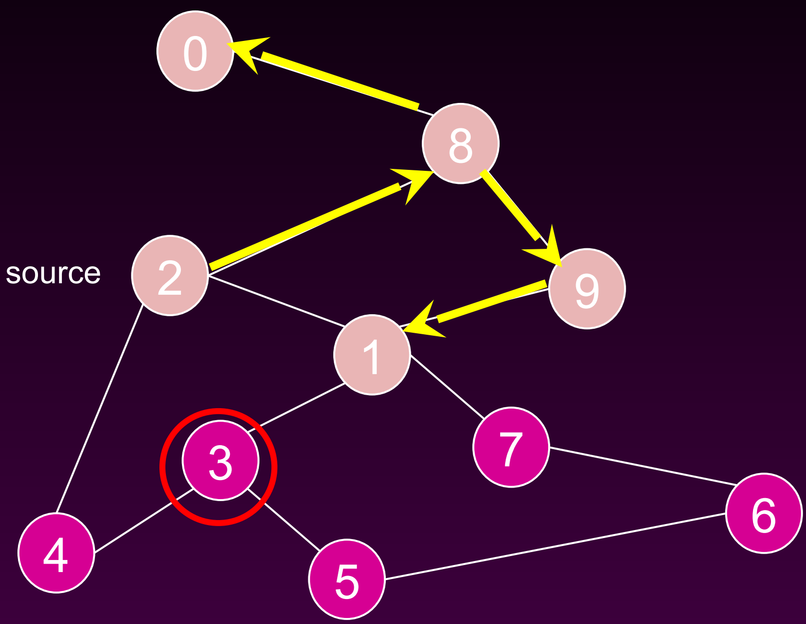
		Pred
0	T	8
1	F	-
2	T	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	T	2
9	T	8

Mark 9 as visited

Mark Pred[9]

Recursive calls

RDFS(2)
RDFS(8)
RDFS(9)
-> visit 1, RDFS(1)



Adjacency List

0	8			
1	3	7	9	2
2	8	1	4	
3	4	5	1	
4	2	3		
5	3	6		
6	7	5		
7	1	6		
8	2	0	9	
9	1	8		

Visited Table (T/F)

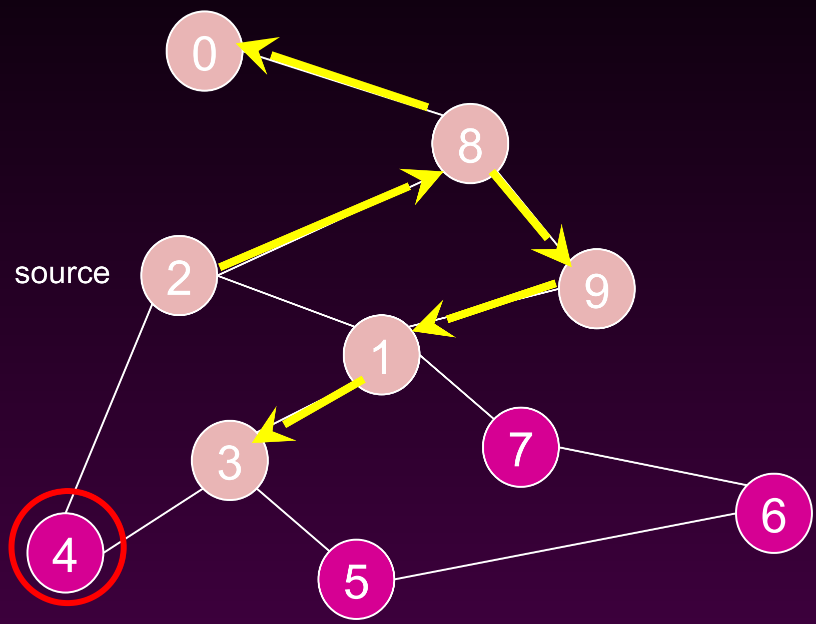
		Pred
0	T	8
1	T	9
2	T	-
3	F	-
4	F	-
5	F	-
6	F	-
7	F	-
8	T	2
9	T	8

Mark 1 as visited

Mark Pred[1]

Recursive calls

RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
visit RDFS(3)



Adjacency List

0	8
1	3 7 9 2
2	8 1 4
3	4 5 1
4	2 3
5	3 6
6	7 5
7	1 6
8	2 0 9
9	1 8

Visited Table (T/F)

