Problem Set-2

joint probability, random variables, cumulative distribution function, (probability) density function/mass function, expectation, variance

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- 1. Let X be a random variable with probability mass function (pmf) p(1) = .5, p(2) = .2 and p(3) = .3. Find the cdf.
- 2. Are the following functions cumulative distribution (cdf) function?
 - (a) $F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 e^{-x} & \text{if } x \ge 0 \end{cases}$
 - (b) $F(x) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1} x, -\infty < x < \infty.$
 - (c) $F(x) = \begin{cases} 0 & \text{if } x < -5 \\ x & \text{if } -5 \le x \le 0.5 \\ 1 & \text{if } x > 0.5 \end{cases}$
- 3. Let X be a random variable having the p.m.f.

$$p(x) = \begin{cases} \frac{c}{(2x-1)(2x+1)} & \text{if } x \in \{1, 2, 3, \cdots\} \\ 0 & \text{otherwise} \end{cases}$$

where c is a real constant.

- (a) Find the value of constant c.
- (b) Find the cdf of X.
- 4. Let X be a random variable with p.m.f

$$P(X=r) = \binom{n}{r} p^r (1-p)^{n-r}, \ r=0,1,2,\cdots,n, \ 0 \le p \le 1.$$

Find the p.m.f of the random variables (a) Y = aX + b and (b) $Y = X^2$.

- 5. Let X be a random variable denoting the outcomes of rolling of a die. Find the expectation and variance of random variable $Y = X^2$.
- 6. Suppose that 3 batteries are randomly chosen from a group of 3 new, 4 used but still working, and 5 defective batteries. If we let X and Y denote, respectively, the number of new and used but still working batteries that are chosen, then find the joint probability mass function of X and Y.
- 7. The quantity $0 \le x \le 1$ is distributed as P(x) = Ax(1-x). What is A? What is the average value of x and the standard deviation s?
- 8. In a study by Cross et al., patients who were involved in problem gambling treatment were asked about co-occurring drug and alcohol addictions. Let the discrete random variable X represent the number of co-occurring addictive substances used by the subjects. The table below summarizes the frequency distribution for this random variable.

Number of Substances Used	Frequency
0	144
1	342
2	142
3	72
4	39
5	20
6	6
7	9
8	2
9	1

- (a) Construct a table of the relative frequency and the cumulative frequency for this discrete distribution.
- (b) What is probability that an individual selected at random used five addictive substances?

- (c) What is the probability that an individual selected at random used more than six addictive substances?
- (d) What is the probability that an individual selected at random used between two and five addictive substances, inclusive?
- (e) Find the mean, variance, and standard deviation of this frequency distribution.
- 9. An exponential random variable X has a probability density function of the form

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & if x \ge 0\\ 0 & \text{otherwise} \end{cases}$$

Find the variance of X.