LOCAL EXTREMA AND SADDLE POINTS

(1) Examine the following functions for local maxima, local minima and saddle points.

   (a) $x^2 - y^2$.

   (b) $x^4 + y^4 - 2x^2 - 2y^2 + 4xy$.

   (c) $xy - x^2 - y^2 - 2x - 2y + 4$.

(2) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = xye^{-(x^2+y^2)}$ for all $(x, y) \in \mathbb{R}$.

   (a) Identify the points of local maxima and minima, and the saddle points.

   (b) Show that f is bounded on $\mathbb{R}^2$.

   (c) Show that the points of local maxima/minima are the points of absolute maxima/minima.

(3) Find a point on the surface $z = xy + 1$ which is nearest to $(0, 0, 0)$.

(4) Find the points of absolute maximum and absolute minimum of the function $f(x, y) = x^2 + y^2 - 2x + 2$ on the region $\{(x, y) : x^2 + y^2 \leq 4 \text{ with } y \geq 0\}$.

(5) Let $D = \{(x, y) \in \mathbb{R}^2 : x > 0 \text{ and } y > 0\}$ and $f : D \to \mathbb{R}$ be given by $f(x, y) = xy + \frac{1000}{x} + \frac{1000}{y}$. Find the infimum of the function $f(x, y)$ on D.

(6) Let $f(x, y) = 3x^4 - 4x^2y + y^2$. Show that f has a local minimum at $(0, 0)$ along every line through $(0, 0)$. Does f have a minimum at $(0, 0)$? Is $(0, 0)$ a saddle point for f?

(7) Let $D = [-2, 2] \times [-2, 2]$ and $f : D \to \mathbb{R}$ be defined as $f(x, y) = 4xy - 2x^2 - y^4$. Find absolute maxima and absolute minima of f in D.