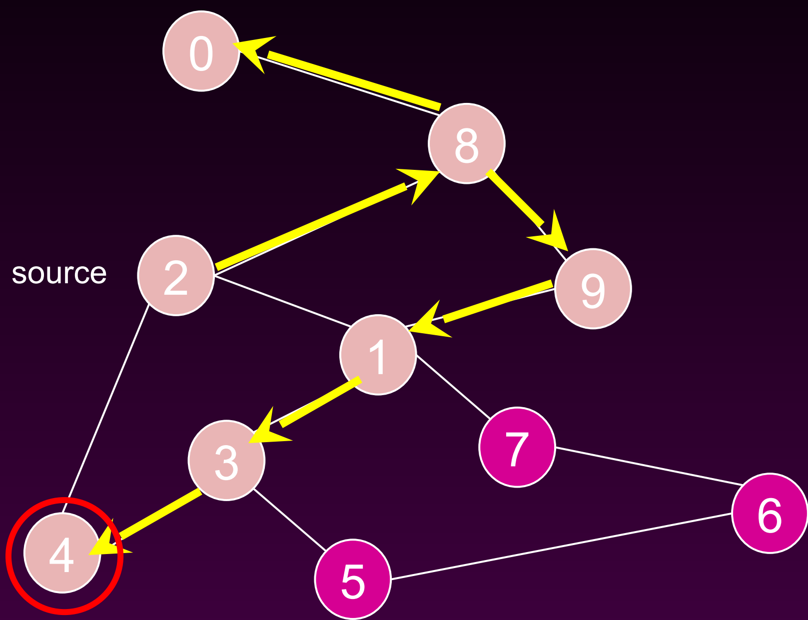
		Pred
0	T	8
1	T	9
2	T	-
3	T	1
4	F	-
5	F	-
6	F	-
7	F	-
8	T	2
9	T	8

Mark 3 as visited

Mark Pred[3]

Recursive calls

RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
visit RDFS(4)



Adjacency List

0	8			
1	3	7	9	2
2	8	1	4	
3	4	5	1	
4	2	3		
5	3	6		
6	7	5		
7	1	6		
8	2	0	9	
9	1	8		

Visited Table (T/F)

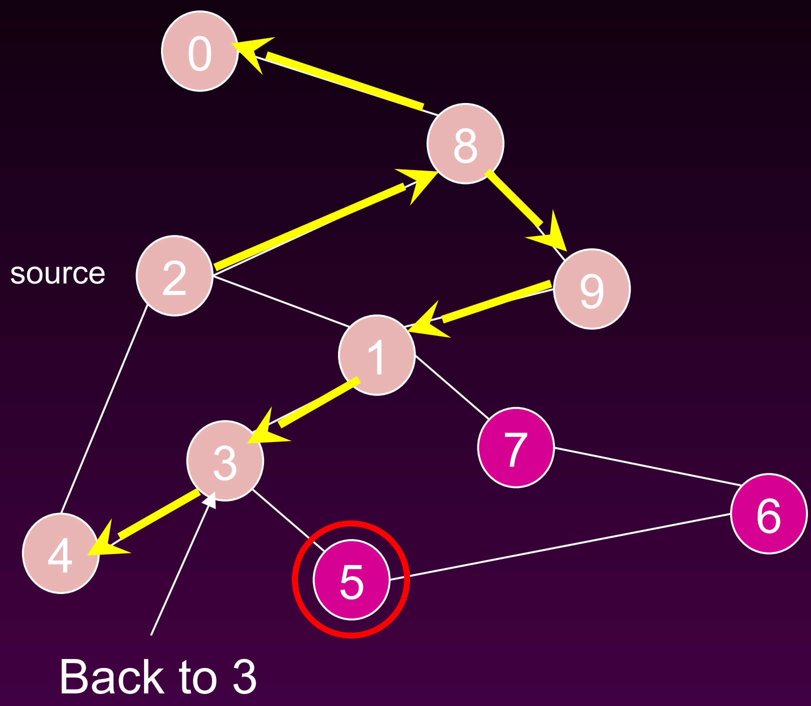
		Pred
0	T	8
1	T	9
2	T	-
3	T	1
4	T	3
5	F	-
6	F	-
7	F	-
8	T	2
9	T	8

Mark 4 as visited

Mark Pred[4]

Recursive calls

RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(4) → STOP all of 4's neighbors have been visited
return back to call RDFS(3)



