CONVERGENCE TESTS II: RATIO, ROOT AND LEIBNIZS TESTS

(1) Determine the values of $\alpha \in \mathbb{R}$ for which $\sum_{n=1}^{\infty} (\frac{\alpha n}{n+1})^n$ converges.

(2) Consider $\sum_{n=1}^{\infty} a_n$, where $a_n > 0$ for all n. Prove or disprove the following statements.

(a) If $\frac{a_{n+1}}{a_n} < 1$ for all n, then the series converges.

(b) If $\frac{a_{n+1}}{a_n} > 11$ for all n, then the series diverges.

(3) Show that the series $\frac{1}{1^2} + \frac{1}{2^3} + \frac{1}{3^2} + \frac{1}{4^3} + \frac{1}{5^2} + \frac{1}{6^3} + \cdots$ converges and that the root test and ratio test are not applicable.

(4) Consider the rearranged geometric series $\frac{1}{2} + 1 + \frac{1}{8} + \frac{1}{4} + \frac{1}{32} + \frac{1}{16} + \frac{1}{128} + \frac{1}{64} + \cdots$. Show that the series converges by the root test and that the ratio test is not applicable.

(5) (a) If $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converges absolutely, show that $\sum_{n=1}^{\infty} a_n b_n$ converges absolutely.

(b) If $\sum_{n=1}^{\infty} a_n$ converges absolutely and (b_n) is a bounded sequence, show that $\sum_{n=1}^{\infty} a_n b_n$ converges absolutely.

(c) Give an example of a convergent series $\sum_{n=1}^{\infty} a_n$ and a bounded sequence (b_n) such that $\sum_{n=1}^{\infty} a_n b_n$ diverges.

(6) n each of the following cases, discuss the convergence/divergence of the series $\sum_{n=1}^{\infty} a_n$ where a_n equals

a) $\frac{n!}{n^n}$	b) $\frac{7^{n+1}}{9^n}$	c) $\frac{n!}{(e)^{n^2}}$	d) $\frac{n^2 2^n}{(2n+1)!}$
e) $(1 - \frac{1}{n})^{n^2}$	f) $\frac{n^2}{3^n}(1 + \frac{1}{n})^{n^2}$	g) $\sin(\frac{(-1)^n}{n^p}), p > 0$	h) $\frac{1}{2^n - n}$
i) $(-1)^n \frac{(\ln n)^3}{n}$	j) $(-1)^n (n^{\frac{1}{n}} - 1)^n$	k) $\frac{2^n + n^2 - \ln n}{n!}$	l) $\frac{\cos(\pi n) \ln n}{n}$
m) $(1 + \frac{2}{n})^{n^2 - \sqrt{n}}$	n) $\frac{n^2 (2\pi + (-1)^n)^n}{10^n}$		