General Relativity
Problem Set III
Lie derivative, Connection, Curvature
Due date: 27/04/2018

1. Find out the Lie derivative of a one-form in a coordinate independent way. Verify your result by using the Lie derivative formulas for a function and a vector field.

2. Find out the Lie derivative of a $(0, 2)$ type tensor.

3. Show that $\mathcal{L}_X i_X \omega = i_X \mathcal{L}_X \omega$ for any r-form ω.

4. Show that connection co-efficient does not transform as a tensor. [Convince yourself that connection coefficients are zero for any flat manifold.]

5. Show that covariant differentiation commutes with contraction.

6. Show that $[\nabla_\mu, \nabla_\nu]V^\rho = R^\rho{}_{\sigma\mu\nu}V^\sigma$

7. Find out the Christoffel connection coefficients of a 2-sphere. Find the Riemann tensor, Ricci tensor components and curvature (Riccci) scalar of 2-sphere.

8. Deduce the equation of geodesic of a 2-sphere and find the geodesic curve.

9. Show that $\Gamma^\mu_{\mu\nu} = \frac{1}{\sqrt{|g|}}\partial_\nu(\sqrt{|g|})$. Where $|g|$ is the modulus of the determinant of the metric tensor.

10. Riemann Normal Coordinates (RNC) denotes a coordinate system where Christoffel symbols vanish but not their derivatives.

    Use RNC to prove the Bianchi identity: $\nabla_{[\rho} R_{\mu\nu]\sigma\alpha} = 0$.