Adjacency List

0	8
1	3 7 9 2
2	8 1 4
3	4 5 1
4	2 3
5	3 6
6	7 5
7	1 6
8	2 0 9
9	1 8

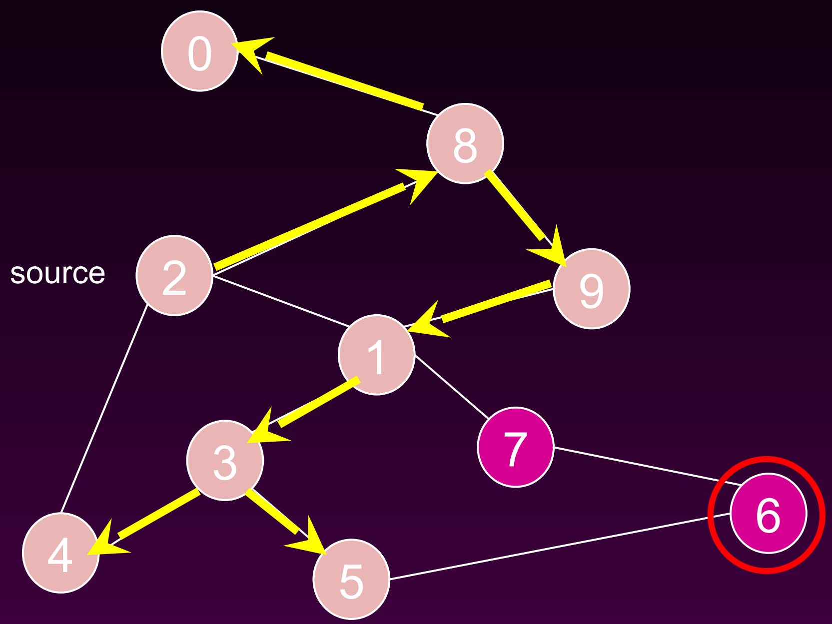
Visited Table (T/F)

		Pred
0	T	8
1	T	9
2	T	-
3	T	1
4	T	3
5	F	-
6	F	-
7	F	-
8	T	2
9	T	8

Pred

Recursive calls

RDFS(2)
  RDFS(8)
    RDFS(9)
      RDFS(1)
        RDFS(3)
          visit 5 -> RDFS(5)



Adjacency List

0	8
1	3 7 9 2
2	8 1 4
3	4 5 1
4	2 3
5	3 6
6	7 5
7	1 6
8	2 0 9
9	1 8

Visited Table (T/F)

		Pred
0	T	8
1	T	9
2	T	-
3	T	1
4	T	3
5	T	3
6	F	-
7	F	-
8	T	2
9	T	8

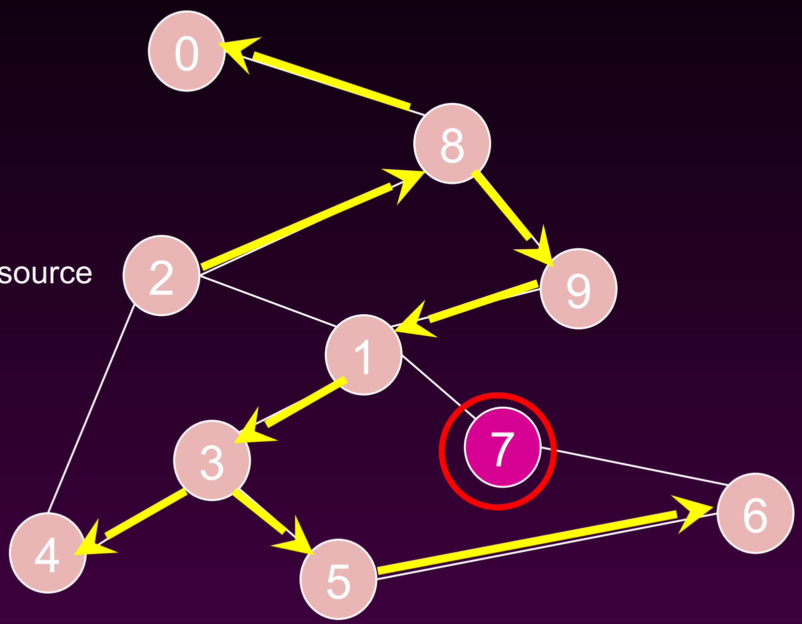
Recursive calls

RDFS(2)
    RDFS(8)
        RDFS(9)
            RDFS(1)
                RDFS(3)
                    RDFS(5)
                        3 is already visited, so visit 6 -> RDFS(6)

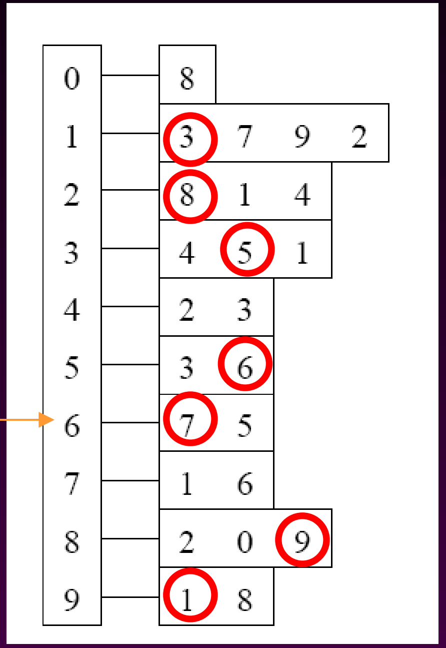
Mark 5 as visited

Mark Pred[5]

source



Adjacency List



Visited Table (T/F)

