

C216-

06797

Binomial distribution + which the random sample follows.

$n = 20$, probability of 'success' (the particular adult has hypertension) is $p = 0.24$

$$1-p = 0.76$$

[1]

(i) Exactly 3 people have hypertension:

$$P(3) = {}^{20}C_3 (0.24)^3 (0.76)^{17}$$

[1]

(ii) fewer than 4 people have hypertension

$$P(X \leq 4) = 1 - P(X \geq 4)$$

$$= \sum_{n=0}^3 {}^{20}C_n (0.24)^n (0.76)^{20-n}$$

[1]

Intuitively

(iii) at least 5 people have hypertension

$$P(X \geq 5) = 1 - P(X < 5)$$

$$= 1 - \sum_{n=0}^4 {}^{20}C_n (0.24)^n (0.76)^{20-n}$$

[2]

वर्षां

2. PMF & Poisson R.V.

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!} ; x = 0, 1, 2, \dots$$

[1]

$$\text{MGF : } E[e^{tx}] \rightarrow [1]$$

$$M(t) = \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda} \lambda^x}{x!}$$

ग्र

$$= \sum_{x=0}^{\infty} e^{-\lambda} \frac{(ae^t)^x}{x!} \rightarrow [1]$$

$$= \sum_{x=0}^{\infty} e^{-\lambda} \frac{e^{(\lambda e^t)^x}}{x!} = \frac{e^{-\lambda} e^{\lambda e^t}}{e^{-\lambda} (1 - e^t)} \\ = e^{\lambda e^t}$$

$$\boxed{M(t) = e^{-\lambda} (1 - e^t)} \rightarrow [1]$$

$$\text{Variance : } E[x^2] - (E[x])^2 = \text{Var}(x)$$

$$E[x] = M'(0) = \left. \frac{d}{dt} M(t) \right|_{t=0}$$

$$E[x^2] = M''(0) \\ \Rightarrow \left. \lambda e^t e^{\lambda e^t} + \lambda^2 e^{2t} \right|_{t=0}$$

$$\Rightarrow \lambda e^t e^{\lambda e^t} \Big|_{t=0}$$

$$\Rightarrow \lambda + \lambda^2$$

[1+1]

$$\Rightarrow \lambda$$

$$\therefore \text{Var}(x) = \lambda + \lambda^2 - \lambda = \lambda$$

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Third moment :-

$$M'''(0) = \frac{d^3}{dt^3} M(t) \Big|_{t=0}$$

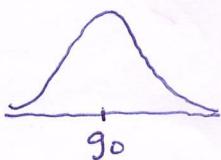
$$2) \left[\lambda e^t e^{2et} + (2e^t)^2 e^{2et} + 2\lambda^2 e^{2t} e^{2et} + (2e^t)^3 e^{2et} \right]_{t=0}$$

$$2) M'''(0) = \lambda + \lambda^2 + 2\lambda^2 + \lambda^3 \\ = \lambda + 3\lambda^2 + \lambda^3$$

[2]

8.

3.



(a) Normal range

90 + 38 to 90 - 38 mg/dL

$\Rightarrow 128 - 52 \text{ mg/dL}$

[1]

In terms of std. normal variable

$$\Phi(-1) = .1586$$

Total fraction people outside normal range is

$$100 \times (2 \times .1586) \approx 31.72\% \text{ or } 32\% \quad [2]$$

$$(b) \Phi(-2) = .02275$$

... per cent of people will be called abnormal now

$$\Rightarrow (2 \times .02275) \times 100 \Rightarrow .045 \times 100 = 4.5\%$$

[2]

(c) The random sample will also have normal distribution.

Ans

$$Z = \frac{X - 90}{\frac{38}{\sqrt{50}}}$$

$$X = \frac{x_1 + \dots + x_n}{n}$$

$$n = 50$$

$$\bar{x} = 90$$

$$\sigma' = \frac{\sigma}{\sqrt{n}} = \frac{38}{\sqrt{50}}$$

$$P\left(\frac{95-90}{\frac{38}{\sqrt{50}}} \leq Z \leq \frac{105-90}{\frac{38}{\sqrt{50}}}\right)$$

$$\Rightarrow P\left(\frac{5\sqrt{50}}{38} \leq Z \leq \frac{15\sqrt{50}}{38}\right)$$

[1]

$$\Rightarrow P(0.93 \leq Z \leq 2.79)$$

8.

$$\therefore \Phi(2.79) - \Phi(0.93)$$

$$= .9974 - .8238$$

$$= .1736$$

[1]

$$\Phi(-0.93) = .17$$

$$\Phi(0.93) = 1 - \Phi(-0.93) = .8238$$

$$\Phi(-2.79) = .002$$

$$1 - \Phi(-2.79) = .9974$$