Homework Set 1

Q1. Show that $7x^2$ is $O(x^3)$. Is it also true that x^3 is $O(7x^2)$?

Q2. How can big-O notation be used to estimate the sum of the first ***n*** positive integers ?

Q3. Arrange the following functions in the increasing order of their growth rates:
(√2) n , 2√n , n2 (log n)2, (n log n)2, n√n , nn, (log n)n

Q4. Give as good as big-O estimate as possible for each of the following functions,

a) $(n^2 + 8)(n + 1)$
b) $(n \log n + n^2)(n^3 + 2)$
c) $(n! + 2^n)(n^3 + \log(n^2 + 1))$

Q5. Show that $3x^2 + 8x\log x$ is Ɵ(x2)

Q6. Show that if f(x) and g(x) are functions from the set of real numbers to the set of real numbers, then f(x) is O(g(x)) if and only if g(x) is Ω(f(x)).

Q7. Devise a recursive algorithm for computing the greatest common divisor of two non-negative integers a and b with a < b if gcd(a; b) = gcd(a; b   a).
by examining the values of this expression for small values of n. Use mathematical induction to prove your result. Compute the running time of your algorithm.

Q8. Use mathematical induction to prove that $n! < n^n$ whenever n is a positive integer greater than

Q9. Prove that $1+1/4+1/9..........+ 1/n^2 < 2- 1/n$,
whenever n is a positive integer greater than 1.

Q10. Use mathematical induction to show that 6 divides $n^3 - n$ whenever n is a non negative integer.
