# Indian Institute of Information Technology Allahabad

## Mid Semester Examination, September 2016

Date of Examination (Meeting): 04.10.2016 (2nd meeting)

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Program Code & Semester: B.Tech. (IT), B.Tech. (ECE), Dual Degree - Semester I

Paper Title: Mathematics - I, Paper Code: SMAT130C

Paper Setter: Abdullah Bin Abu Baker & Sumit Kumar Upadhyay

Max Marks: 40 Duration: 2 hours

Attempt each question on a new page, and attempt all the parts of a question at the same place. Numbers indicated on the right in [ ] are full marks of that particular problem. All the notations used are standard and same as used in lectures.

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1. (a) Evaluate $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$. [2]

   (b) Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a continuous function such that $f(c) > 0$ for some $c \in \mathbb{R}$. Show that there exists a $\delta > 0$ such that $f(x) > 0$ for all $x \in (c - \delta, c + \delta)$. [2]

2. Find the number of real solutions of the equation $x^{17} - e^{-x} + 5x + \cos x = 0$. [4]

3. Let $f$ be differentiable on $[a, b]$. Show that there exist $c_1, c_2, c_3 \in (a, b)$ such that $c_2 \neq c_3$ and $f'(c_2) + f'(c_3) = 2f'(c_1)$. [5]

4. Find the intervals of decrease/increase, intervals of concavity/convexity, points of local minima/local maxima, points of inflection for the function $f(x) = \frac{2x^2+1}{x^2+1}$. [5]

5. (a) Let $f : [0, 1] \longrightarrow \mathbb{R}$ and $n$ be a fixed non-negative integer. Suppose $f^{(n+1)}$ exists on $[0, 1]$ and $f^{(n+1)}(x) = 0$ for all $x \in [0, 1]$. Show that $f$ is polynomial of degree less than or equal to $n$. [3]

   (b) Show that for $0 \leq x \leq 1$,

$$\log(1+x) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^n}{n}.$$ [5]

6. (a) Let $f : [a, b] \longrightarrow \mathbb{R}$ be a continuous function such that $f(x) \geq 0$ for all $x \in [a, b]$ and $\int_a^b f(x)dx = 0$. Show that $f(x) = 0$ for all $x \in [a, b]$. [4]

   (b) Does there exists an integrable function $f$ on $[a, b]$ such that $f(x) \geq 0$ for all $x \in [a, b]$ and $\int_a^b f(x)dx = 0$ but $f(c) \neq 0$ for some $c \in [a, b]$. [2]

   (c) Let $f : [0, 1] \longrightarrow \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x, & x \text{ is rational} \\ 0, & x \text{ is irrational}. \end{cases}$$

   Evaluate the upper and lower Riemann integrals of $f$ and show that $f$ is not integrable. [8]