Answers

1. **a.)** $\sim p \leftrightarrow \sim q$

   Swimming at the New Jersey shore is not allowed if and only if sharks have not been spotted near the shore.

   b.) $\sim p \vee (p \wedge q)$

   Swimming at the New Jersey shore is not allowed or swimming at the New Jersey shore is allowed and sharks have been spotted near the shore.

   c.) $\sim p \rightarrow \sim q$

   If swimming at the New Jersey shore is not allowed, then sharks have not been spotted near the shore.

   d.) $q \rightarrow p$

   If sharks have been spotted near the shore then Swimming at the New Jersey is allowed.

2. **a.)** $r \wedge \sim q$ **b.)** $p \wedge q \wedge r$ **c.)** $r \leftrightarrow (p \vee q)$ **d.) (**$q \rightarrow p) \leftrightarrow r$

   e.) $r \rightarrow p$

3. **a.) Converse:** If I stay up late then I will sleep until noon.

   Contrapositive: If i don't stay up late then I will not sleep until noon.

   Inverse: If I don't sleep until noon then I haven't stayed up late.

   b.) Converse: If I come to class then there will be a quiz.

   Contrapositive: If I do not come to class then there will not be a quiz.

   Inverse: If there is not going to be a quiz then i don't come to the class.

   c.) Converse: A positive integer is a prime if it has no divisors other than 1 and itself.

   Contrapositive: If a positive integer has a divisor other than 1 and itself then it is not prime.

   Inverse: If a positive integer is not prime, then it has a divisor other than1 and itself.

   d.) Converse: I will stay at home only if it snows tonight.

   Contrapositive: I will not stay at home only if it doesn't snow tonight.

   Inverse: If it doesn't snow tonight then I will not stay home.

4. (Hint: Draw Truth table using basic connectives)
5. (Hint: Dual is the negation of a connective. Let say dual of **False** is **True**, **True** is **False**, ^ is ⋁, ⋁ is ^**)**

6. (Hint: Tautology is All true values in the table.)
7. (Hint: If final result is 1 then it is a tautology.)
8. (Hint: Verify using Truth Table or Reducing the proposition.)
9. (Hint: Satisfiable is either tautology or contingency. Either all values are true by the end or at least one is true. All values should not be false.)
10. (Hint: De Morgan's law:

$$\sim(p \wedge q) = \sim p \vee \sim q$$

$$\sim(p \vee q) = \sim p \wedge \sim q$$

11. $\forall x\, N(x)$ means: Every computer on campus is connected to the network.
12. (Hint: Use De Morgan's law for Universal quantifiers)
13. **a.)** $\forall x\, (S(x) \rightarrow P(x))$

    b). $\forall x\, (S(x) \vee P(x))$

14. **a.)** $\forall\ x\, (D(x) \rightarrow F(x))$

    b.) $\forall\ x\, (B(x) \rightarrow F(x))$

    c.) $\sim \exists x\, T(x)$ (Domain: Dog)

    d.) $\neg \exists x\, S(x)$(Domain: People)

    e.) $\exists\ x\, (P(x) \wedge S(x) \wedge F(x)))$

    f.) $\exists x \neg S(x)$ (S(x) be the predicate "x obeys the speed limit" and Domain is drivers)

    g.) $\exists x \neg A(x)$ (A(x) be the predicate "x has a good attitude" and Domain is everyone in class)

15. a.) $\forall x(S(x) \rightarrow A(x, \text{Professor Srijit}))$

    b.) $\exists x(S(x) \wedge \forall y(F(y) \rightarrow \sim A(x,y)))$

    c.) $\exists x(F(x) \wedge \forall y(S(y) \rightarrow \sim A(y,x)))$

    d.) $\forall y(F(y) \rightarrow \exists x(S(x) \vee A(x,y)))$

    e.) $\exists x(F(x) \wedge \forall y((F(y) \wedge (y \neq x)) \rightarrow A(x,y)))$

    f.) $\forall x(F(x) \rightarrow (A(x, \text{Professor Mikael}) \vee A(\text{Professor Mikael}, x)))$