DOUBLE INTEGRALS

(1) Let $R = [0, \frac{\pi}{2}] \times [0, \frac{\pi}{2}]$. Evaluate $\iint\limits_R \sin x \cos y dxdy$.

(2) Let R be the region in $\mathbb{R}^2$ bounded by the curves $y = 2x^2$ and $y = 1 + x^2$. Evaluate $\iint\limits_R (2x^2 + y)dxdy$.

(3) Evaluate the following iterated integrals by interchanging the order of integration.

(a) $\int_0^1 \int_y^1 \cos x^2 dxdy$ (b) $\int_0^1 \int_{x^2}^1 x^3 e^{y^3} dydx$.

(4) Find the volume of the solid enclosed by the surfaces $z = 6 - x^2 - y^2$, $z = 2x^2 + y^2 - 1$, $x = -1$, $x = 1$, $y = -1$ and $y = 1$.

(5) Let D be the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $y + z = 1$ and $z = 0$. Find the volume of D.

(6) Let R be the region in $\mathbb{R}^2$ bounded by the straight lines $y = x$, $y = 3x$ and $x + y = 4$. Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(u, v) = (u - v, u + v)$. Find the set $S \subset \mathbb{R}^2$ satisfying $T(S) = R$.

(7) Let R be the region in $\mathbb{R}^2$ bounded by the curve defined in the polar co-ordinates $r = 1 - \cos\theta$, $0 \leq \theta \leq \pi$ and the x-axis. Consider the transformation $T : [0, \pi] \times [0, 1] \to \mathbb{R}^2$ defined by $T(r, \theta) = (r\cos\theta, r\sin\theta)$. Let S be the subset of $[0, \pi] \times [0, 1]$ satisfying $T(S) = R$. Sketch the regions S and R.

(8) Using the change of variables $u = x + y$ and $v = x - y$, show that $\int_0^1 \int_0^x (x - y)dydx = \int_0^1 \int_v^{2-v} \frac{v}{2} dudv$.

(9) Let R be the region bounded by $x = 0$, $x = 1$, $y = x$ and $y = x + 1$. Show that $\iint\limits_R \frac{dxdy}{\sqrt{xy - x^2}} = \int_0^1 \frac{du}{\sqrt{u}} \int_0^1 \frac{dv}{\sqrt{v}}$.

(10) Convert $\int_0^1 \int_{x^2}^x dydx$ into an iterated integral involving polar coordinates.

(11) Find the volume of the solid in the first octant bounded below by the surface $z = \sqrt{x^2 + y^2}$ and above by $x^2 + y^2 + z^2 = 8$ as well as the planes $y = 0$ and $y = x$.