Indian Institute of Information Technology Allahabad
Convex Optimization (SMAT430C)
Quiz I

Duration: **45 Minutes** Date: February 14, 2017
Full Marks: 20 Time: 15:30 – 16:15 IST

Attempt all the Questions. Numbers indicated on the right in [ ] are full marks of that particular problem. All notations are standard and same as used in lectures. Please be precise in your answer.

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1. State whether the following statements are true or false. In either case write the precise reason in one or two lines. [2+1+1+1]

   (a) A set is convex if and only if it is midpoint convex.

   (b) The matrix $\begin{pmatrix} 1 & 0 \\ 2 & 2 \end{pmatrix}$ is positive semidefinite.

   (c) A finite nonempty set in $\mathbb{R}^n$ is always open.

   (d) Let K be a proper cone, and $\preceq_K$ a generalized inequality. Then $\preceq_K$ is reflexive.

2. Find the distance between two parallel hyperplanes $\{x \in \mathbb{R}^n : a^T x = b_1\}$ and $\{x \in \mathbb{R}^n : a^T x = b_2\}$. [2]

3. Let C be an affine set and $x \in C$. Prove that $C - x$ is a subspace. [3]

4. Find minimum and minimal element(s) of the set $\{x \in \mathbb{R}^2 : ||x||_2 \leq 1\}$. [3]

5. Prove that a closed convex set is the intersection of all halfspaces that contain it. (Hint: Use Separating Hyperplane Theorem). [3]

6. Find the dual cone of $\{Ax : x \succeq 0\}$, where $A \in \mathbb{R}^{n \times n}$. [4]