## INDIAN INSTITUTE OF INFORMATION TECHNOLOGY, ALLAHABAD

Quiz-I, September 2017
15/09/2017
Max Marks: 20
Duration: 1 hour
Note: Use of calculator is allowed. Be sure to carefully justify your answers. Total 6 problems are there each carrying 5 marks. The table of standard normal cdf values is provided.

1. Based on data collected by the National Center for Health Statistics and made available to the public in the Sample Adult database, an estimate of the percentage of adults who have at some point in their life been told they have hypertension is 23.53 percent. If we select a simple random sample of 20 U.S. adults and assume that the probability that each has been told that he or she has hypertension is .24 , find the probability that the number of people in the sample who have been told that they have hypertension will be i) exactly 3 , ii) fewer than 4 , iii) at least 5 . [5]
2. Write down the probability mass function of Poisson random variable with parameter $\lambda$. Calculate the moment generating function of it. Hence calculate the variance of Poisson random variable. What is the third moment of Poisson distribution? [8]
3. Assume blood-glucose levels in a population of adult women are normally distributed with mean $90 \mathrm{mg} / \mathrm{dL}$ and standard deviation $38 \mathrm{mg} / \mathrm{dL}$.
(a) Suppose the abnormal range were defined to be glucose levels outside of 1 standard deviation of the mean (i.e., either at least 1 standard deviation above the mean, or at least 1 standard deviation below mean). Individuals with abnormal levels will be retested. What percentage of individuals would be called abnormal and need to be retested? What is the normal range of glucose levels in units of $\mathrm{mg} / \mathrm{dL}$ ?
(b) Suppose the abnormal range were defined to be glucose levels outside of 2 standard deviations of the mean. What percentage of individuals would now be called abnormal?
(c) What is the probability that a random sample of 50 women drawn from the population will have mean glucose-levels between $95 \mathrm{mg} / \mathrm{dL}$ and $105 \mathrm{mg} / \mathrm{dL}$ ? [7]

