UNIVARIATE AND MULTIVARIATE CALCULUS - ASSESSMENT I
SECTION C

Question.

(1) Write down the definition of convergence of sequence in terms of quantifiers. [1]

Solution. Let (x_n) be a sequence. We say that (x_n) is convergent if

$\exists\, l \in \mathbb{R}\ (\forall\, \epsilon > 0\ (\exists\, N \in \mathbb{N}\ (\forall\, n \geq N\ (|x_n - l| < \epsilon))))$. [1]

(2) Let $A \subset \mathbb{R}$ be nonempty. Define $-A = \{-x : x \in A\}$. Show that

$\sup(-A) = -\inf A$.

Solution. Let $\inf A = \alpha$. Then $\alpha \leq x$ for all $x \in A$. [1]

$\implies -x \leq -\alpha$ for all $x \in A$. Hence, $-\alpha$ is an upper bound of $-A$. [1]

Let β be any upper bound of $-A$. Then $-x \leq \beta$ for all $x \in A$.

It follows that $-\beta \leq x$ for all $x \in A$. Hence, $-\beta$ is a lower bound of A. [1]

By definition of infimum, $-\beta \leq \alpha$ or $-\alpha \leq \beta$. [1]