

Indian Institute of Information Technology Allahabad
Mathematics - I (SMAT130C)
Quiz 02

Duration: 1 Hour
Full Marks: 20

Date: November 26, 2016
Time: 15:00 – 16:00 IST

Attempt all the Questions. Numbers indicated on the right in [] are full marks of that particular problem. Please be precise in your answer. You are not allowed to write anything on the question paper.

1. Determine the values of p for which $\int_0^{\infty} \frac{1-e^{-x}}{x^p} dx$ converges. [6]
2. Sketch the graph of the polar equation $r^2 = 8 \cos 2\theta$. Find the area of the region that lies inside the curve $r^2 = 8 \cos 2\theta$ and outside the circle $r = 2$. [Express the answer in an integral expression. There is no need to evaluate the integral]. [4]
3. Let $f(x, y) = 3x^4 - 4x^2y + y^2$. Show that f has a local minimum at $(0, 0)$ along every line through $(0, 0)$. Is $(0, 0)$ a saddle point for f . [4]

4. Let

$$f(x, y) = \begin{cases} \frac{y}{|y|} \sqrt{x^2 + y^2}, & \text{if } y \neq 0 \\ 0, & \text{if } y = 0. \end{cases}$$

Show that all directional derivatives of f exist at $(0, 0)$, but f is not differentiable at $(0, 0)$. [6]

*****Do not expect so easy question paper for the end semester examination*****