Problem Set 08: Test for maxima and minima

(1) Let $h : \mathbb{R} \longrightarrow \mathbb{R}$ be defined as $h(x) = f(x)g(x)$, where f and g are non-negative functions. Show that h has a local maximum at a if f and g have a local maximum at a.

(2) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined by $f(x) = (\sin x - \cos x)^2$. Find the maximum value of f on $\mathbb{R}$.

(3) Let $f : [-2, 0] \longrightarrow \mathbb{R}$ be defined by $f(x) = 2x^3 + 2x^2 - 2x - 1$. Find the maximum and minimum values of f on $[-2, 0]$.

(4) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be such that $f'(x) = 14(x-2)(x-3)^2(x-4)^3(x-5)^4$. Find all the points of local maxima and local minima.

(5) Let $x_1, x_2, \ldots, x_n \in \mathbb{R}$ and $f(x) = \sqrt{(x-x_1)^2 + (x-x_2)^2 + \cdots + (x-x_n)^2}, x \in \mathbb{R}$. Find the point of absolute minimum of the function f.

(6) Find the points of local maxima and local minima of $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $f(x) = 2x^4 e^{-x^2}$.

(7) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a twice differentiable function with the following properties: $f(-1) = 4$, $f(0) = 2$, $f(1) = 0$, $f'(x) > 0$ for $|x| > 1$, $f'(x) < 0$ for $|x| < 1$, $f'(1) = 0$, $f'(-1) = 0$, $f''(x) < 0$ for $x < 0$ and $''(x) > 0$ for $x > 0$. Sketch the graph of f.

(8) Sketch the graphs of the following functions after finding the intervals of decrease/increase, concavity/convexity, points of local minima/local maxima, points of inflection and asymptotes.

a) $f(x) = \frac{x^2+x-5}{x-1}$ b) $f(x) = \frac{2x^2-1}{x^2-1}$ c) $f(x) = \frac{x^2}{x^2+1}$

d) $f(x) = \frac{2x^3}{x^2-4}$ e) $f(x) = 3x^4 - 8x^3 + 12$.