Problem Set-1

Probability basics, conditional probability, Bayes' theorem, joint probability
Date: 10/08/2018

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1. Let E, F, G be three events. Find expressions for the events that of E, F, G

   (a) only F occurs

   (b) both E and F but not G occur

   (c) at least two events occur

   Suppose 84% of hypertensive and 23% of normotensive are classified as hypertensive by an automated blood-pressure machine. What are the predictive value positive and predictive value negative of the machine, assuming 20% of the adult population is hypertensive?

2. Let E and F be two independent events. Then show that E^c and F^c are independent.

   You enter a chess tournament where your probability of winning a game is 0.3 against half the players (call them type 1), 0.4 against a quarter of the players (call them type 2), and 0.5 against the remaining quarter of the players (call them type 3). You play a game against a randomly chosen opponent. What is the probability of winning?

3. Alice is taking a probability class and at the end of each week she can be either up-to-date or she may have fallen behind. If she is up-to-date in a given week, the probability that she will be up-to-date (or behind) in the next week is 0.8 (or 0.2, respectively). If she is behind in a given week, the probability that she will be up-to-date (or behind) in the next week is 0.6 (or 0.4, respectively). Alice is (by default) up-to-date when she starts the class. What is the probability that she is up-to-date after three weeks?

4. A total of 28 percent of American males smoke cigarettes, 7 percent smoke cigars, and 5 percent smoke both cigars and cigarettes. What percentage of males smoke neither cigars nor cigarettes?

   An insurance company believes that people can be divided into two classes ? those that are accident prone and those that are not. Their statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability .4, whereas this probability decreases to .2 for a non-accident-prone person. If we assume that 30 percent of the population is accident prone, what is the probability that a new policy holder will have an accident within a year of purchasing a policy?

5. Two digits are chosen at random without replacement from the set of integers $1, 2, 3, 4, 5, 6, 7, 8$. Find the probability that both digits are greater than 5.

   An urn contains b black balls and r red balls. One of the balls is drawn at random, but when it is put back in the urn c additional balls of the same color are put in with it. Now suppose that we draw another ball. Find the probability that the first ball drawn was black given that the second ball drawn was red.

6. Three cards are drawn from an ordinary 52-card deck without replacement (drawn cards are not placed back in the deck). Find the probability that none of the three cards is a heart. What is the probability of 1st is not a heart and 2nd is a heart?

   For any two disjoint events A_1 and A_2, Show

$$P(A_1 \cup A_2|B) = P(A_1|B) + P(A_2|B)$$