0	T	8
1	T	9
2	T	-
3	T	1
4	T	3
5	T	3
6	T	5
7	F	-
8	T	2
9	T	8

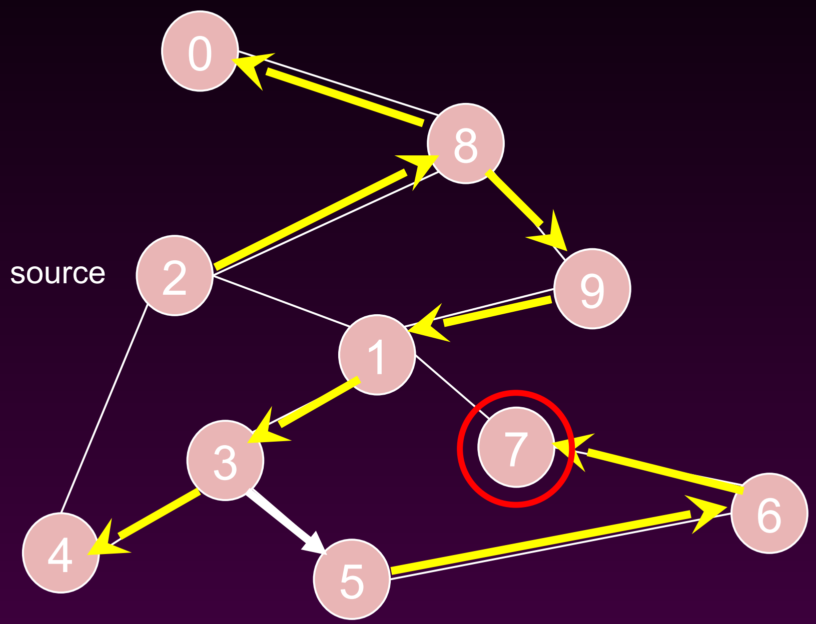
Pred

Recursive calls

RDFS(2)
    RDFS(8)
        RDFS(9)
            RDFS(1)
                RDFS(3)
                    RDFS(5)
                        RDFS(6)
                            visit 7 -> RDFS(7)

Mark 6 as visited

Mark Pred[6]



Adjacency List

0	8
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6	7 5
7	1 6
8	2 0 9
9	1 8

Visited Table (T/F)

