Homework Assignment Set2

May 1, 2022

Q1 : Show that $7x^2$ is $O(x^3)$. Is it also true that x^3 is $O(7x^2)$?

Q2 : Let f(x) = $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$, where $a_0, a_1, .., a_{n-1}, a_n$ are real numbers. Then f(x) is O(x^n). Is it true? Justify.

Q3 : How can *big-O* notation be used to estimate the sum of the first n positive integers ?

Q4 : Estimate the *big-O* for the $factorial$ function and the *logarithm* of the factorial function.

$factorial f(n)$ or $n!$ is defined by $(n! = 1 * 2 * 3.. * n)$,
where n is a positive integer and $0! = 1$

For example,
$1! = 1, 2! = 1 * 2 = 2, 3! = 1 * 2 * 3 = 6, 4! = 1 * 2 * 3 * 4 = 24$

Function n! grows rapidly. For instance, 20! = 2,432,902,008,176,640,000.

Q5 : Give a *big-O* estimate for $f(x) = (x + 1)log(x^2 + 1) - 3x^2$

[Hint: If $\mathrm{f_1(x) = O(g_1(x))}$ and $\mathrm{f_2(x) = O(g_2(x))}$,
then $\mathrm{(f_1 + f_2)(x) = O(max(g_1(x),\ g_2(x)))}$

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then $\mathrm{(f_1 f_2)(x)}$ is $\mathrm{O(g_1(x) g_2(x))}$]

Q6 : Let k be a positive integer. Show that $1^k + 2^k + .. + n^k$ is $O(n^{k+1})$.

Q7 : Give as good as *big-O* estimate as possible for each of the following functions,

a) $(n^2 + 8)(n + 1)$
b) $(nlogn + n^2)(n^3 + 2)$
c) $(n! + 2^n)(n^3 + log(n^2 + 1))$

Q8 : Show that if $f(x)$ and $g(x)$ are functions from the set of real numbers to the set of real numbers, then $f(x)$ is $O(g(x))$ if and only if $g(x)$ is $\Omega(f(x))$.

Q9 : Show that $3x^2 + 8xlogx$ is $\theta(x^2)$.

Q10 : Give a recursive algorithm for finding the minimum of finite set of integers.

Q11 : Devise a recursive algorithm for computing the greatest common divisor of two non-negative integers a and b with a < b if $gcd(a, b) = gcd(a, b-a)$. by examining the values of this expression for small values of n. Use mathematical induction to prove your result.

Q12 : Use mathematical induction to prove that $n! < n^n$ whenever n is a positive integer greater than 1.

Q13 : Show that $1^2 - 2^2 + 3^2 - .... + (-1)^{n-1} * n^2 = (-1)^{n-1} * \frac{n*(n+1)}{2}$ whenever n is a positive integer.

Q14 : Prove that $1 + \frac{1}{4} + \frac{1}{9} + ... + \frac{1}{n^2} < 2 - \frac{1}{n}$ Whenever n is a positive integer greater than 1.

Q15 : Use mathematical induction to show that 6 divides $n^3 - n$ whenever n is a non negative integer.

Q16 : Give a recursive algorithm for finding the string w^i the concatenation of i copies of w where w is a bit string.

Q17 : Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive 0s.How many such bit string are there of length five?

Q18 : Suppose that the number of bacteria in a colony triples every hour

a) Set up a recurrence relation for the number of the bacteria after n hours have elapsed.

b) If 100 bacteria are used to begin a new colony, how many bacteria will be in colony in 10 hours?

Q19 :

a) Find a recurrence relation for the number of permutations of a set with n elements.

b) Use this recurrence relation to find the number of permutations of a set with n elements using iteration.

Q20 :

a) Find a recurrence relation for the number of bit strings of length n that do not contain three consecutive 0s.

b) What are the initial conditions.

c) How many bit strings of length seven do not contain three consecutive 0s?

Q21 :

a) Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one stair or two stairs at a time.

b) What are the initial conditions.

c) How many ways can this person climb a flight of eight stairs?

Q22 :

a) Find a recurrence relation for the number of ternary strings that contain either two consecutive 0s or two consecutive 1s.

b) What are the initial conditions.

c) How many ternary strings of length six contain two consecutive 0s or two consecutive 1s ?