TRIPLE INTEGRALS

(1) Let D denote the solid bounded by the surfaces $y = x$, $y = x^2$, $z = x$ and $z = 0$. Evaluate $\iiint\limits_D y\,dxdydz$.

(2) Let D denote the solid bounded below by the plane $z + y = 2$, above by the cylinder $z + y^2 = 4$ and on the sides $x = 0$ and $x = 2$. Evaluate $\iiint\limits_D x\,dxdydz$.

(3) Suppose $\int_0^4 \int_{\sqrt{x}}^2 \int_0^{2-y} dzdydx = \iiint\limits_D dxdydz$ for some region $D \subset \mathbb{R}^3$.

(a) Sketch the region D.

(b) Sketch the projections of D on the xy, yz and xz planes.

(c) Write $\int_0^4 \int_{\sqrt{x}}^2 \int_0^{2-y} dzdydx$ as iterated integrals of other orders.

(4) Let $D = \{(x, y, z) \in \mathbb{R}^3 : \frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{9} \leq 1\}$ and $E = \{(u, v, w) \in \mathbb{R}^3 : u^2 + v^2 + w^2 \leq 1\}$. Show that $\iiint\limits_D dxdydz = \iiint\limits_E 24dudvdw$.

(5) In each of the following cases, describe the solid D in terms of the cylindrical coordinates.

(a) Let D be the solid that is bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$.

(b) Let D be the solid that lies within the cylinder $x^2+(y-1)^2 = 1$ below the paraboloid $z = x^2 + y^2$and above the plane $z = 0$.

(c) Let S denote the torus generated by revolving the circle $\{(x, z) : (x - 2)^2 + z^2 = 1\}$ about the z-axis. Let D be the solid that is bounded above by the surface S and below by $z = 0$.

(6) Let D be the solid that lies inside the cylinder $x^2 + y^2 = 1$, below the cone $z = \sqrt{4(x^2 + y^2)}$ and above the plane $z = 0$. Evaluate $\iiint\limits_D x^2dxdydz$.

(7) Evaluate $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{x^2+y^2}^4 x\,dzdydx$.

(8) Describe the following regions in terms of the spherical coordinates.

(a) The region that lies inside the sphere $x^2 + y^2 + (z - 2)^2 = 4$ and outside the sphere $x^2 + y^2 + z^2 = 1$.

(b) The region that lies below the sphere $x^2 + y^2 + z^2 = z$ and above the cone $z = \sqrt{x^2 + y^2}$.

(c) The region that is enclosed by the cone $z = \sqrt{3(x^2 + y^2)}$ and the planes $z = 1$ and $z = 2$.

(9) Let D denote the solid bounded above by the plane $z = 4$ and below by the cone $z = \sqrt{x^2 + y^2}$. Evaluate $\iiint\limits_D \sqrt{x^2 + y^2 + z^2}dxdydz$.

(10) Let D denote the solid enclosed by the spheres $x^2+y^2+(z-1)^2 = 1$ and $x^2+y^2+(z-1)^2 = 3$. Using spherical coordinates, set up iterated integrals that gives the volume of D.