# Discrete Mathematics
# Propositional Logic

1. Let p and q be proposition

   p : Swimming at the New Jersey shore is allowed.

   q : Sharks have been spotted near the shore.

   Express the following statements as an English sentence.

   **a.)** $\sim p \leftrightarrow \sim q$ **b.)** $\sim p \vee (p \wedge q)$ **c.)** $\sim p \rightarrow \sim q$ **d.)** $q \rightarrow p$

2. Let p, q and r be proposition

   p : You get an A on the final exam.

   q : You do every given assignment.

   r : You get an A in Discrete mathematics.

   Write these propositions using p, q and r and the logical connectives.

   **a.)** You get an A in Discrete mathematics, but you do not do every given assignment.

   **b.)** You get an A on the final exam, You do every given assignment and You get an A in Discrete mathematics.

   **c.)** You get an A in Discrete mathematics if and only if you either do every given assignment or you get an A in the final exam.

   **d.)** If you do every given assignment, you get A on the final exam if and only if you get an A in discrete mathematics.

   **e.)** To get an A on the final exam, it is necessary for you to get an A in Discrete mathematics.

3. State the converse, contrapositive, and inverse of each of the conditional statements.

   **a.)** When I stay up late, it is necessary that I sleep until noon.

   **b.)** I go to the class whenever there is going to be a quiz.

   **c.)** A positive integer is prime only if it has no divisors other than 1 and itself.

   **d.)** If it snows tonight, then I will stay at home.

4. Construct a Truth Table for each of these compound propositions.

   **a.)** $p \rightarrow \sim q$ **b.)** $(p \wedge q \wedge r) \leftrightarrow (q \wedge r)$

   **c.)** $(p \vee \sim q) \wedge (p \vee \sim s)$ **d.)** $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$

   **e.)** $(p \oplus q) \vee (p \oplus \sim q)$ **f.)** $(p \leftrightarrow q) \oplus (p \leftrightarrow \sim q)$

   **g.)** $\sim(\sim p \leftrightarrow \sim q) \leftrightarrow (p \leftrightarrow r)$ **h.)** $(p \wedge q) \wedge r$

   **i.)** $(p \vee q) \wedge \sim r$

5. Find the dual of each of these compound propositions.

   **a.)** $p \vee \sim q$ **b.)** $p \wedge \sim q \wedge \sim r$

   **c.)** $(p \vee \mathbf{F}) \wedge (q \vee \mathbf{T})$ **d.)** $(p \wedge q \wedge r) \vee s$

6. Check if each of these conditional statements is a tautology by using truth tables.
   **a.)** $(p \wedge q) \rightarrow p$ **b.)** $(p \wedge q) \rightarrow (p \rightarrow q)$
   **c.)** $(\sim p \wedge (p \vee q)) \rightarrow q$ **d.)** $(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$
   **e.)** $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ **f.)** $(p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r)$
7. Show that each of these following statement is a tautology without using truth table.
   **a.)** $\sim(p \rightarrow q) \rightarrow p$ **b.)** $(p \wedge (p \rightarrow q)) \rightarrow q$
   **c.)** $\sim p \rightarrow (p \rightarrow q)$ **d.)** $\sim(p \rightarrow q) \rightarrow \sim q$
8. Check if the following statements are logically equivalent or not.
   **a.)** $p \rightarrow q$ **and** $\sim q \rightarrow \sim p$ **b.)** $(p \rightarrow q) \wedge (p \rightarrow r)$ **and** $p \rightarrow (q \wedge r)$
   **c.)** $p \leftrightarrow q$ **and** $\sim p \leftrightarrow \sim q$ **d.)** $\sim p \rightarrow (q \rightarrow r)$ **and** $q \rightarrow (p \vee r)$
   **e.)** $(p \rightarrow q) \rightarrow r$ **and** $p \rightarrow (q \rightarrow r)$ **f.)** $\sim(p \oplus q)$ **and** $p \leftrightarrow q$
9. Check whether following statements are satisfiable or not.
   **a.)** $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ **b.)** $(p \wedge q) \rightarrow (p \rightarrow q)$
   **c.)** $(p \vee q) \wedge (\sim p \vee r) \rightarrow (q \vee r)$ **d.)** $(p \wedge (p \rightarrow q)) \rightarrow q$
10. Use De Morgan's law to find the negation of these statements.
    **a.)** Mikael has a phone and he has a laptop.
    **b.)** Saumya will go to the concert or Soham will go the concert.
    **c.)** Rahul is not intelligent but hard working.
    **d.)** Carlos will bicycle or run tomorrow.
11. What does the statement $\forall x\, N(x)$ means if $N(x)$ is "Computer x is connected to the network" and the domain consists of all computers on campus?
12. Show that if each of these following are logically equivalent.
    **a.)** $\forall x\, p(x) \vee \forall x\, q(x)$ **and** $\forall x\, (p(x) \vee q(x))$
    **b.)** $\exists x\, p(x) \wedge \exists x\, q(x)$ **and** $\exists x\, (p(x) \wedge q(x))$
13. Let $P(x)$ be the predicate "x must take a discrete mathematics course" and let $Q(x)$ be the predicate "x is a computer science student". Use quantifiers to express each of these statements:
    **a.)** Every computer science student must take a discrete mathematics course.
    **b.)** Everybody must take a discrete mathematics course or be a computer science student.
14. Express each of these using quantifiers.
    **a.)** All dogs have fleas.
    **b.)** Every bird can fly.
    **c.)** There is no dog that can talk.
    **d.)** No one can keep a secret.
    **e.)** There exists a pig that can swim and catch fish.
    **f.)** Some drivers do not obey the speed limit.

**g.)** There is someone in this class who does not have a good attitude.

15. Let S(x) be the predicate " **x** is a student", F(x) the predicate " **x** is a faculty member", and A(x,y) the predicate "**x** has asked **y** a question," where the domain consist of all people associated with your school. Use quantifiers to express each of these statements.

    **a.)** Every student has asked Professor Srijit a question.

    **b.)** Some students has not asked any faculty member a question.

    **c.)** There is a faculty member who has never been asked a question by a student.

    **d.)** Some student has asked every faculty member a question.

    **e.)** There is a faculty member who has asked every other faculty member a question.

    **f.)** Every faculty member has either asked Professor Mikael a question or been asked a question by Professor Millar.