Homework Assignment Set 1

Q1. Consider the algebraic expression $(7x+y)(5a-b)^{3.}$ Draw the corresponding tree diagram and find the scope of the exponential operation.

Q2. Find $Floor(\log_2 0.001)$

Q3. Plot the graphs of the exponential function $f(x) = 2^x$, the logarithmic function $g(x) = \log_2 x$ and the linear function $h(x) = x$ on the same coordinate axis.
(a) Describe a geometric property of graphs f(x) and g(x)
(b) For any positive number c, how are f(c), g(c) and h(c) related?

Q4. Use the principle of mathematical induction to prove the following assertions: **[Moderate]**

a. $x^{2n+1}+y^{2n+1}$ is divisible by x+y for all n in $\mathbf{N_0}$.

b. 1/sqrt(1) + 1/sqrt(2) + . . . + 1/sqrt(n) > 2(sqrt(n+1) - 1) for all n in **N**.

Q5. Find a loop invariant for each of the following loops: **[Moderate]**

a.
```
int n, x, y, t;

n = 0;
x = 1 + rand() % 9;
y = 1 + rand() % 9;
while (n < 10) {
   t = 1 + rand() % 9;
   x *= t;
   y /= t;
   ++n;
}
```

b.
```
#define NITER 100

double x, s, t;
int i;

i = 0; s = t = 1;
do {
  ++i;
  t /= (double)i;
  s += t;
} while (i < NITER);
```

Q6. Consider the sequence of integers given by: **[Moderate]**

$a_1 = 1,$
$a_2 = 1,$
$a_n = 6a_{n-2} - a_{n-1}$ for n >= 3.

a. Write a recursive function to compute a_{20}.

b. Write an iterative function to compute a_{20}.

c. Suppose that a mathematician tells you that

$$a_n = \left(2^{n+1} + (-3)^{n-1}\right)/5 \text{ for all } n>=1.$$

Use this formula to compute a_{20}.

Compare the timings of these three approaches for computing a_{20}. In order to measure time, use the built-in function `clock()` defined in `<time.h>`.

Q7. Write a program to determine the smallest positive integer n with the following property. Let

$n = a_k a_{k-1} \ldots a_1 a_0$

be the decimal representation of n with $a_k > 0$. Look at the integer:

$n' = a_0 a_k a_{k-1} \ldots a_2 a_1$

(the cyclic right shift of n). The desired property of n is that n' must be a proper integral multiple of n. **[Hard]**

Q8. Write a program to find the smallest positive integer n with the property that the decimal expansion of 2^n starts with the four digits `2005`, i.e., $2^n = 2005\ldots$ (Hint: Take log.) **[Hard]**