		Pred
0	T	8
1	T	9
2	T	-
3	T	1
4	T	3
5	T	3
6	T	5
7	T	6
8	T	2
9	T	8

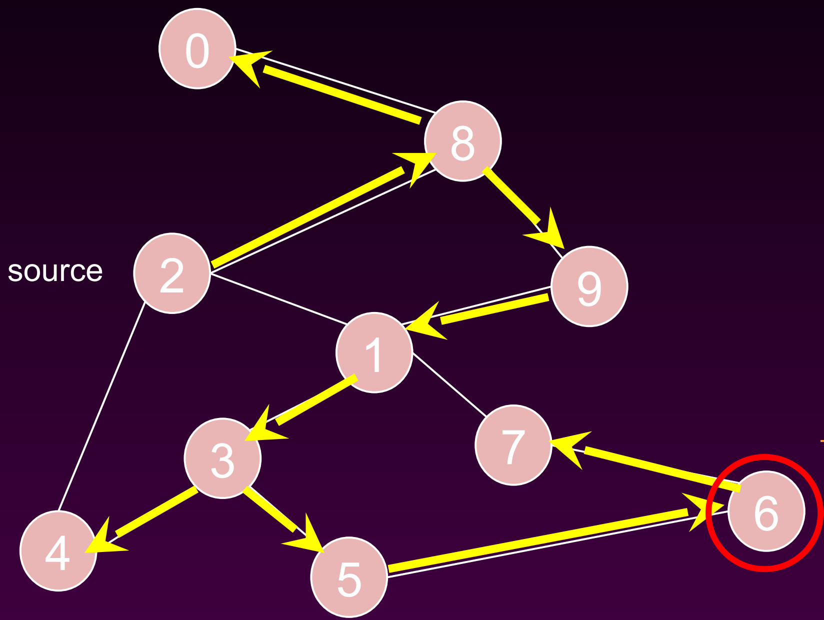
Pred

Recursive calls

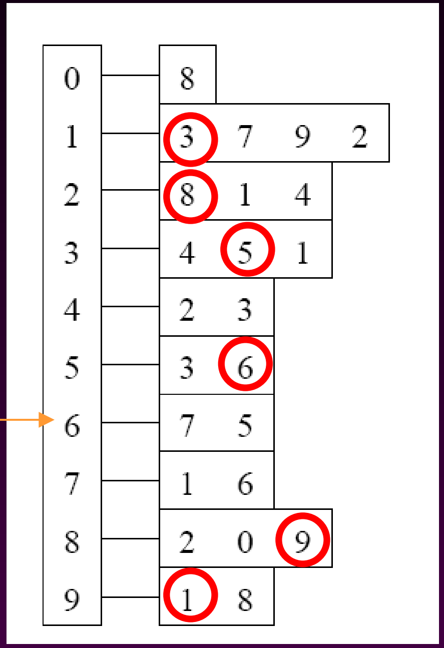
RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(5)
RDFS(6)
RDFS(7) -> Stop no more unvisited neighbors

Mark 7 as visited

Mark Pred[7]



Adjacency List



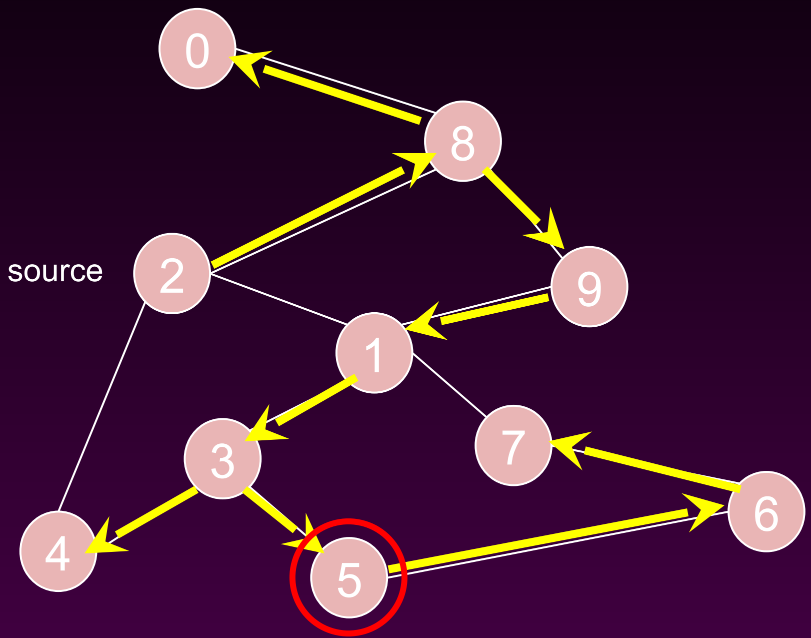
Visited Table (T/F)

		Pred
0	T	8
1	T	9
2	T	-
3	T	1
4	T	3
5	T	3
6	T	5
7	T	6
8	T	2
9	T	8

Pred

RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(5)
RDFS(6) -> Stop

Recursive calls



Adjacency List

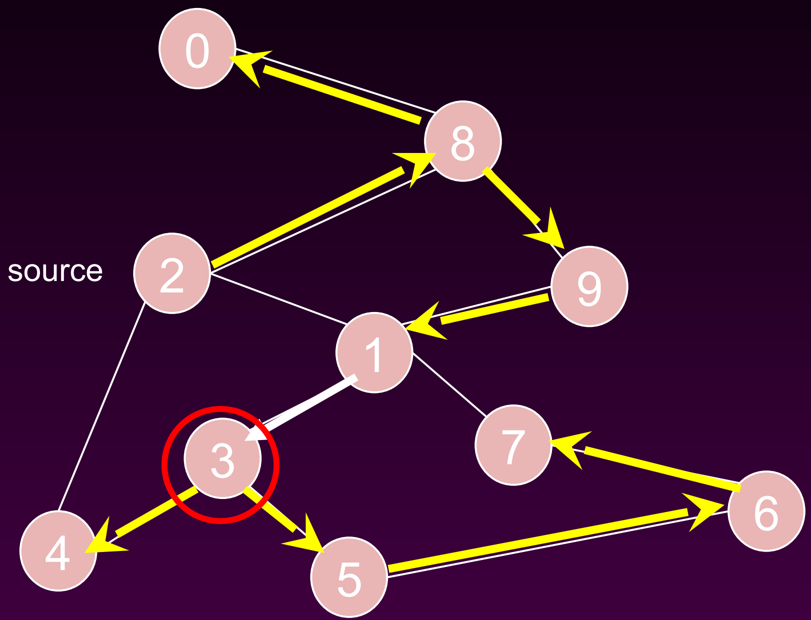
0	8			
1	3	7	9	2
2	8	1	4	
3	4	5	1	
4	2	3		
5	3	6		
6	7	5		
7	1	6		
8	2	0	9	
9	1	8		

