Indian Institute of Information Technology Allahabad
Convex Optimization (SMAT430C): Quiz II

Attempt as many **Questions** as you can. Do not worry about marks. Time allotted is **1 hour**.

1. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a continuous function. Let $c \in \mathbb{R}$ be a point such that for $t \leq c$, f is decreasing, and for $t \geq c$, f is increasing. Prove that f is quasiconvex. (Note: f need not be convex).

2. Find $\sup\{a^T x : ||x||_2 \leq 5\}$, where a is a nonzero vector in $\mathbb{R}^n$.

3. Let the pair x and (λ, ν) be primal and dual feasible respectively. If the duality gap associated with this pair is zero, prove that x is primal optimal and (λ, ν) is dual optimal.

4. Find the local extreme values of $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.

5. A vector $g \in \mathbb{R}^n$ is a *subgradient* of $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ at $x \in \text{dom } f$ if for all $y \in \text{dom } f$ we have $f(y) \geq f(x) + g^T(y - x)$. If f is convex and differentiable, then its gradient at x is a subgradient.

   A function f is called *subdifferentiable* at x if there exists at least one subgradient at x. The set of subgradients of f at the point x is called the *subdifferential* of f at x, and is denoted by $\delta f(x)$.

   Consider the absolute value function $f(x) = |x|,\ x \in \mathbb{R}$. Find $\delta|x|$.

6. Consider the problem

   minimize $\quad -xy$
   subject to $\quad x + y^2 \leq 2,$
   $\quad x,\ y \geq 0.$

   Find the Lagrangian associated with the above problem. Derive tha KKT conditions.