UNIVARIATE AND MULTIVARIATE CALCULUS - ASSESSMENT I
SECTION D

Question. Given nonempty subsets A and B of positive real numbers, define

$$AB = \{xy : x \in A, y \in B\}.$$

Show that $\sup(AB) = \sup A \sup B$.

Solution. Let $\sup A = \alpha$ and $\sup B = \beta$. Then

$x \leq \alpha$ for all $x \in A$ and $y \leq \beta$ for all $y \in B$. [1]

$\implies xy \leq \alpha\beta$ for all $x \in A$ and $y \in B$. Thus, $\alpha\beta$ is an upper bound of AB. [1]

Let γ be any upper bound of AB. Then $xy \leq \gamma$ for all $x \in A$ and $y \in B$.

In particular, $x \leq \frac{\gamma}{y}$ for all $x \in A$. This implies that $\alpha \leq \frac{\gamma}{y}$ by definition of supremum. [2]

$\implies y \leq \frac{\gamma}{\alpha}$ for all $y \in B$. Hence, $\beta \leq \frac{\gamma}{\alpha}$ or $\alpha\beta \leq \gamma$. [1]