Visited Table (T/F)

		Pred
0	T	8
1	T	9
2	T	-
3	T	1
4	T	3
5	T	3
6	T	5
7	T	6
8	T	2
9	T	8

RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1)
RDFS(3)
RDFS(5) -> Stop

Recursive calls



Adjacency List

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7	1 6
8	2 0 9
9	1 8

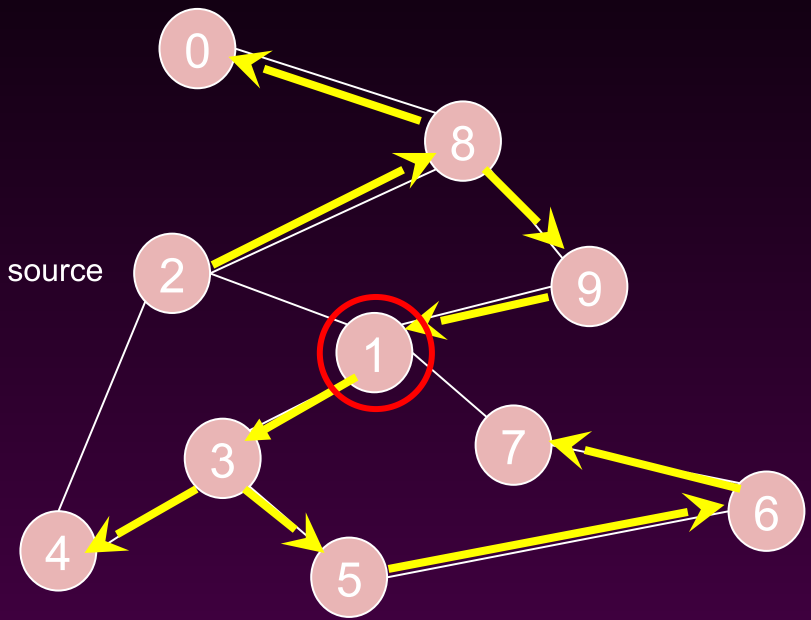
Visited Table (T/F)

		Pred
0	T	8
1	T	9
2	T	-
3	T	1
4	T	3
5	T	3
6	T	5
7	T	6
8	T	2
9	T	8

Pred

Recursive calls

RDFS(2)
  RDFS(8)
    RDFS(9)
      RDFS(1)
        RDFS(3) -> Stop



Adjacency List

0	8
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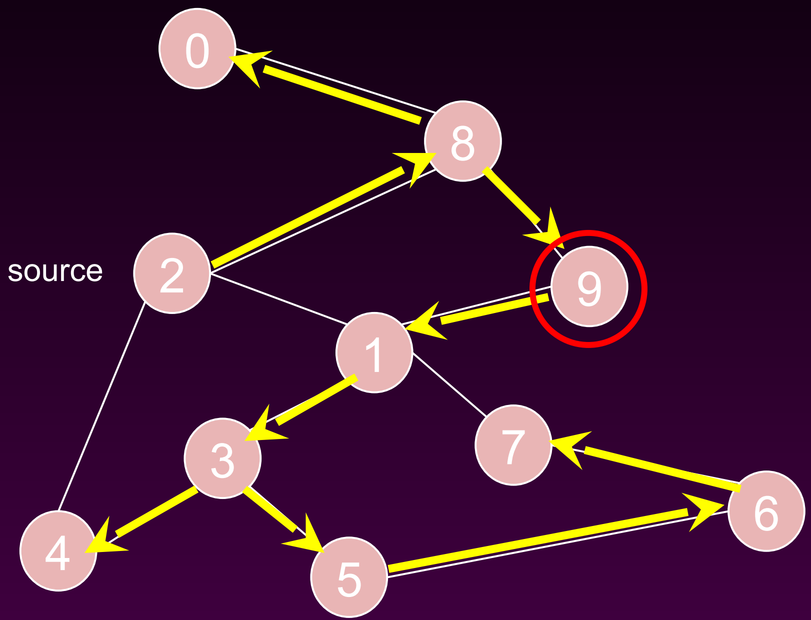
6	7 5
7	1 6
8	2 0 9
9	1 8

Visited Table (T/F)

