UNIVARIATE AND MULTIVARIATE CALCULUS - ASSESSMENT I SECTION B

Question. Find the infimum of the set $A = \{\frac{m}{m+n} : m, n \in \mathbb{N}\}.$

Solution. First we note that $0 < \frac{m}{m+n}$. Thus, 0 is a lower bound of A. [1]

Let $\alpha > 0$. We will show that α is not a lower bound. By Archimedean property, there exists $n \in \mathbb{N}$ such that $n\alpha > 1$. [2]

 $\implies \frac{1}{n+1} < \frac{1}{n} < \alpha$. We observe that $\frac{1}{n+1} \in A$. Hence, α is not a lower bound of A. [2]