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

| Z | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -3.9 | . 00005 | . 00005 | . 00004 | . 00004 | . 00004 | . 00004 | . 00004 | . 00004 | . 00003 | . 00003 |
| -3.8 | . 00007 | . 00007 | . 00007 | . 00006 | . 00006 | . 00006 | . 00006 | . 00005 | . 00005 | . 00005 |
| -3.7 | . 00011 | . 00010 | . 00010 | . 00010 | . 00009 | . 00009 | . 00008 | . 00008 | . 00008 | . 00008 |
| -3.6 | . 00016 | . 00015 | . 00015 | . 00014 | . 00014 | . 00013 | . 00013 | . 00012 | . 00012 | . 00011 |
| -3.5 | . 00023 | . 00022 | . 00022 | . 00021 | . 00020 | . 00019 | . 00019 | . 00018 | . 00017 | . 00017 |
| -3.4 | . 00034 | . 00032 | . 00031 | . 00030 | . 00029 | . 00028 | . 00027 | . 00026 | . 00025 | . 00024 |
| -3.3 | . 00048 | . 00047 | . 00045 | . 00043 | . 00042 | . 00040 | . 00039 | . 00038 | . 00036 | . 00035 |
| -3.2 | . 00069 | . 00066 | . 00064 | . 00062 | . 00060 | . 00058 | . 00056 | . 00054 | . 00052 | . 00050 |
| -3.1 | . 00097 | . 00094 | . 00090 | . 00087 | . 00084 | . 00082 | . 00079 | . 00076 | . 00074 | . 00071 |
| -3.0 | . 00135 | . 00131 | . 00126 | . 00122 | . 00118 | . 00114 | . 00111 | . 00107 | . 00104 | . 00100 |
| -2.9 | . 00187 | . 00181 | . 00175 | . 00169 | . 00164 | . 00159 | . 00154 | . 00149 | . 00144 | . 00139 |
| -2.8 | . 00256 | . 00248 | . 00240 | . 00233 | . 00226 | . 00219 | . 00212 | . 00205 | . 00199 | . 00193 |
| -2.7 | . 00347 | . 00336 | . 00326 | . 00317 | . 00307 | . 00298 | . 00289 | . 00280 | . 00272 | . 00264 |
| -2.6 | . 00466 | . 00453 | . 00440 | . 00427 | . 00415 | . 00402 | . 00391 | . 00379 | . 00368 | . 00357 |
| -2.5 | . 00621 | . 00604 | . 00587 | . 00570 | . 00554 | . 00539 | . 00523 | . 00508 | . 00494 | . 00480 |
| -2.4 | . 00820 | . 00798 | . 00776 | . 00755 | . 00734 | . 00714 | . 00695 | . 00676 | . 00657 | . 00639 |
| -2.3 | . 01072 | . 01044 | . 01017 | . 00990 | . 00964 | . 00939 | . 00914 | . 00889 | . 00866 | . 00842 |
| -2.2 | . 01390 | . 01355 | . 01321 | . 01287 | . 01255 | . 01222 | . 01191 | . 01160 | . 01130 | . 01101 |
| -2.1 | . 01786 | . 01743 | . 01700 | . 01659 | . 01618 | . 01578 | . 01539 | . 01500 | . 01463 | . 01426 |
| -2.0 | . 02275 | . 02222 | . 02169 | . 02118 | . 02068 | . 02018 | . 01970 | . 01923 | . 01876 | . 01831 |
| -1.9 | . 02872 | . 02807 | . 02743 | . 02680 | . 02619 | . 02559 | . 02500 | . 02442 | . 02385 | . 02330 |
| -1.8 | . 03593 | . 03515 | . 03438 | . 03362 | . 03288 | . 03216 | . 03144 | . 03074 | . 03005 | . 02938 |
| -1.7 | . 04457 | . 04363 | . 04272 | . 04182 | . 04093 | . 04006 | . 03920 | . 03836 | . 03754 | . 03673 |
| -1.6 | . 05480 | . 05370 | . 05262 | . 05155 | . 05050 | . 04947 | . 04846 | . 04746 | . 04648 | . 04551 |
| -1.5 | . 06681 | . 06552 | . 06426 | . 06301 | . 06178 | . 06057 | . 05938 | . 05821 | . 05705 | . 05592 |
| -1.4 | . 08076 | . 07927 | . 07780 | . 07636 | . 07493 | . 07353 | . 07215 | . 07078 | . 06944 | . 06811 |
| -1.3 | . 09680 | . 09510 | . 09342 | . 09176 | . 09012 | . 08851 | . 08691 | . 08534 | . 08379 | . 08226 |
| -1.2 | . 11507 | . 11314 | . 11123 | . 10935 | . 10749 | . 10565 | . 10383 | . 10204 | . 10027 | . 09853 |
| -1.1 | . 13567 | . 13350 | . 13136 | . 12924 | . 12714 | . 12507 | . 12302 | . 12100 | . 11900 | . 11702 |
| -1.0 | . 15866 | . 15625 | . 15386 | . 15151 | . 14917 | . 14686 | . 14457 | . 14231 | . 14007 | . 13786 |
| -0.9 | . 18406 | . 18141 | . 17879 | . 17619 | . 17361 | . 17106 | . 16853 | . 16602 | . 16354 | . 16109 |
| -0.8 | . 21186 | . 20897 | . 20611 | . 20327 | . 20045 | . 19766 | . 19489 | . 19215 | . 18943 | . 18673 |
| -0.7 | . 24196 | . 23885 | . 23576 | . 23270 | . 22965 | . 22663 | . 22363 | . 22065 | . 21770 | . 21476 |
| -0.6 | . 27425 | . 27093 | . 26763 | . 26435 | . 26109 | . 25785 | . 25463 | . 25143 | . 24825 | . 24510 |
| -0.5 | . 30854 | . 30503 | . 30153 | . 29806 | . 29460 | . 29116 | . 28774 | . 28434 | . 28096 | . 27760 |
| -0.4 | . 34458 | . 34090 | . 33724 | . 33360 | . 32997 | . 32636 | . 32276 | . 31918 | . 31561 | . 31207 |
| -0.3 | . 38209 | . 37828 | . 37448 | . 37070 | . 36693 | . 36317 | . 35942 | . 35569 | . 35197 | . 34827 |
| -0.2 | . 42074 | . 41683 | . 41294 | . 40905 | . 40517 | . 40129 | . 39743 | . 39358 | . 38974 | . 38591 |
| -0.1 | . 46017 | . 45620 | . 45224 | . 44828 | . 44433 | . 44038 | . 43644 | . 43251 | . 42858 | . 42465 |
| -0.0 | . 50000 | . 49601 | . 49202 | . 48803 | . 48405 | . 48006 | . 47608 | . 47210 | . 46812 | . 46414 |

