## Problem Set-2

joint probability, random variables, cumulative distribution function, (probability) density function/mass function, expectation, variance

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1. Let $X$ be a random variable with probability mass function (pmf) $p(1)=.5, p(2)=.2$ and $p(3)=.3$. Find the cdf.
2. Are the following functions cumulative distribution (cdf) function?
(a) $F(x)= \begin{cases}0 & \text { if } x<0 \\ 1-e^{-x} & \text { if } x \geq 0\end{cases}$
(b) $F(x)=\frac{1}{2}+\frac{1}{\pi} \tan ^{-1} x,-\infty<x<\infty$.
(c) $F(x)= \begin{cases}0 & \text { if } x<-5 \\ x & \text { if }-5 \leq x \leq 0.5 \\ 1 & \text { if } x>0.5\end{cases}$
3. Let $X$ be a random variable having the p.m.f.

$$
p(x)= \begin{cases}\frac{c}{(2 x-1)(2 x+1)} & \text { if } x \in\{1,2,3, \cdots\} \\ 0 & \text { otherwise }\end{cases}
$$

where $c$ is a real constant.
(a) Find the value of constant $c$.
(b) Find the cdf of $X$.
4. Let $X$ be a random variable with p.m.f

$$
P(X=r)=\binom{n}{r} p^{r}(1-p)^{n-r}, r=0,1,2, \cdots, n, 0 \leq p \leq 1
$$

Find the p.m.f of the random variables (a) $Y=a X+b$ and (b) $Y=X^{2}$.
5. Let $X$ be a random variable denoting the outcomes of rolling of a die. Find the expectation and variance of random variable $Y=X^{2}$.
6. Suppose that 3 batteries are randomly chosen from a group of 3 new, 4 used but still working, and 5 defective batteries. If we let $X$ and $Y$ denote, respectively, the number of new and used but still working batteries that are chosen, then find the joint probability mass function of $X$ and $Y$.
7. The quantity $0 \leq x \leq 1$ is distributed as $P(x)=A x(1-x)$. What is A? What is the average value of $x$ and the standard deviation $s$ ?
8. In a study by Cross et al., patients who were involved in problem gambling treatment were asked about co-occurring drug and alcohol addictions. Let the discrete random variable $X$ represent the number of co-occurring addictive substances used by the subjects. The table below summarizes the frequency distribution for this random variable.

| Number of Substances Used | Frequency |
| :---: | :---: |
| 0 | 144 |
| 1 | 342 |
| 2 | 142 |
| 3 | 72 |
| 4 | 39 |
| 5 | 20 |
| 6 | 6 |
| 7 | 9 |
| 8 | 2 |
| 9 | 1 |

(a) Construct a table of the relative frequency and the cumulative frequency for this discrete distribution.
(b) What is probability that an individual selected at random used five addictive substances?
(c) What is the probability that an individual selected at random used more than six addictive substances?
(d) What is the probability that an individual selected at random used between two and five addictive substances, inclusive?
(e) Find the mean, variance, and standard deviation of this frequency distribution.
9. An exponential random variable $X$ has a probability density function of the form

$$
f(x)= \begin{cases}\lambda e^{-\lambda x} & \text { if } x \geq 0 \\ 0 & \text { otherwise }\end{cases}
$$

Find the variance of $X$.