		Pred
0	T	8
1	T	9
2	T	-
3	T	1
4	T	3
5	T	3
6	T	5
7	T	6
8	T	2
9	T	8

RDFS(2)
RDFS(8)
RDFS(9)
RDFS(1) -> Stop

Recursive calls



Adjacency List

0	8
1	3 7 9 2
2	8 1 4
3	4 5 1
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6	7 5
7	1 6
8	2 0 9
9	1 8

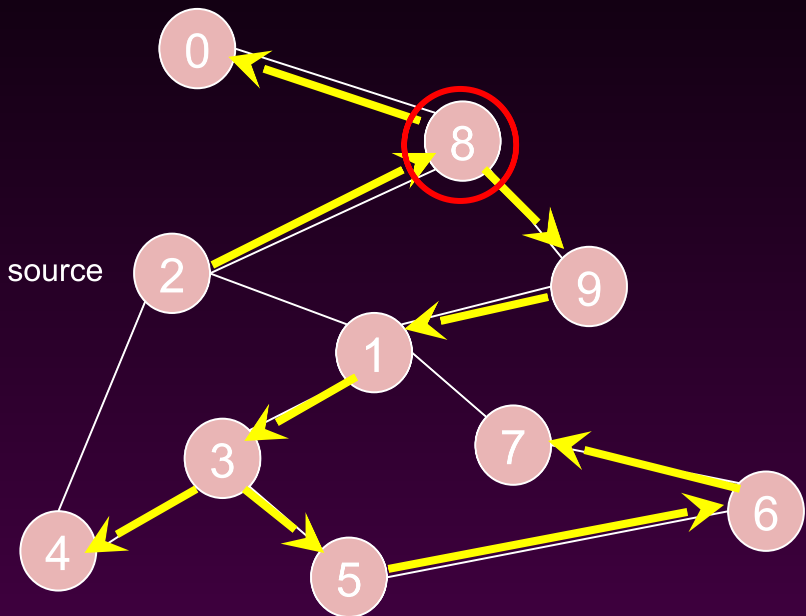
Visited Table (T/F)

		Pred
0	T	8
1	T	9
2	T	-
3	T	1
4	T	3
5	T	3
6	T	5
7	T	6
8	T	2
9	T	8

Pred

RDFS(2)
RDFS(8)
RDFS(9) -> Stop

Recursive calls



Adjacency List

0	8			
1	3	7	9	2
2	8	1	4	
3	4	5	1	
4	2	3		
5	3	6		
6	7	5		
7	1	6		
8	2	0	9	
9	1	8		

Visited Table (T/F)

		Pred
0	T	8
1	T	9
2	T	-
3	T	1
4	T	3
5	T	3
6	T	5
7	T	6
8	T	2
9	T	8

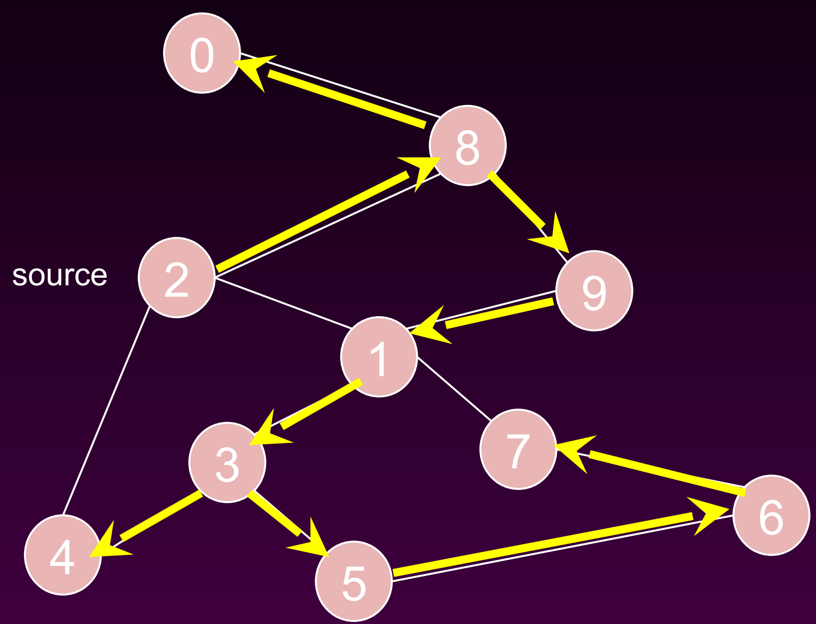
Pred

RDFS(2)

