UNIVARIATE AND MULTIVARIATE CALCULUS - ASSESSMENT I
SECTION A

Question. Let A and B be nonempty subsets of real numbers. Show that $\sup(A \cup B) = \max\{\sup A, \sup B\}$.

Solution. Let $\sup A = \alpha$ and $\sup B = \beta$. Then

$x \leq \alpha$ for all $x \in A$ and $y \leq \beta$ for all $y \in B$. [1]

$\Longrightarrow z \leq \max\{\alpha, \beta\}$ for all $z \in A \cup B$. Thus, $\max\{\alpha, \beta\}$ is an upper bound of $A \cup B$. [1]

Let γ be any upper bound of $A \cup B$. Then $z \leq \gamma$ for all $z \in A \cup B$. In particular, $z \leq \gamma$ for all $z \in A$ and $z \leq \gamma$ for all $z \in B$. [1]

$\Longrightarrow \alpha \leq \gamma$ and $\beta \leq \gamma$ (by definition of supremum) [1]

Therefore, $\max\{\alpha, \beta\} \leq \gamma$. [1]