Problem Set-1

Probability basics, conditional probability, Bayes' theorem, joint probability

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1. Let E, F, G be three events. Find expressions for the events that of E, F, G
   (a) only F occurs
   (b) both E and F but not G occur
   (c) at least one event occurs
   (d) at least two events occur
   (e) all three events occur
   (f) none occurs
   (g) at most one event occurs
   (h) at most two events occur

2. Let $S = \{0, 1, 2, \cdots\}$ and $E \subseteq S$. Then in each of the following cases, verify P is a probability on S.
   (a) $P(E) = \sum_{x \in E} \frac{e^{-\lambda}\lambda^x}{x!}, \lambda > 0.$
   (b) $P(E) = \sum_{x \in E} p(1-p)^x, p > 0$
   (c) $P(E) = 0$, if E has finite number of elements, and $P(E) = 1$, if E has infinite number of elements.

3. Throw a die 20 times. What is the probability that you get a 6 on 10th trial?

4. Let E and F be two independent events. Then show that
   (a) E^c and F are independent.
   (b) E and F^c are independent.
   (c) E^c and F^c are independent.

5. Two digits are chosen at random without replacement from the set of integers $1, 2, 3, 4, 5, 6, 7, 8$. Find the probability that both digits are greater than 5.

6. Consider two independent fair coins tosses, in which all four possible outcomes are equally likely. Let H1 = 1st toss is a head, H2 = 2nd toss is a head, and D = the two tosses have different results. Find $P(H1)$, $P(H2)$, $P(H1 \cap H2)$, $P(H1|D)$, $P(H2|D)$, and $P(H1 \cap H2|D)$.

7. An urn contains b black balls and r red balls. One of the balls is drawn at random, but when it is put back in the urn c additional balls of the same color are put in with it. Now suppose that we draw another ball. Find the probability that the first ball drawn was black given that the second ball drawn was red.

8. Suppose 84% of hypertensive and 23% of normotensive are classified as hypertensive by an automated blood-pressure machine. What are the predictive value positive and predictive value negative of the machine, assuming 20% of the adult population is hypertensive?

9. The primary aim of a study by Carter et al. was to investigate the effect of the age at onset of bipolar disorder on the course of the illness. One of the variables investigated was family history of mood disorders. The table shows the frequency of a family history of mood disorders in the two groups of interest (Early age at onset defined to be 18 years or younger and Later age at onset defined to be later than 18 years). Suppose we pick a person at random from this sample. What is the probability that this person will be 18 years old or younger?

Family History of Mood Disorders	Early=18(E)	Later > 18(L)	Total
Negative (A)	28	35	63
Bipolar Disorder (B)	23	32	55
Unipolar (C)	44	49	93
Unipolar and Bipolar (D)	56	62	118
Total	151	178	339

Suppose we pick a subject at random from the 339 subjects and find that he is 18 years or younger (E). What is the probability that this subject will be one who has no family history of mood disorders (A)?

Joint probability: What is the probability that a person picked at random from the 339 subjects will be Early (E) and will be a person who has no family history of mood disorders (A)?

10. A bag contains ten balls. Among them six are red and four are white. Three balls are drawn at random and not replaced. Find the probability mass function for the number of red balls drawn.

11. Suppose the random variable X has density function

$$F(x) = \begin{cases} Ae^{-x} & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

Find the value of the constant A and hence calculate the probability that X lies in the interval $1 < X \leq 2$.

12. The joint density function of X and Y is given by

$$\begin{aligned} f(x,y) &= 2e^{-x}e^{-2y}; \qquad 0 < x < \infty,\ 0 < y < \infty \\ &= 0 \qquad otherwise \end{aligned}$$

Compute $(a)P\{X > 1, Y < 1\}$; $(b)P\{X < Y\}$.