RDFS(8) -> Stop

Recursive calls

Example Finished



Adjacency List

0	8
1	3 7 9 2
2	8 1 4
3	4 5 1
4	2 3
5	3 6
6	7 5
7	1 6
8	2 0 9
9	1 8

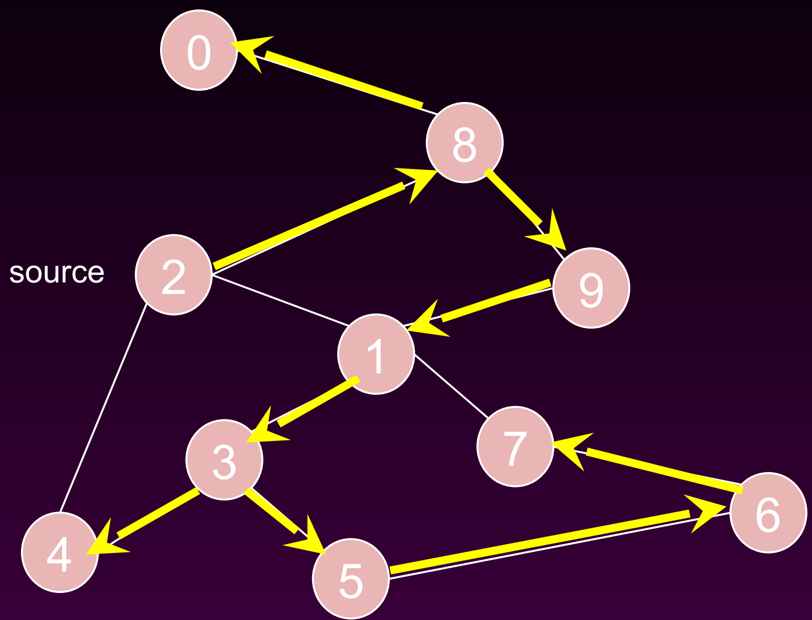
Visited Table (T/F)

		Pred
0	T	8
1	T	9
2	T	-
3	T	1
4	T	3
5	T	3
6	T	5
7	T	6
8	T	2
9	T	8

RDFS(2) -> Stop

Recursive calls finished

DFS Path Tracking



DFS find out path too

Adjacency List

0	8
1	3 7 9 2
2	8 1 4
3	4 5 1
4	2 3
5	3 6
6	7 5
7	1 6
8	2 0 9
9	1 8

Visited Table (T/F)

		Pred
0	T	8
1	T	9
2	T	-
3	T	1
4	T	3
5	T	3
6	T	5
7	T	6
8	T	2
9	T	8

Algorithm *Path*(w)

1. **if** $pred[w] \neq -1$
2. **then**
3. *Path*$(pred[w])$;
4. output w

Try some examples.

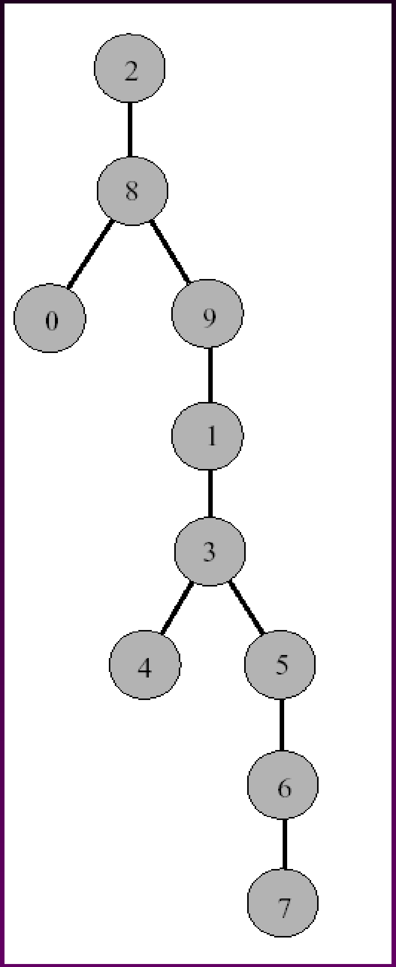
Path(0) ->

Path(6) ->

Path(7) ->

DFS Tree

Resulting DFS-tree.
Notice it is much “deeper”
than the BFS tree.



Captures the structure of the recursive calls

- when we visit a neighbor w of v, we add w as child of v
- whenever DFS returns from a vertex v, we climb up in the tree from v to its parent

Time Complexity of DFS
(Using adjacency list)

- We never visited a vertex more than once
- We had to examine all edges of the vertices
  - We know $\Sigma_{vertex\ v}$ degree(v) = 2m where m is the number of edges
- So, the running time of DFS is proportional to the number of edges and number of vertices (same as BFS)
  - O(n + m)
- You will also see this written as:
  - O(|v|+|e|) |v| = number of vertices (n) |e| = number of edges (m)