Problem Set-2

Dated: 23/08/2018 & Submission deadline is on: 30/8/2018

1. Let X be a random variable having the p.m.f.

$$p(x) = \begin{cases} \frac{c}{(2x-1)(2x+1)} & \text{if } x \in \{1, 2, 3, \cdots\} \\ 0 & \text{otherwise} \end{cases}$$

where c is a real constant.

   (a) Find the value of constant c.

   (b) Find the cumulative distribution function of X.

   (c) For the positive integers m and n such that $m < n$, evaluate $P(X < m+1), P(X \geq m), P(m \leq X < n)$ and $P(m < X \leq n)$.

   (d) Find the conditional probabilities $P(\{X > 1\}|\{1 \leq X < 4\})$ and $P(\{1 < X < 6\}|\{X \geq 3\})$.

2. The primary aim of a study by Carter et al. was to investigate the effect of the age at onset of bipolar disorder on the course of the illness. One of the variables investigated was family history of mood disorders. The table shows the frequency of a family history of mood disorders in the two groups of interest (Early age at onset defined to be 18 years or younger and Later age at onset defined to be later than 18 years). Suppose we pick a person at random from this sample. What is the probability that this person will be 18 years old or younger?

Family History of Mood Disorders	Early=18(E)	Later $>$ 18(L)	Total
Negative (A)	28	35	63
Bipolar Disorder (B)	23	32	55
Unipolar (C)	44	49	93
Unipolar and Bipolar (D)	56	62	118
Total	151	178	339

Suppose we pick a subject at random from the 339 subjects and find that he is 18 years or younger (E). What is the probability that this subject will be one who has no family history of mood disorders (A)?

Joint probability: What is the probability that a person picked at random from the 339 subjects will be Early (E) and will be a person who has no family history of mood disorders (A)?

3. A bag contains ten balls. Among them six are red and four are white. Three balls are drawn at random and not replaced. Find the probability mass function for the number of red balls drawn.

4. The exponential distribution for random variable X is given by

$$f(x) = \lambda e^{-\lambda x}, \qquad x \geq 0$$

Find the mean of x. Find the moment generating function $M_X(t)$ of the p.d.f.

5. Suppose the random variable X has distribution function

$$F(x) = \begin{cases} Ae^{-x} & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

Find the value of the constant A and hence calculate the probability that X lies in the interval $1 < X < 2$.

6. Roll a dice ($n = 1, 2, ...6$). Two events s_1 and s_2 are defined as follows:

$$s_1 = \begin{cases} 1 & \text{if } n = 2, 4, 6 \\ -1 & \text{if } n = 1, 3, 5 \end{cases}$$

$$s_2 = \begin{cases} 1 & \text{if } n = 3, 6 \\ -1 & \text{if } n = 1, 2, 4, 5 \end{cases}$$

Show that $< s_1 s_2 >=< s_1 >< s_2 >$. Show that $P(s_1, s_2) = P_1(s_1)P_2(s_2)$. So s_1 and s_2 are uncorrelated.

7. Repeat 1 with the following s_1 and s_2 to show that the events are correlated. Find $Cov(s_1, s_2)$ and correlation coefficient.

$$s_1 = \begin{cases} 1 & \text{if } n = 1, 2, 3 \\ -1 & \text{if } n = 4, 5, 6 \end{cases}$$

$$s_2 = \begin{cases} 1 & \text{if } n = 2, 4, 6 \\ -1 & \text{if } n = 1, 3, 5 \end{cases}$$

8. (i) Suppose a group of 100 men aged $60-64$ in Dehradun received a new flu vaccine from a health center in 2014. From the 2014 life table of the health center, it is found that the approximate probability that a man, aged between $60-64$, dies in the next year is 0.02. How likely are, at least 5 out of 100 men who received flu vaccine and aged $60-64$ to die within the next year?
(ii) What is the probability that amongst the 60 to 64-year old men who got flu vaccination exactly 25 survive and at least 10 die within the next year? (*you don't need to calculate the exact numerical values of the probabilities*)