UNIVARIATE AND MULTIVARIATE CALCULUS - ASSESSMENT I
IFE

Question.

Let ℓ be a lower bound of a set A. Show that $\inf A = \ell$ if and only if there exists a sequence (a_n) in A such that $x_n \to \ell$.

Solution. ($\Longrightarrow$) For $\epsilon = \frac{1}{n}$, there exist $x_n \in A$ such that $x_n < \ell + \frac{1}{n}$, i.e., $\ell \leq x_n < \ell + \frac{1}{n}$. By Sandwich theorem, $x_n \to \ell$. [2]

$\Longleftarrow$ Let $\varepsilon > 0$. As $a_n \to \ell$, $\exists\ N \in \mathbb{N}$ such that $|a_n - \ell| < \varepsilon\ \forall\ n \geq N$. [1]

This implies that $a_N < \ell + \varepsilon$. We note that $a_N \in A$. Therefore, $\inf A = \ell$. [2]