Problem Set-3

Special probability mass functions and density functions, covariance and correlation

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1. What is the probability of getting at least one 6 in throwing of a dice 10 times?

2. Toss a fair coin M times. (a) Move one step forward (in one particular direction) each time you get a head (H). What is the probability $P(n)$ that you are n steps away from where you have started?

   (b) If you move one step forward for each H and one step backward for each tail (T), what is $P(n)$? What is the mean and variance of the probability mass function (PMF)? Find the mode of PF for $M = 3$?

3. Let X be a Binomial random variable with parameters n, and p. Show that

$$P(X = x + 1) = \frac{p}{1 - p}\left(\frac{n - x}{x + 1}\right)P(X = x)$$

   .

4. In a 10-over cricket match, the runs that can be scored by a poor batsman is given by a Poisson distribution with parameter $\lambda = 10$. On the other hand, the runs that a good batsman can score is given by a Poisson distribution with parameter $\lambda = 30$. If a batsman scores 20 runs in the match, would you judge him as good or poor?

5. Let X be a Poisson random variable with parameter $\lambda > 0$. then show that $E(2^X) = \frac{1}{P(X=0)}$

6. An investigator notices that children develop chronic bronchitis in the first year of life in about 3 out of 20 households where both parents are chronic bronchitis, as compared to the national incidence rate of chronic bronchitis, which is 5% in the first year of life. How likely are infants in at least 3 out of 20 households will develop chronic bronchitis if probability of developing the disease in any one household is .05?

7. A probability class has 300 students and each student has probability 1/3 of getting an A, independently of any other student. What is the mean of X, the number of students that get an A?

8. If X is a normal random variable with mean μ and variance σ^2 , and if a, b are scalars, then show that the random variable

$$Y = aX + b$$

   is also normal with mean $a\mu + b$ and variance $a^2\sigma^2$.

9. What is the probability that a z picked at random from the population of z's will have a value between -2.5 and 2.5?

10. Two continuous random variables X and Y have a joint probability distribution function

$$f(x, y) = A(x + y),$$

    where A is a constant and $0 \le x \le 1; 0 \le y \le 1$.
    (a) Determine A.
    (b) Calculate the correlation ($\mathrm{Cov}(X, Y)$) between X and Y.

11. Roll a dice ($n = 1, 2, ...6$). Two events s_1 and s_2 are defined as follows:

$$s_1 = \begin{cases} 1 & \text{if } n = 2, 4, 6 \\ -1 & \text{if } n = 1, 3, 5 \end{cases}$$

$$s_2 = \begin{cases} 1 & \text{if } n = 3, 6 \\ -1 & \text{if } n = 1, 2, 4, 5 \end{cases}$$

Show that $< s_1 s_2 >=< s_1 >< s_2 >$. Show that $P(s_1, s_2) = P_1(s_1)P_2(s_2)$. So s_1 and s_2 are uncorrelated.

12. Repeat 1 with the following s_1 and s_2 to show that the events are correlated. Find $Cov(s_1, s_2)$ and correlation coefficient.

$$s_1 = \begin{cases} 1 & \text{if } n = 1, 2, 3 \\ -1 & \text{if } n = 4, 5, 6 \end{cases}$$

$$s_2 = \begin{cases} 1 & \text{if } n = 2, 4, 6 \\ -1 & \text{if } n = 1, 3, 5 \end{cases}$$

Table of Standard Normal Cumulative Probabilities

z	-3.0									
$\Phi(z)$	0.001									
z	-2.9	-2.8	-2.7	-2.6	-2.5	-2.4	-2.3	-2.2	-2.1	-2.0
$\Phi(z)$	0.002	0.003	0.003	0.005	0.006	0.008	0.011	0.014	0.018	0.023
z	-1.9	-1.8	-1.7	-1.6	-1.5	-1.4	-1.3	-1.2	-1.1	-1.0
$\Phi(z)$	0.029	0.036	0.045	0.055	0.067	0.081	0.097	0.115	0.136	0.159
z	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0.0
$\Phi(z)$	0.184	0.212	0.242	0.274	0.309	0.345	0.382	0.421	0.460	0.500
z	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\Phi(z)$	0.500	0.540	0.579	0.618	0.655	0.691	0.726	0.758	0.788	0.816
z	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$\Phi(z)$	0.841	0.864	0.885	0.903	0.919	0.933	0.945	0.955	0.964	0.971
z	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9
$\Phi(z)$	0.977	0.982	0.986	0.989	0.992	0.994	0.995	0.997	0.997	0.998
z	3.0									
$\Phi(z)$	0.999									