

# Indian Institute of Information Technology Allahabad

## Mid Semester Examination, March 2017

Date of Examination (Meeting): 05.03.2017 (1st meeting)

---

Program Code & Semester: B.Tech. (IT) & Dual Degree B.Tech.-M.Tech. IV Semester

Paper Title: Convex Optimization, Paper Code: SMAT430C

Paper Setter: Abdullah Bin Abu Baker & Anand Kumar Tiwari

Max Marks: 35

Duration: 2 hours

Attempt each question on a new page, and attempt all the parts of a question at the same place. Numbers indicated on the right in [ ] are full marks of that particular problem.

**Notations:**  $\mathbb{N}$ : Set of natural numbers,  $\mathbb{Z}$ : Set of integers,  $K_1$  and  $K_2$  are Cones,  $\inf$  is the infimum,  $\mathbb{R}_+^n = \{x \in \mathbb{R}^n : x \succeq 0\}$ ,  $\mathbb{R}_{++} = \{x \in \mathbb{R} : x > 0\}$ .

---

1. Prove or disprove the following statements.

- (a) Let  $A, B \subset \mathbb{R}$  such that  $A \subseteq B$ . Then  $\inf B \leq \inf A$ . [2]
- (b) The set  $\{x \in \mathbb{R} : -\frac{1}{2}x^2 + x + 1 \leq 0\}$  is convex. [2]
- (c) The average value of a continuous and convex function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  on any line segment is less than or equal to the average of its values at the endpoints of the segment. [3]
- (d) The cone  $\mathbb{R}_+^n$  is self-dual. [3]
- (e)  $K_1 \subseteq K_2 \implies K_1^* \subseteq K_2^*$ . [2]

2. Let  $f : \mathbb{R} \times \mathbb{R}_{++} \rightarrow \mathbb{R}$  be defined as  $f(x, y) = \frac{x^2}{y}$ . Determine whether  $f$  is convex or not. Is  $f$  quasiconvex. [4]

3. Show that the conjugate of a function is always a convex function. Derive the conjugate of the exponential function on  $\mathbb{R}$ . [6]

4. Find minimum and minimal element(s) of the set  $\{x \in \mathbb{R}^2 : \|x\|_\infty \leq 1\}$ . [3]

5. Find the supremum and infimum of the set  $\{\frac{m}{|m|+n} : n \in \mathbb{N}, m \in \mathbb{Z}\}$ . [2]

6. Prove that a function is convex if and only if its epigraph is a convex set. [8]