

Indian Institute of Information Technology Allahabad
Mathematics - I (SMAT130C)
Quiz 01

Duration: 30 Minutes
Full Marks: 20

Date: September 03, 2016
Time: 11:00 – 11:30 IST

Attempt all the Questions. Numbers indicated on the right in [] are full marks of that particular problem. All the notations used are standard and same as used in lectures. Please be precise in your answer.

1. Prove the following statements.

- (a) Let A be a nonempty subset of \mathbb{R} , and $\alpha \in \mathbb{R}$ be the least upper bound of A . Then there exists a sequence (a_n) in A such that $a_n \rightarrow \alpha$. [5]
- (b) Let (x_n) be a sequence of real numbers. If $x_n \rightarrow x$, then $|x_n| \rightarrow |x|$. Is the converse true? [3]
- (c) Let (a_n) be a sequence in \mathbb{R} . If $\sum_{n=1}^{\infty} a_n$ converges, then $a_n \rightarrow 0$. [2]

2. Let (x_n) be sequence defined by

$$x_n = n^\alpha (1 + \beta)^{-n} \sin n$$

for all $n \in \mathbb{N}$, where α and β are fixed positive real numbers. Show that (x_n) converges. (Don't try with L'Hopital's Rule!). [5]

3. Let $y \in (0, 1)$. Discuss the convergence/divergence of the series

$$\sum_{n=1}^{\infty} [(n+1)y^n + \sin n]. \quad [5]$$